Simulation of Lattice Gauge Action from the Overlap Operator

- Gauge Field Tensor and Gauge Action from the Overlap Dirac Operator
- Monte Carlo Simulation of Lattice QCD

Lattice06, Tucson, July 24, 2006

Gauge Operators

- Gauge action usually constructed from the links
  - Plaquette – Wilson
  - Improved -- Iwasaki, Lüscher-Weisz, DBW2
  - Fixed Point Action – Hasenfratz-Niedermayer
- Gauge field tensor from
  - Clover leaf
  - Improvement
- Smearing of links (APE, HYP, Stout,...)

Topological Charge from the Overlap Dirac Operator

- Neuberger's overlap operator

$$D_{ov} = 1 + \varepsilon(H); \;\; H = \gamma_5 D_W(m_0)$$

- Index theorem on the lattice (Hasenfratz, Laliena, Niedermayer, Lüscher)

$$\text{index } D_{ov} = Tr\gamma_5 (1 - \frac{a}{2} D_{ov})$$

- Local version (Kikukawa & Yamada, Fujikawa, Suzuki, Adams)

$$q_L(x) = tr\gamma_5 (1 - \frac{a}{2} D_{ov}(x,x)) \xrightarrow[a \to 0]{} a^4 q(x) + O(a^6)$$

- Study of topological structure of the vacuum
  - Sub-dimensional long range order of coherent charges (Horvàth et al; Thacker talk)
  - Negativity of the local topological charge correlator (Horvàth et al)

Gauge-Fermion Connection

- Suppose O is gauge covariant, i.e.

$$O(gUg^{-1}) = gO(U)g^{-1}$$

then,

$$\mathrm{tr}_c O(gUg^{-1})(x,x) = \mathrm{tr}_c g(x)O(U)(x,x)g^{-1}(x) = \mathrm{tr}_c O(U)(x,x)$$

is gauge invariant.

- Taking the continuum limit (a → 0) (Horvath, hep-lat/0607031)

$$\mathrm{tr}_{cs}\gamma_5 D_{ov}(x,x) \xrightarrow[a\to 0]{} a^4 F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} + O(a^6)$$

$$\mathrm{tr}_{cs}(D_{ov}(x,x) - D^0_{ov}(x,x)) \xrightarrow[a\to 0]{} a^4 F^a_{\mu\nu}F^a_{\mu\nu} + O(a^6)$$

- Furthermore, one can obtain gauge covariant operators, e.g.

$$\mathrm{tr}_s \sigma_{\mu\nu} D_{ov}(x,x) \xrightarrow[a\to 0]{} a^2 F_{\mu\nu} + O(a^4)$$

- Following Suzuki, we obtain the following result

$$\mathrm{tr}_s \sigma_{\mu\nu} a D_{ov}(x,x) \xrightarrow[a\to 0]{}$$

$$\int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{-2}{(Z^+Z)^{3/2}} \left[(m + r\sum_{\lambda}(c_\lambda - 1))c_\mu c_\nu + 2rc_\mu s_\nu^2 \right] a^2 F_{\mu\nu} + O(a^4)$$

where,

$$c_\mu = \cos k_\mu,\ s_\mu = \sin k_\mu,\ Z^+Z = \sum_\nu s_\nu^2 + \left[m + \sum_\lambda (c_\lambda - 1) \right]^2$$

$$r = 1,\ \mathrm{m}_0 = 1.368,\quad c = -0.08156$$

Liu, Alexandru, Horvath; Alexandru, Horvath, Liu

Lattice QCD Action

- Gauge action:
  - $\mathrm{tr}_{cs}(D_{ov}(x,x) - D_{ov}^{0}(x,x)) \xrightarrow[a\to 0]{} a^4 F_{\mu\nu}^{a} F_{\mu\nu}^{a}(x) + O(a^6);$

    $$S_g = \frac{1}{2cg^2}\mathrm{Tr}(D_{ov} - D_{ov}^{0})$$
  - γ_5 hermicity and G - W relation

$$S_g = \frac{1}{2cg^2}\mathrm{Tr}\, D_{ov} = \frac{1}{4cg^2}\mathrm{Tr}(D_{ov} + D_{ov}^{+}) = \frac{1}{4cg^2}\mathrm{Tr}(D_{ov}^{+} D_{ov})$$

- Partition function

$$Z = \int DUD\overline{\psi}D\psi\, e^{\frac{-1}{4cg^2}\mathrm{Tr}\, D_{ov}^{+} D_{ov} + \sum_{f=1}^{N_f}\overline{\psi}_f D_{ov}(m_f)\psi_f}$$

MC Simulation

- Auxiliary fermion:

$$Z = \int DUD\overline{\psi}_f D\psi_f D\overline{\psi}_g D\psi_g \; e^{\overline{\psi}_g (e^{\frac{-1}{4cg^2} D_{ov}^{+} D_{ov}}) \psi_g + \sum_{f=1}^{N_f} \overline{\psi}_f D_{ov}(m) \psi_f}$$

$$Z = \int DU \det e^{\frac{-1}{4cg^2} D_{ov}^{+} D_{ov}} \prod_{f=1}^{N_f} \det D_{ov}(m_f)$$

- Pseudofermions:

$$Z = \int DUd\phi_f^* d\varphi_f \; e^{-\sum_{f=1}^{Nf} \phi_f^* \frac{e^{\frac{1}{4cN_f g^2} D_{ov}^{+} D_{ov}}}{D_{ov}(m_f)} \phi_f}$$

MC Simulation

- Two examples with HMC algorithm:
  - Polynomial approximation: normality $\left[D_{ov}, D_{ov}^{+}\right] = 0$

$$\phi_f^* e^{\frac{D_{ov}^{+} D_{ov}}{4cN_f g^2}} D_{ov}^{-1}(m_f)\phi_f = \phi_f^* e^{\frac{D_{ov}^{+} D_{ov}}{8cN_f g^2}} D_{ov}^{-1} e^{\frac{D_{ov}^{+} D_{ov}}{8cN_f g^2}} (m_f)\phi_f$$

Chebyshev polynomials

$$e^{\frac{D_{ov}^{+} D_{ov}}{8cN_f g^2}} \approx \sum_{i=1}^{M} c_i (D_{ov}^{+} D_{ov})^i; \quad e^x = (e^{x/N})^N$$

MC Simulation

- Rational Polynomial approximation:
  - two flavors

$$\phi^* e^{\frac{1}{6cg^2} D_{ov}^+ D_{ov}} (D_{ov}^+ D_{ov}(m))^{-1} \phi \approx \phi^* \sum_{i=1}^{N} \frac{a_i}{D_{ov}^+ D_{ov} + b_i} \phi,$$

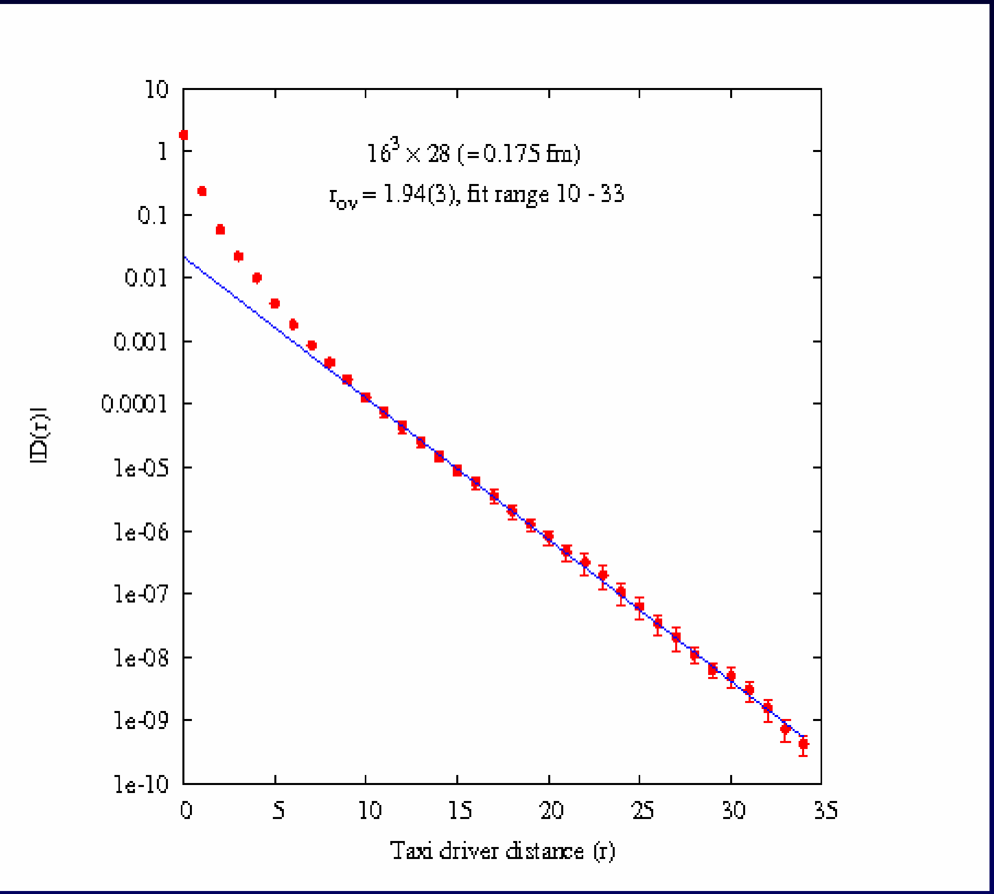
  - one flavor

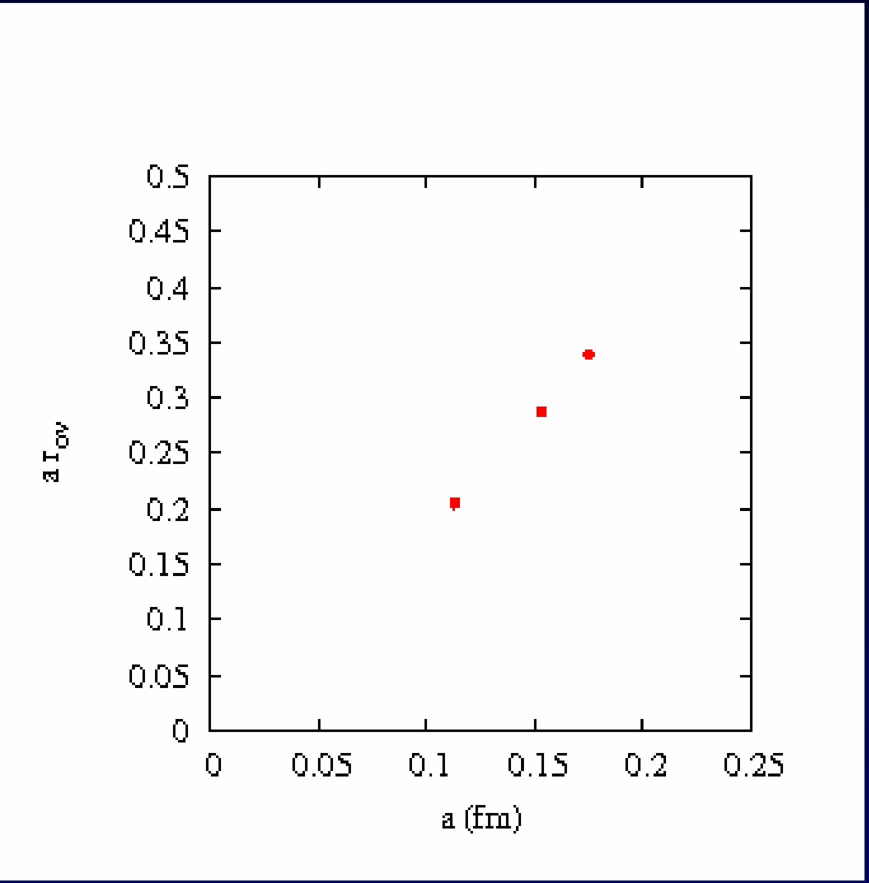
$$\phi^* e^{\frac{1}{12cg^2} D_{ov}^+ D_{ov}} (D_{ov}^+ D_{ov}(m))^{-1/2} \phi \approx \phi^* \sum_{i=1}^{N} \frac{c_i}{D_{ov}^+ D_{ov} + d_i} \phi,$$

Remarks

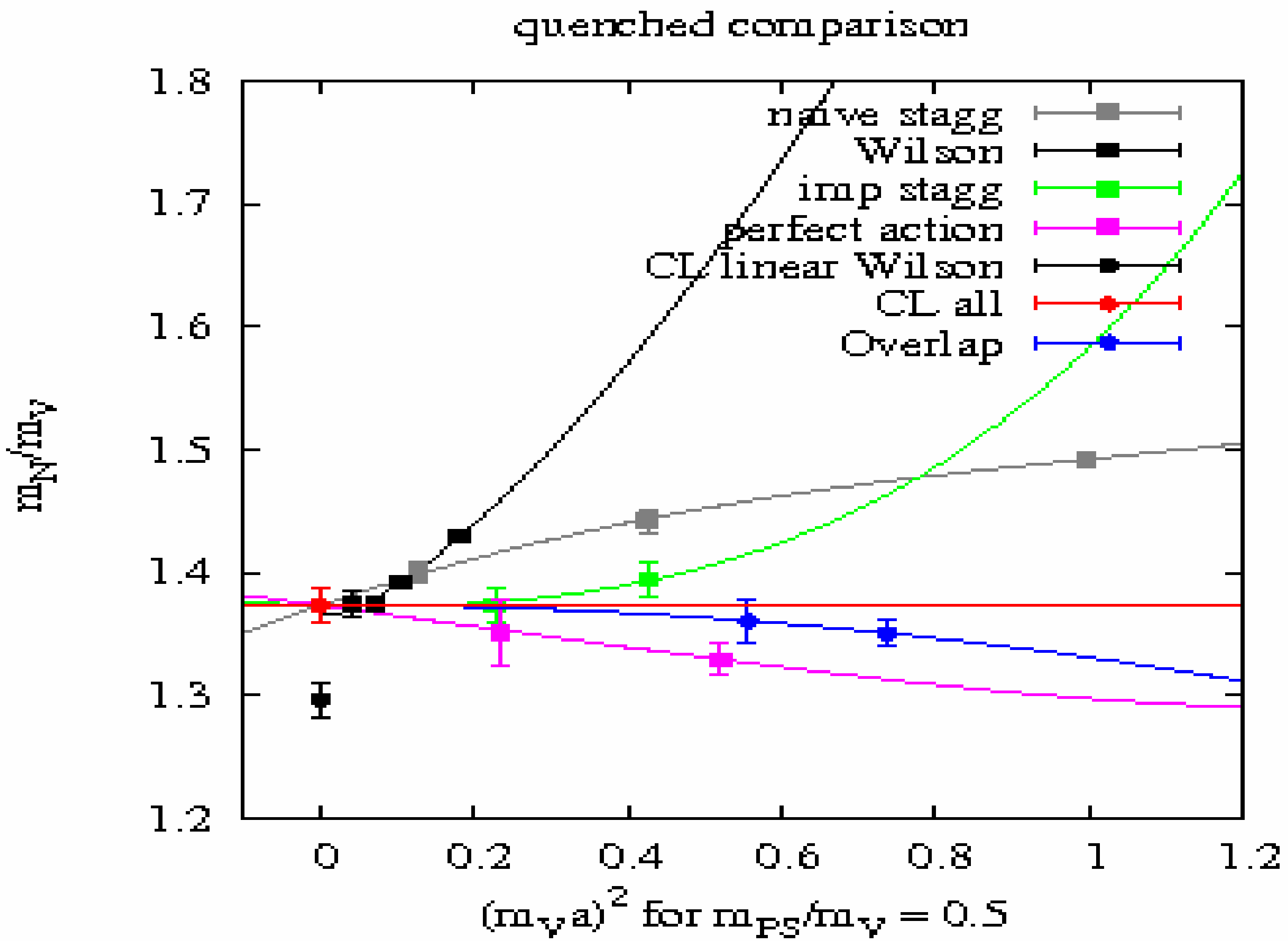
- Gauge action and fermion action based on the same ovrelap operator with the same pseudofermion approximation.
- Gauge action is not ultar-local (chiral smearing).
- Scaling and locality of overlap operator.
- Reflection positivity (negativity of local topological charge correlator)

$$\langle q(x)q(0)\rangle_{|x|\neq 0} \leq 0$$

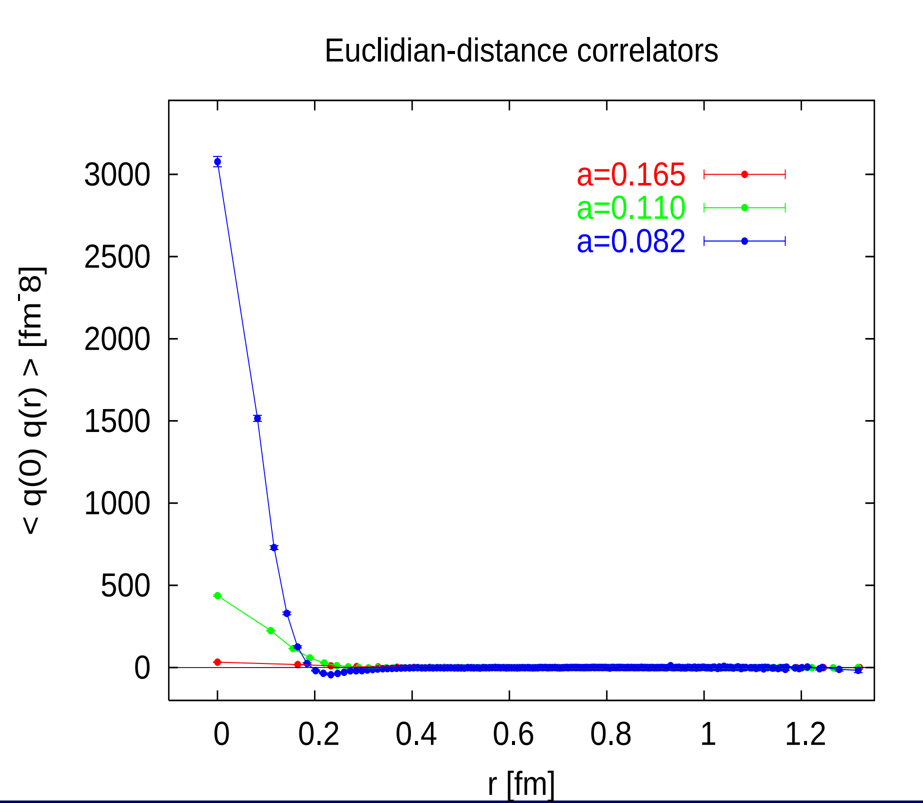


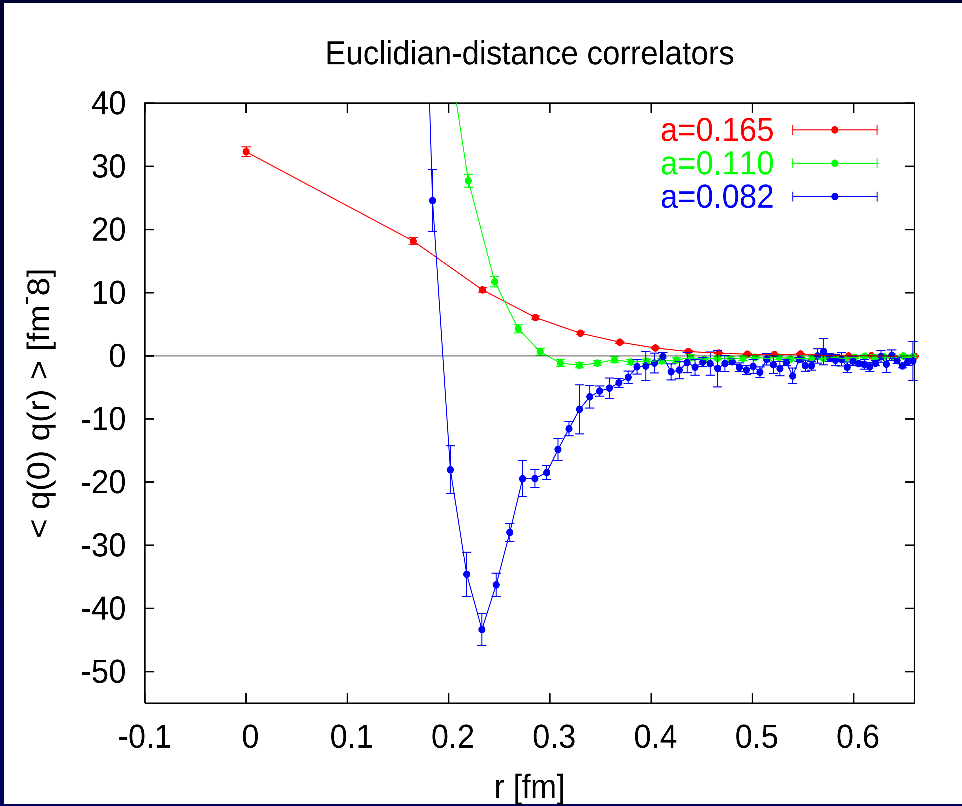


T. Draper et al. (χ collaboration, hep-lat/0510075)



quenched comparison
naive stagg
Wilson
imp stagg
perfect action
CL linear Wilson
CL all
Overlap
1.8
1.7
1.6
1.5
1.4
1.3
1.2
0
0.2
0.4
0.6
0.8
1
1.2
m_N/m_V
$(m_V a)^2$ for $m_{PS}/m_V = 0.5$





I. Horvàth et al., PL B617, 49 (2005) [hep-lat/0504005]