Dynamical Overlap Simulations in the Schwinger Model with a Preconditioned Fermionic Force

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Outline:

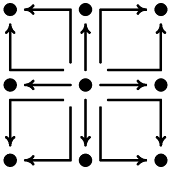
- Schwinger model
- Dirac operators D_{HF} and D_{ov-HF}
- HMC with a preconditioned force
- Algorithmic results for Acceptances and Reversibilities
- Measurements of observables Locality and Chiral Condensate
- Conclusions

The Schwinger model:

- first investigated by Julien Schwinger (1962)
- U(1) gauge theory (QED) in two (Euclidean) dimensions
- here: two flavor degenerate mass theory particularly suitable for numerical simulations
- computationally cheap toy model (for QCD)
- asymptotically free
- super-renormalizable
- chiral condensate $\Sigma = \langle \psi \bar{\psi} \rangle \sim m^{\frac{1}{3}}$
  (unlike QCD, symmetry breaking not spontaneous)

The hypercube Dirac operator D_{HF}:

- hypercube operator D_{HF} couples diagonal next-to-next neighbors



- truncated "perfect" lattice operator (Bietenholz/Hip 2000)
- approximates chiral symmetry
- Hermitian version

$$Q_{HF} = \gamma_5 \left(D_{HF} - 1 \right)$$

The overlap hypercube operator D_{ov-HF}:

$$D^{(0)}_{ov-HF} = 1 + \frac{\gamma_5 Q_{HF}}{\sqrt{Q_{HF}^2}}, \qquad D_{ov-HF}(m) = \left(1 - \frac{m}{2}\right) D^{(0)}_{ov-HF} + m$$

- fulfills GW relation

$$D^{(0)}\gamma_5 + \gamma_5 D^{(0)} = D^{(0)}\,\gamma_5\,D^{(0)}$$

$\rightarrow$ retains chiral symmetry

- computationally expensive
- Hermitian operator

$$Q_{ov-HF} = \gamma_5 D_{ov-HF}$$

- compared to Neuberger's version, D_{Wilson} is replaced by D_{HF}

The HMC force:

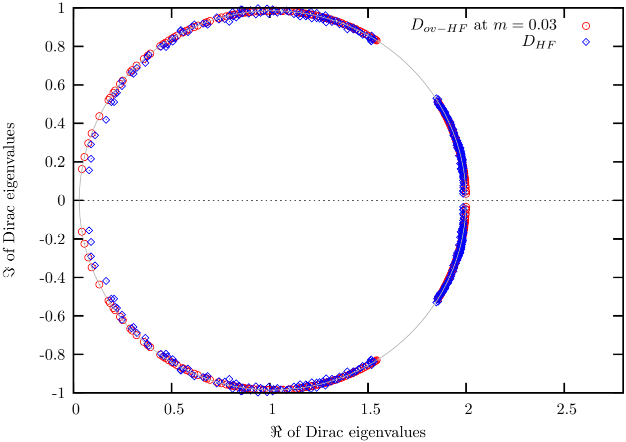
- regular HMC fermionic force term

$$\bar{\psi}\, Q^{-1}_{ov-HF} \left(Q^{-1}_{ov-HF} \frac{\partial Q_{ov-HF}}{\partial \alpha_{x,\mu}} + \frac{\partial Q_{ov-HF}}{\partial \alpha_{x,\mu}} Q^{-1}_{ov-HF} \right) Q^{-1}_{ov-HF}\, \psi$$

- $\alpha_{x,\mu}$: gauge link variables
- force term expensive to calculate, and

  problematic due to the sign function $\frac{Q_{HF}}{\sqrt{Q^2_{HF}}}$ in Q_{ov-HF}

Dirac spectrum:



- typical Dirac spectra at $m = 0.03$, $\beta = 6$
- D_{HF} is quite close to the spectrum of D_{ov-HF}
  $\rightarrow$ D_{HF} is an approximate GW operator

The HMC with a preconditioned force (Alg I):

- we use a simplified force: $Q_{ov-HF} \rightarrow Q_{HF}$

$$\bar{\psi}\, Q_{HF}^{-1} \left(Q_{HF}^{-1} \frac{\partial Q_{HF}}{\partial \alpha_{x,\mu}} + \frac{\partial Q_{HF}}{\partial \alpha_{x,\mu}} Q_{HF}^{-1} \right) Q_{HF}^{-1}\, \psi$$

(Christian/Jansen/Pollakowski/Nagai, PoS Lat05)

- computationally much cheaper, but very low acceptance
- does not break area conservation
- deviations from the trajectory are corrected by an exact accept/reject step based on the full (high precision) Q_{ov-HF} operator

The HMC with an improved preconditioned force (Alg II):

- Q_{ov-HF} is determined by Chebyshev polynomials to some precision ϵ $Q_{ov-HF}(\epsilon)$
- choose some $\epsilon' \gg \epsilon$ and construct the force as

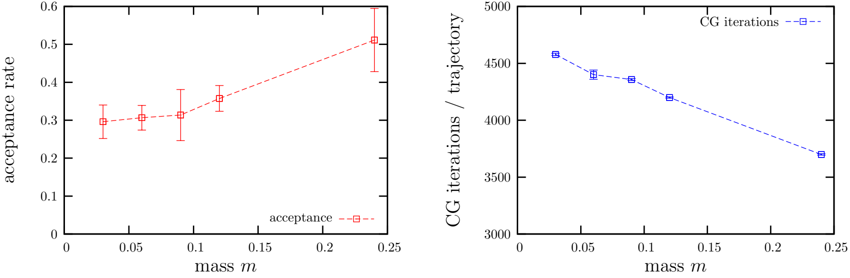
$$\bar{\psi}\, Q^{-1}_{ov-HF}(\epsilon') \left[Q^{-1}_{ov-HF}(\epsilon') \frac{\partial Q_{HF}}{\partial \alpha_{x,\mu}} + \frac{\partial Q_{HF}}{\partial \alpha_{x,\mu}} Q^{-1}_{ov-HF}(\epsilon') \right] Q^{-1}_{ov-HF}(\epsilon')\, \psi$$

- our choice: $\epsilon' = 5 \times 10^{-3}$ (force), $\epsilon = 10^{-16}$ (acc./rej. step)
- computationally still cheap, improves acceptance by a factor ~ 10
- not based on Hamiltonian dynamics,
  however area conservation persists

Simulation details:

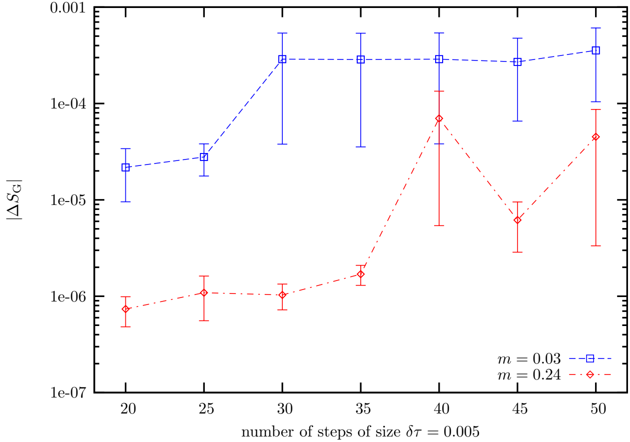
- 16×16 lattice, $\beta = 5$ and $m = 0.03, 0.06, 0.09, 0.12$ and 0.24
- integration scheme: Sexton-Weingarten with partial $(\delta\tau)^3$ error cancellation, multiple times scales
- evaluation of the D_{ov-HF}: project out lowest 2 eigenvalues of Q_{HF}^2 and approximate $1/\sqrt{Q_{HF}^2}$ by Chebyshev polynomials
- the following are obtained by algorithm Alg II ($\epsilon' = 0.005$)
- simulations were carried out on IBM p690 clusters of "Norddeutscher Verbund für Hoch- und Höchstleistungsrechnen" (HLRN)

Acceptance, CG iteration numbers:



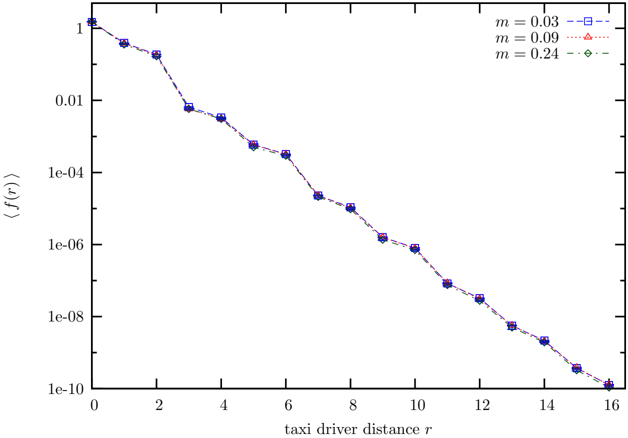
- measured acceptance, CG iteration per trajectory at trajectory length 0.125 with 20 steps at $\beta = 5$
- decent acceptance rates are possible
- as usual, heavier fermions are easier/cheaper to compute

Reversibility:



- $|\Delta S_G|$ for $m = 0.03$ and $m = 0.24$
- reversibility improves for higher masses
- possibly exponential increase, however data still preliminary
  $\rightarrow$ no conclusive Lyapunov exponent yet

Locality for D_{ov-HF}:



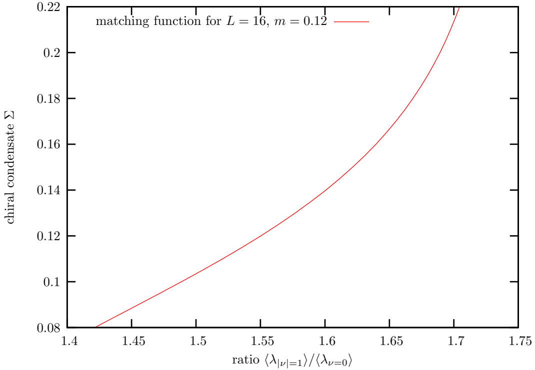
$$f(r) = \max_x \left\{ D_{xy}\eta_y \,\middle|\, \|x-y\|_1 = r \right\}, \quad \eta_y = \delta_{y0}\begin{pmatrix}1\\0\end{pmatrix}$$

- exponential decay → operator is local

(Hernández/Jansen/Lüscher 1999)

Chiral condensate:

- λ_ν: lowest eigenvalue of the Dirac operator in the sector of topological charge $\pm\nu$
- determined by matching the ratio of $\langle\lambda_{|\nu|=1}\rangle/\langle\lambda_{\nu=0}\rangle$ to Σ



(Wilke/Guhr/Wettig 1997)

- $m_{ov-HF} = 0.12$: $\langle\lambda_{|\nu|=1}\rangle/\langle\lambda_{\nu=0}\rangle = 1.6 \pm 0.1 \rightarrow \Sigma = 0.144 \pm 0.055$

Conclusions:

- investigated the overlap hypercube operator in the Schwinger model
- our overlap operator uses the hypercube operator instead of the Wilson operator as the kernel
- Alg II: low precision D_{ov-HF} in HMC force
  $\rightarrow$ decent acceptance rate
- showed numerical results for acceptance and reversibility in order to demonstrate the viability of the approach
- determined locality and the chiral condensate as observables