Chiral Lattice Gauge Theories from Warped Domain Walls and Ginsparg-Wilson Fermions

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Problems with regulating a χGT

- Nielsen-Ninomiya theorem: cannot have a local lattice formulation without doublers that maintains exact γ_5 symmetry
  - Doublers come in left-right pairs.
  - Dirac mass terms can lift extra fermions: Kinetic term mixes left and right
  - Hard breaking of chiral symmetry.
- GW fermions have exact chiral symmetry
  - Definition of left and right involves gauge fields.
  - Phase of measure not defined *a priori*.
  - Ambiguity cancels for vector like theories.

Domain Wall fermions

- Four dimensional ‘defects’ or walls in five dimensions
  - Massive fermion on an interval

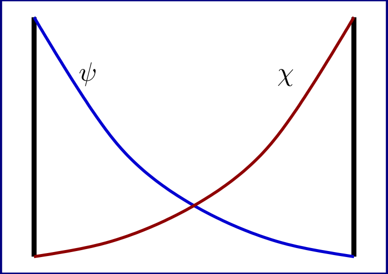
$$S = \int d^4x \int_R^{R'} dz \{ -i\bar\psi\partial_\mu\bar\sigma^\mu\psi - i\chi\partial_\mu\sigma^\mu\bar\chi + \psi\partial_z\chi - \bar\chi\partial_z\bar\psi + M\psi\chi + M\bar\chi\bar\psi \}$$

  - Kaluza Klein Decomposition

$$(\partial_z - M)\bar\chi = m_n\bar\chi \qquad -(\partial_z + M)\psi = m_n\psi$$

  - Two chiral modes forced to the walls by the heavy mass M:

$\psi \propto e^{-Mz}$



$\bar\chi \propto e^{Mz}$

Higgsing the right fermion

- Defects come in pairs: walls with left handed fermions come paired with walls with right handed partners
- Parts of the same 5-d fermion: have the same charge!
- Introduce Higgs field, H localized on the right wall
- Introduce neutral massive Majorana fermions, $S_{L,R}$, on the right wall
- Add Yukawa coupling

$$y\chi H S_L + m S_L S_L + m S_R S_R$$

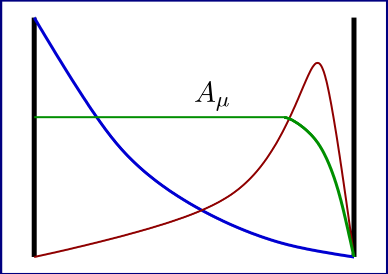
- Makes right handed mode heavy
- Makes gauge boson massive

5-D gauge bosons

- Make gauge boson dynamical with Neumann boundary conditions on left wall

$$\int d^4x \int_R^{R'} dz \, \frac{1}{4g_5^2} \left(F_{\mu\nu}F^{\mu\nu} + 2F_{\mu 5}F^{\mu 5} \right)$$

- Lowest (and all even) Kaluza-Klein modes massive
- Odd Kaluza-Klein modes are not Higgsed



- Does not result in a 4-D gauge theory: all odd Kaluza-Klein masses controlled by the size of the fifth dimension

Curved 5-d Space

- Curvature in *5D* provides an additional scale: AdS_5

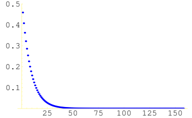
$$ds^2 = (\frac{R}{z})^2(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$$

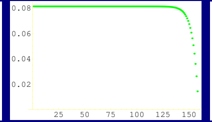
- Fluctuation in the gauge field cost lower energy for large z
- Lightest gauge boson mode lighter than the scale set by size of 5^{th} dimension

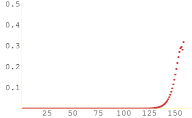
$$m_1^2 = \frac{1}{R'^2 \log(R'/R)} \qquad m_n^2 \propto \frac{1}{R'^2}$$

- Small effect on fermion zero modes for large M.
- $R' \to 0 \qquad \Rightarrow \qquad m_{n\neq 1} \to \infty$
- $R'/R \to \infty \qquad \Rightarrow \qquad m_1 \to 0$

Obtaining a Chiral Gauge Theory







- To obtain Chiral Gauge Theory

$$\frac{m_{KK}}{\Lambda} \to \infty \qquad \frac{m}{\Lambda} \to 0$$

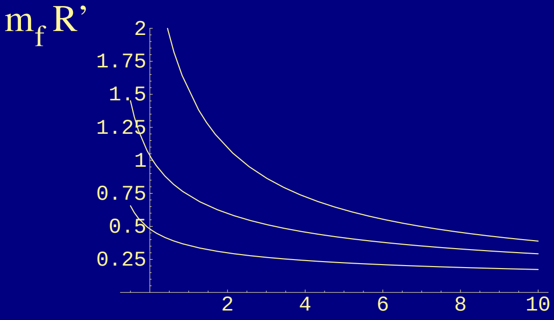
- Need

$$g_1\left(\frac{1}{aR}\right) \sim \frac{8\pi^2}{\beta_0 a}$$

- Continuum argument need small a
- Strong coupling for individual gauge groups

Potential problem

- Right handed mode may move in from wall → massless.
- Not in one-loop perturbation theory



- Longitudinal mode of gauge field couples strongly to the fermions near the right wall

$$y(z) = \frac{4\pi}{\sqrt{b}} \frac{z \log(z/R)}{\log(R'/R)}$$

Two dimensional theory

- Fermions are left $\bar{\psi}_+ \partial_- \psi_+$ or right moving $\bar{\psi}_- \partial_+ \psi_-$.
- Mass terms Dirac $\bar{\psi}_+ \psi_-$ or Majorana $\psi_+ \psi_-$.
- U(1) gauge theory with
  - left chiral fermions with charges 3 and 4
  - right chiral with charge 5

  is chiral and anomaly free.
- Breaks global fermion number because of anomaly
- maintains a $3_-^1\, 4_-^3\, \bar{5}_+^3$ global symmetry.
- Has all the problems of the four dimensional theory.

Domain walls and AdS_3

- Energy scale redshifts along the extra dimension: AdS symmetry implies local scale is $1/z$
- Scaling of masses and gauge coupling different in two dimension.
- Higgs' mechanism at the 'UV wall'.
- Lightest gauge mode lighter than typical:

$$m_{A_n} \sim \frac{n\pi}{R'} \qquad m_{A_0}^2 = \frac{2}{R'^2} \frac{1}{\ln(R'/R)} \left(1 + \mathcal{O}\left(\frac{1}{\ln(R'/R)}\right)\right)$$

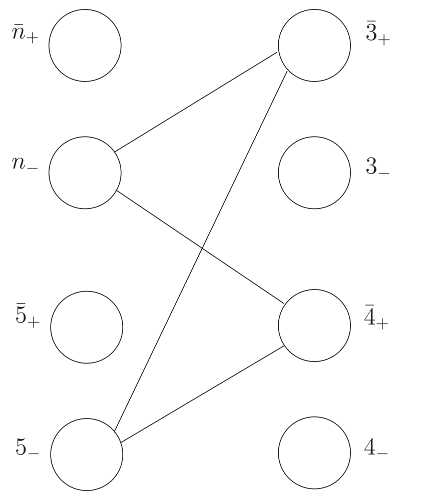
- Gauge coupling (dimensionful) smaller than local scale.

Lifting the fermions

- Clifford algebra different in two dimensions.
- Fermions get masses in left light pairs.
- Cannot completely lift all right fermions.
- Start with two neutral Dirac fermions $n_{\pm}^{1,2}$ and add Majorana masses to leave $\Im n^1$ and $\Re n^2$ massless.
- Couple n_{-}^{1} and n_{-}^{2} to the charged right handed modes using Higgs' mechanism.
- $\Im n_{+}^{1} + i\Re n_{+}^{2}$ remains light but neutral right fermion.
- Domain wall construction breaks even the $3^1 4^3 \bar{5}^3$ global symmetry.

GW fermions

- Replace domain walls with GW fermions
- Start with vectorlike n, 3, 4, 5.
- Add unitary Higgs (nonlinear σ-model).
- Add Majorana and Dirac-like Yukawa.
- Leave n_+, 3_-, 4_-, 5_+ uncoupled.



Chiral Gauge Theory

- Kinetic term splits exactly between left and right modes:

$$\bar{\psi}D\psi = \bar{\psi}_+ D\psi_+ + \bar{\psi}_- D\psi_-$$

- Fermion measure splits exactly

$$\mathcal{D}\psi = \mathcal{D}\psi_+ \mathcal{D}\psi_-$$

- The entire partition function splits for each gauge configuration!

$$Z = Z_{normal}(n_+, 3_-, 4_-, 5_+) Z_{mirror}(n_-, 3_+, 4_+, 5_-)$$

- The 'mirror' partition function has no zero modes at $g = 0$ and $y = \infty$.
- Decoupling of mirror fermions and unbroken gauge symmetry only depends on phase structure of nonlinear sigma-model
- Split is gauge invariant if chiral theory anomaly free.
- Majorana terms break fermion number.
- Maintains $3^1 4^3 \bar{5}^3$ symmetry.

Future Work

- The continuum limit of the AdS_3 model:
  - Restoration of the $3^1_- 4^3_- 5^3_+$ symmetry.
  - Decoupling of the logitudinal gauge mode.
- Phase structure of the GW theory
- Extensions to four dimensions
- Extensions to nonabelian theories
- Study small g limit for theories with global anomalies.