Infrared divergence of the color-Coulomb self-energy in Coulomb gauge QCD

Y. Nakagawa[1]

Collaboration with

T. Saito[1], **H. Toki**[1], **A. Nakamura**[2]

[1] *Research Center for Nuclear Physics,*
Osaka University, Ibaraki, Osaka 567-0047, Japan
[2] *Research Institute for Information Science and Education,*
Hiroshima University, Higashi-Hiroshima 739-8521, Japan

Lattice 2006, Tucson, 7/27/2006

Contents

1 Introduction

Confinement mechanism

- Hadrons are considered as bound states of quarks and gluons.

- Physical states are color singlet states.

$\Longrightarrow$ The energy of a color non-singlet state diverges.
$\Longrightarrow$ absence of isolated quarks and gluons in nature

▶ There are several approaches to understand the mechanism of color confinement where topological objects (collective infrared gluonic degrees of freedom) or unphysical degrees of freedom are responsible for the confinement.

- color monopoles in maximal abelian gauge (dual superconducting scenario)
- center vortices in maximal center gauge (center vortex model)
- ghost in Landau gauge

In Coulomb gauge, what are the relevant degrees of freedom for the confinement?

2 Coulomb gauge QCD

Coulomb gauge Hamiltonian

▶ The Coulomb gauge Hamiltonian can be expressed as the sum of the gluonic part and the instantaneous part:

$$H = \frac{1}{2}\int d^3x \left\{(E_i^{\mathrm{tr}})^2 + B_i^2\right\} + \frac{1}{2}\int d^3y \int d^3z \rho^a(\vec{y},t)\mathcal{V}^{ab}(\vec{y},\vec{z};A^{\mathrm{tr}})\rho^b(\vec{z},t)$$

- color charge density

$$\rho^a = \underbrace{gf^{abc}A_i^{b,\mathrm{tr}}E_i^{c,\mathrm{tr}}}_{gluons} + \underbrace{igq^\dagger T^a q}_{quarks}$$

- kernel of the instantaneous interaction

$$\mathcal{V}^{ab}(\vec{y},\vec{z};A^{\mathrm{tr}}) \equiv \int d^3x \mathcal{G}^{ac}(\vec{y},\vec{x};A^{\mathrm{tr}})(-\nabla_{\vec{x}}^2)\mathcal{G}^{cb}(\vec{x},\vec{z};A^{\mathrm{tr}})$$

▶ $\mathcal{G}$ is the Green's function of the Faddeev-Popov (FP) operator:

$$M^{ab} = -\partial_i D_i^{ab} = -\delta^{ab}\partial_i^2 - gf^{abc}A_i^{c,\mathrm{tr}}\partial_i$$

Color-Coulomb potential

(D. Zwanziger, NPB518, 237 (1998))

▶ We can show by the perturbative analysis that the instantaneous interaction causes an anti-screening effect.

⟹ We expect that the instantaneous interaction provides a confining force.

▶ The color-Coulomb potential is defined as the instantaneous interaction energy between color charges:

$$V_C(\vec{x}-\vec{y}) = g^2 \vec{T}^a \cdot \vec{T}^b \langle \mathcal{V}^{ab}(\vec{x},\vec{y};A^{\mathrm{tr}}) \rangle$$

▶ At the tree level, the color-Coulomb potential is the $1/R$ Coulomb potential.

Coulomb gauge QCD

- Gribov ambiguity and instantaneous interaction

$\Longrightarrow$ conjecture: Instantaneous interaction provides a long-range attractive force?
(V.N. Gribov, NPB138, 1 (1978))

- fundamental modular region and Gribov-Zwanziger scenario
(D. Zwanziger, NPB412, 657 (1994))

- $\boldsymbol{g^2 D_{00}}$ is a renormalization group invariant. (independent of the ultraviolet cutoff and the renormalization point and of the regularization and the renormalization scheme)

$\Longrightarrow$ $\boldsymbol{I}$ and $\boldsymbol{P}$ are renormalization group invariants.
(D. Zwanziger, NPB518, 237 (1998))

- Coulomb gauge is a renormalizable gauge (finite limit of the interpolating gauge).
(L. Baulieu and D. Zwanziger, NPB548 (1999) 527)

- gluon propagators in SU(2) lattice simulations

$\Longrightarrow$ $\tilde{D}_{00}(\vec{k})$ is strongly enhanced at $\vec{k} = 0$, while $\tilde{D}^{\mathrm{tr}}(\vec{k})$ vanishes.

(A. Cucchieri and D. Zwanziger, PRD65 (2001) 014001)

- (scheme independent) invariant color charge

(A. Cucchieri and D. Zwanziger, PRD65 (2001) 014002)

- Zwanziger's inequality $V_C(R) \geq V(R)$

$\Longrightarrow$ If the color-Coulomb potential is non-confining, then the physical potential is also non-confining, i.e., "No confinement without color-Coulomb confinement".

(D. Zwanziger, PRL90 (2003) 102001)

- gluon propagators and ghost propagator in SU(2) lattice simulations

$\Longrightarrow$ The ghost form factor acquires an infrared singularity.

(K. Langfeld and L. Moyaerts, PRD70 (2004) 074507)

Lattice QCD simulations — confinement phase

J. Greensite and S. Olejnik, PRD67, 094503 (2003)

A. Nakamura and T. Saito, PTP115, 189 (2006)

▶ The color-Coulomb potential behaves as a linearly rising potential at large distances.

▶ The color-Coulomb string tension is 2-3 times larger than that of the physical potential.
$\Longrightarrow$ Zwanziger's inequality

▶ The instantaneous interaction provides a strong confining force.

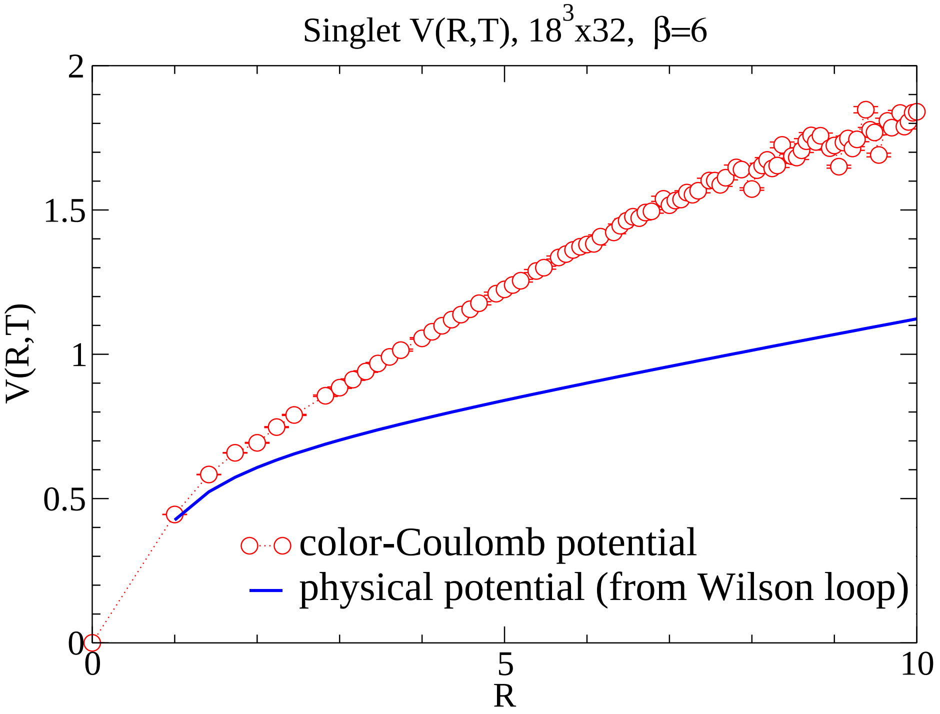

Figure 1: (from PTP115:189 (2006))

Lattice QCD simulations — deconfinement phase

J. Greensite and S. Olejnik, PRD69, 074506 (2004)

A. Nakamura and T. Saito, PTP115, 189 (2006)

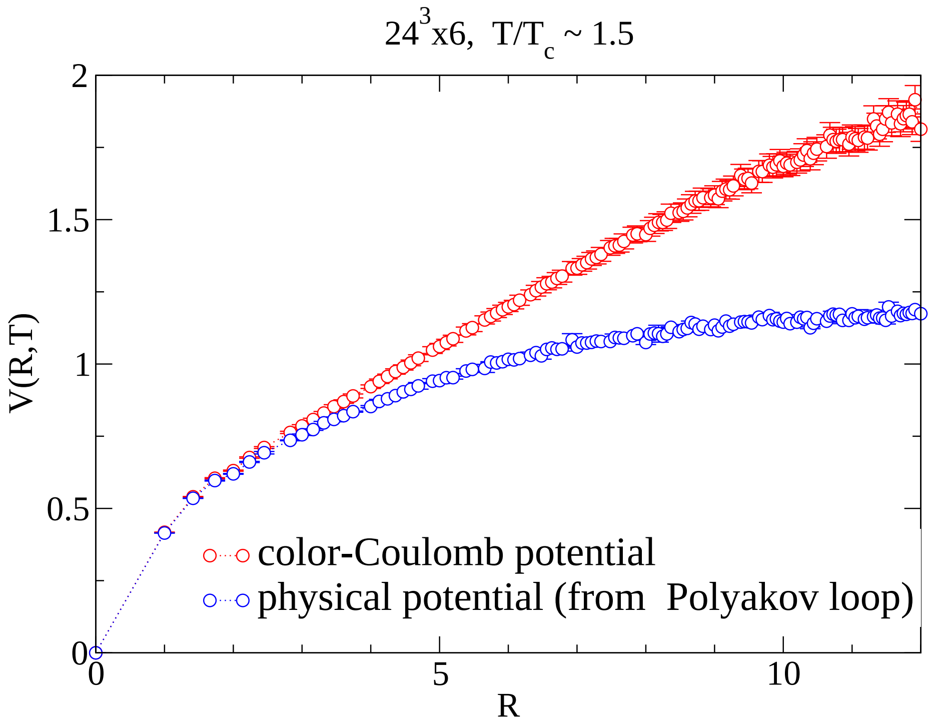

Figure 2: (from PTP115:189 (2006))

- The color-Coulomb potential is a confining potential even in the deconfinement phase.
$\Longrightarrow$ The color-Coulomb string tension does not serve as an order parameter for confinement/deconfinement phase transition.

- In Coulomb gauge, the confinement is attributed to the instantaneous interaction, whereas the confinement/deconfinement phase transition will be caused by the non-instantaneous interaction.

3 Eigenvalue distribution of the FP operator

Motivation for our study

Now, we have two questions:

▶ Why is the color-Coulomb potential confining in the confinement phase?

▶ Why is the color-Coulomb potential confining even in the deconfinement phase? In other words, why does not the color-Coulomb potential show a critical behavior?

$\Longrightarrow$ To address these questions, we study the eigenvalue distribution of the FP operator.

Color-Coulomb self-energy

(J. Greensite, S. Olejnik and D. Zwanziger, JHEP 05, 070 (2005))

- The color-Coulomb potential is defined as the instantaneous interaction energy between color charges:

$$\begin{aligned} V_C(\vec{x}-\vec{y}) &= g^2\vec{T}^a\cdot\vec{T}^b\langle\mathcal{V}^{ab}(\vec{x},\vec{y};A^{\mathrm{tr}})\rangle \\ &= g^2\vec{T}^a\cdot\vec{T}^b\left\langle\int d^3x\,\mathcal{G}^{ac}(\vec{x},\vec{z};A^{\mathrm{tr}})(-\nabla^2_{\vec{z}})\mathcal{G}^{cb}(\vec{z},\vec{y};A^{\mathrm{tr}})\right\rangle \end{aligned}$$

- $\mathcal{G}$ is the Green's function of the Faddeev-Popov (FP) operator:

$$M^{ab} = -\partial_i D_i^{ab} = -\delta^{ab}\partial_i^2 - gf^{abc}A_i^{c,\mathrm{tr}}\partial_i$$

- The Green's function of the FP operator can be expanded in terms of the eigenvectors $\phi_n^a(\vec{x})$ and the eigenvalues λ_n of the FP operator:

$$\mathcal{G}^{ab}(\vec{x},\vec{y};A^{\mathrm{tr}}) = \sum_{n=1}^{8V_3}\frac{\phi_n^{*a}(\vec{x})\phi_n^b(\vec{y})}{\lambda_n}$$

- The color-Coulomb potential can be written as

$$V_C(\vec{x}-\vec{y}) = g^2 \vec{T}^a \cdot \vec{T}^b \left\langle \sum_{n,m=1}^{8V_3} \frac{\phi_n^{*a}(\vec{x})\phi_m^b(\vec{y})}{\lambda_n \lambda_m} \int d^3z \phi_n^c(\vec{z})(-\nabla_{\vec{z}}^2)\phi_m^{*c}(\vec{z}) \right\rangle$$

▶ We set $\vec{y} = \vec{x}$ and consider the color-Coulomb self-energy

$$V_C(\vec{x}-\vec{x}) = g^2 \frac{C_D}{N^2-1} \left\langle \sum_{n,m=1}^{8V_3} \frac{\phi_n^{*a}(\vec{x})\phi_m^a(\vec{x})}{\lambda_n \lambda_m} \int d^3z \phi_n^c(\vec{z})(-\nabla_{\vec{z}}^2)\phi_m^{*c}(\vec{z}) \right\rangle$$

▶ By Fourier transforming $V_C(\vec{x}-\vec{x})$ and using the orthonormal condition

$$\int d^3x \phi_n^{*a}(\vec{x})\phi_m^a(\vec{x}) = \delta_{nm},$$

we have

$$\int \frac{d^3p}{(2\pi)^3} \tilde{V}_C(\vec{p}) = \frac{g^2 C_D}{8V_3} \left\langle \sum_{n=1}^{8V_3} \frac{F_n}{\lambda_n^2} \right\rangle$$

where F_n are the expectation values of the negative Laplacian in the FP eigenmodes

$$F_n = \int d^3x \phi_n^{*a}(\vec{x})(-\nabla^2)\phi_n^a(\vec{x})$$

- We define the normalized density of the FP eigenvalues

$$\rho(\lambda) \equiv \frac{N(\lambda, \lambda + \Delta\lambda)}{8V_3\Delta\lambda}$$

where $N(\lambda, \lambda + \Delta\lambda)$ is the number of eigenvalues in the range $[\lambda, \lambda + \Delta\lambda]$.

- Then, we have

$$\int_0^\Lambda \frac{d|\vec{p}|}{4\pi}|\vec{p}|^2\tilde{V}_C(|\vec{p}|) = g^2 C_D \int_0^{\lambda max} d\lambda \frac{\langle\rho(\lambda)F(\lambda)\rangle}{\lambda^2}$$

where Λ is the ultraviolet cutoff and $\lambda_{\max}$ is the corresponding maximum value of the FP eigenvalue. At the tree level, V_C is the Coulomb potential. Therefore at sufficiently large $\vec{p}$, $\tilde{V}_C(|\vec{p}|) \sim 1/|\vec{p}|^2$, and the integration in the left-hand side is finite in the upper limit as long as we keep the cutoff finite.

$$\int_0^{\Lambda} \frac{d|\vec{p}|}{4\pi} |\vec{p}|^2 \tilde{V}_C(|\vec{p}|) = g^2 C_D \int_0^{\lambda max} d\lambda \frac{\langle \rho(\lambda) F(\lambda) \rangle}{\lambda^2}$$

- If the condition

$$\lim_{\lambda \to 0} \frac{\langle \rho(\lambda) F(\lambda) \rangle}{\lambda} > 0$$

is satisfied in the infinite volume limit, the integration in the right-hand side diverges in the lower limit. In other words,

> If the criterion is satisfied, the color-Coulomb potential is more singular than the Coulomb potential ($\sim 1/|\vec{p}|^2$) in the infrared region.

This is the necessary condition for the color-Coulomb potential being a confining potential.

Zero-th order in the coupling or an abelian theory

$$\left(M^{ab} = -\partial_i D_i^{ab} = -\delta^{ab}\partial_i^2 - g f^{abc} A_i^{c,\mathrm{tr}}\partial_i \quad \longrightarrow \quad M = -\partial_i^2\right)$$

- Eigenvalue equation: $-\nabla^2\phi(\vec{x}) = \lambda\phi(\vec{x})$

- Eigenvalue: $\lambda = \vec{k}^2$

- Eigenvectors: plane waves

- Eigenvalue density: $\rho(\lambda) = \sqrt{\lambda}/4\pi^2$

- expectation value of the negative Laplacian: $F(\lambda) = \lambda$

▶ confinement criterion is not satisfied:

$$\lim_{\lambda\to 0}\frac{\rho(\lambda)F(\lambda)}{\lambda} = \lim_{\lambda\to 0}\frac{\sqrt{\lambda}}{4\pi^2} = 0$$

Simulation details — Action and gauge fixing

- The gauge configurations

are generated by the heat-bath Monte Carlo technique with the Wilson plaquette action:

$$S_W = \beta \sum_{x} \sum_{\mu<\nu} \left[1 - \frac{1}{3} \mathrm{Tr}(U_{x;\mu\nu}) \right]$$

($\beta = 6/g^2$ is the lattice coupling constant.)

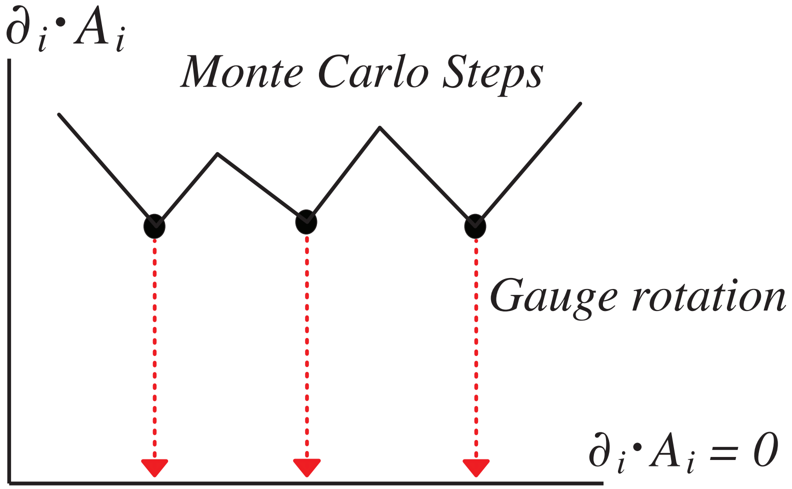

Figure 3: Wilson-Mandula method

- We adopt the Iterative method

to fix the gauge: we minimize the functional

$$F_U[g] = \sum_{i,x} \mathrm{ReTr} \left(1 - \frac{1}{3} g^\dagger(x) U_i(x) g(x+\hat{i}) \right)$$

with respect to the gauge rotation $g(x)$ and find the local minimum of $F_U[g]$.

Simulation details — Eigenvalue equation

- Construct the lattice FP operator

$$\begin{aligned} M^{ab}_{xy} = \quad & \sum_i \mathfrak{Re}\mathrm{Tr}\Big[\{T^a, T^b\}\Big(U_i(x) + U_i(x-\hat{i})\Big)\delta_{x,y} \\ & - 2T^bT^aU_i(x)\delta_{y,x+\hat{i}} - 2T^aT^bU_i(x-\hat{i})\delta_{y,x-\hat{i}}\Big] \end{aligned}$$

- Solve the eigenvalue equation

$$M\phi_n = \lambda_n\phi_n$$

using the Lanczos method and obtain the lowest 1000 eigenvalues and corresponding eigenvectors.

▶ Calculate

$$\boldsymbol{\rho(\lambda) = \frac{N(\lambda, \lambda + \Delta\lambda)}{8V_3\Delta\lambda}} \quad \text{and} \quad \boldsymbol{F_\lambda = \int d^3x \phi_\lambda^{*a}(\vec{x})(-\nabla^2)\phi_\lambda^a(\vec{x}),}$$

and check whether the confinement criterion

$$\boldsymbol{\lim_{\lambda \to 0} \frac{\langle \rho(\lambda) F(\lambda) \rangle}{\lambda} > 0}$$

is satisfied.

- Previous work: J. Greensite, S. Olejnik and D. Zwanziger, JHEP05, 070 (2005)

$\Longrightarrow$ FP eigenvalue density in SU(2) lattice simulation

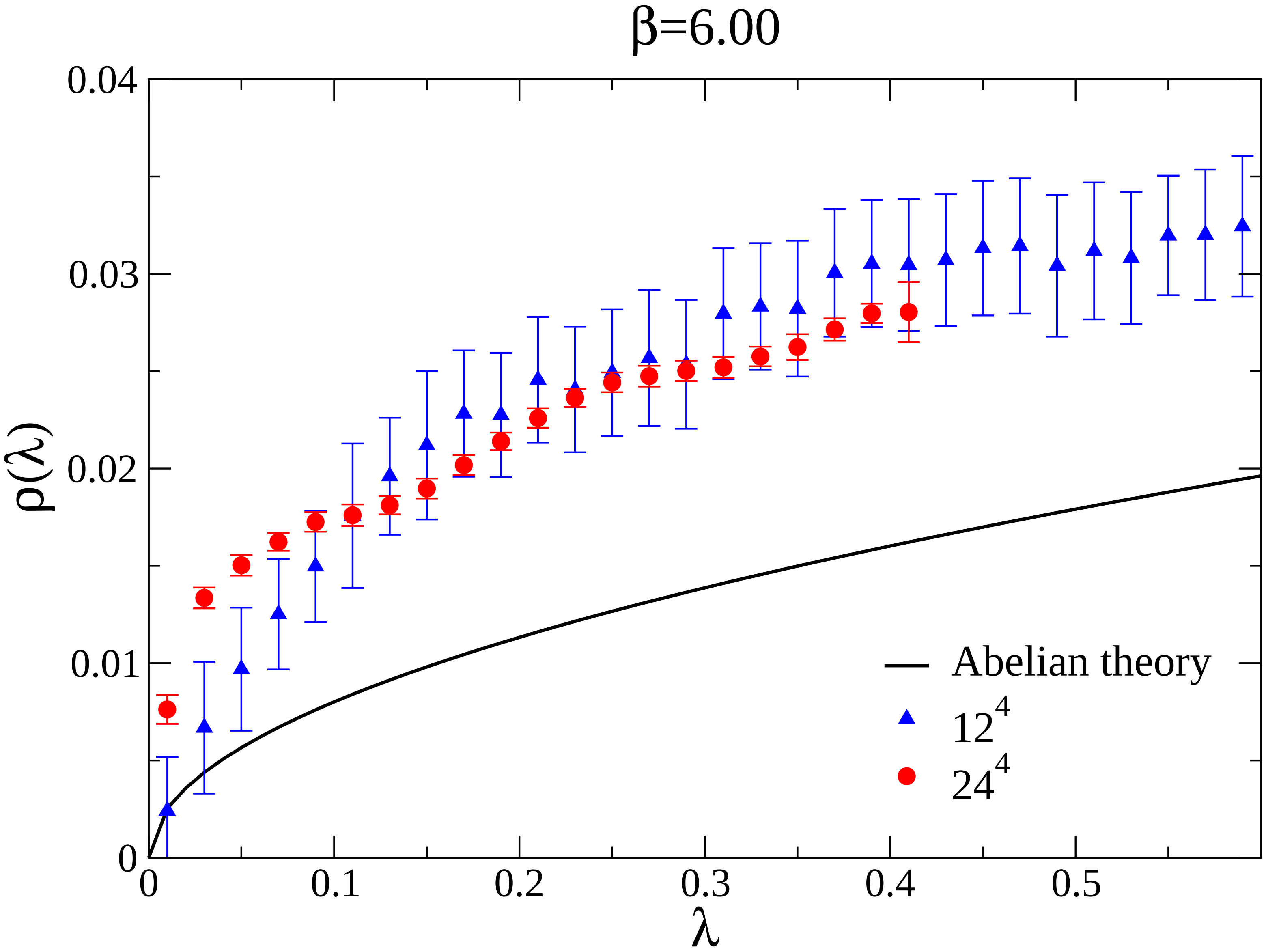

Figure 4: $\rho(\lambda)$ in the confinement phase. The FP eigenvalue density near $\lambda = 0$ increases at large lattice volume.

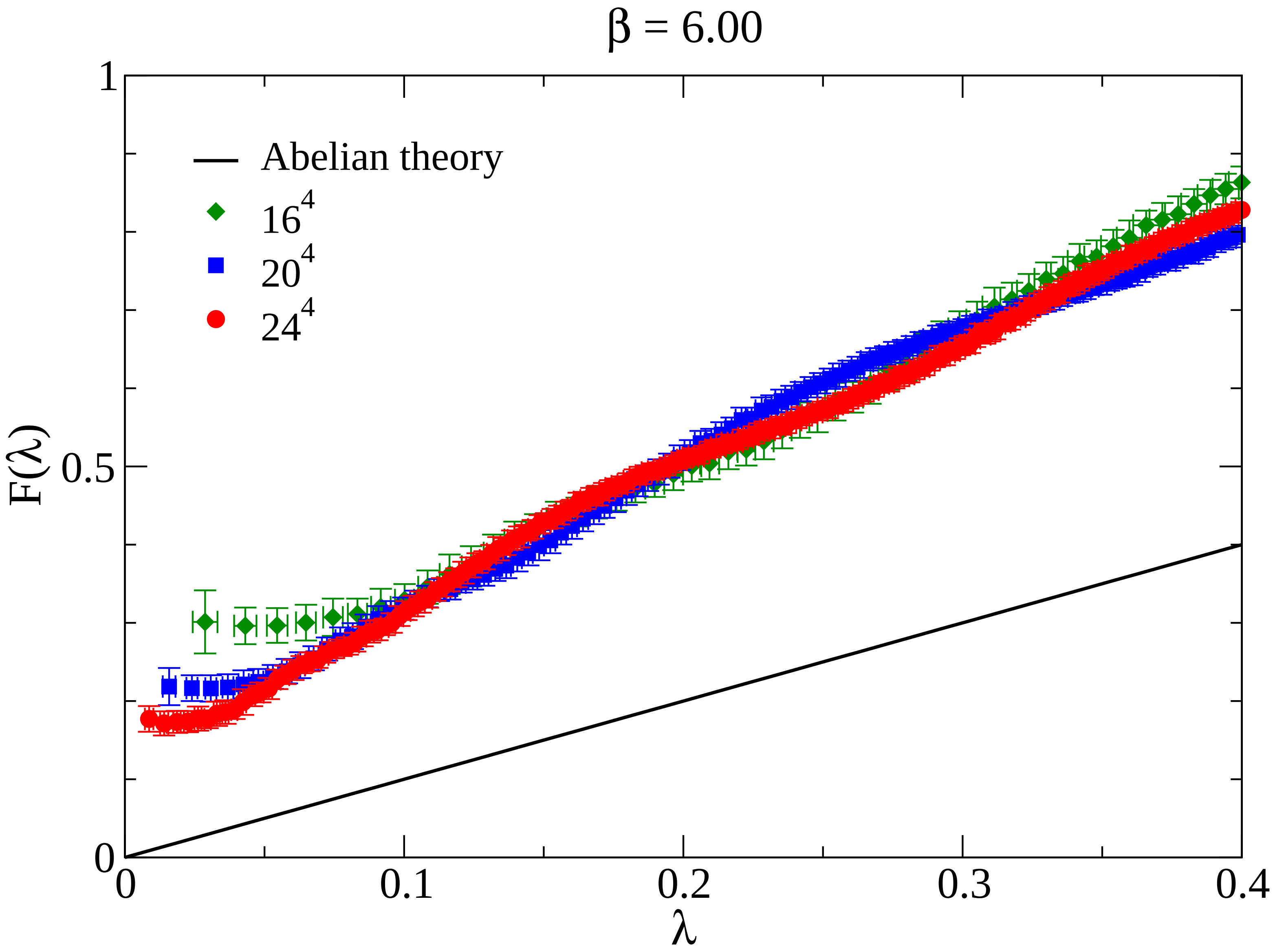

Figure 5: $F(\lambda)$ in the confinement phase. We expect that $F(\lambda)$ has a finite value at $\lambda = 0$ in the infinite volume limit.

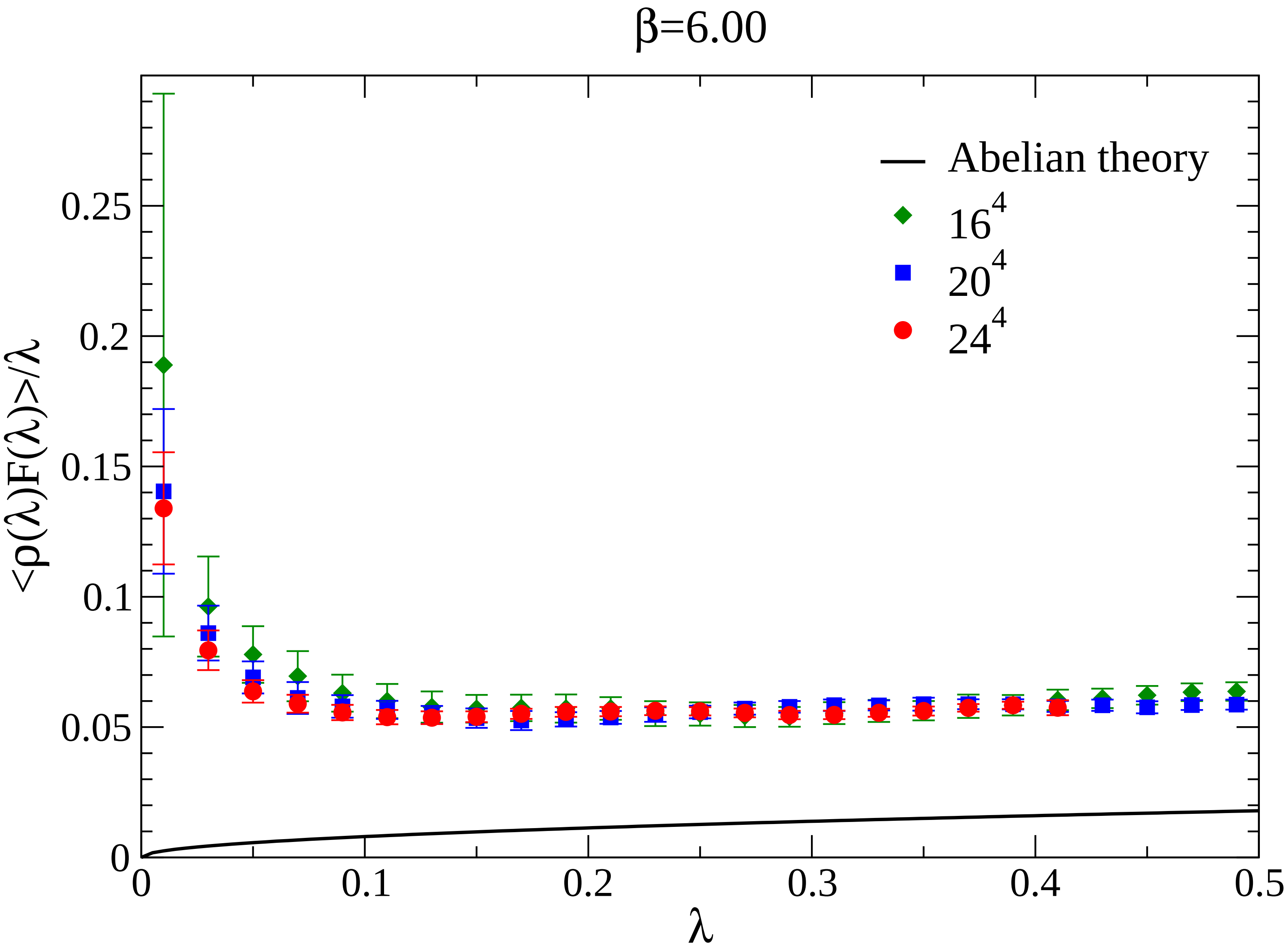

Figure 6: $\rho(\lambda)F(\lambda)/\lambda$ in the confinement phase. We see the enhancement near $\lambda = 0$ and we conclude that the necessary condition $\lim_{\lambda\to 0}\rho(\lambda)F(\lambda)/\lambda > 0$ is satisfied in the confinement phase.

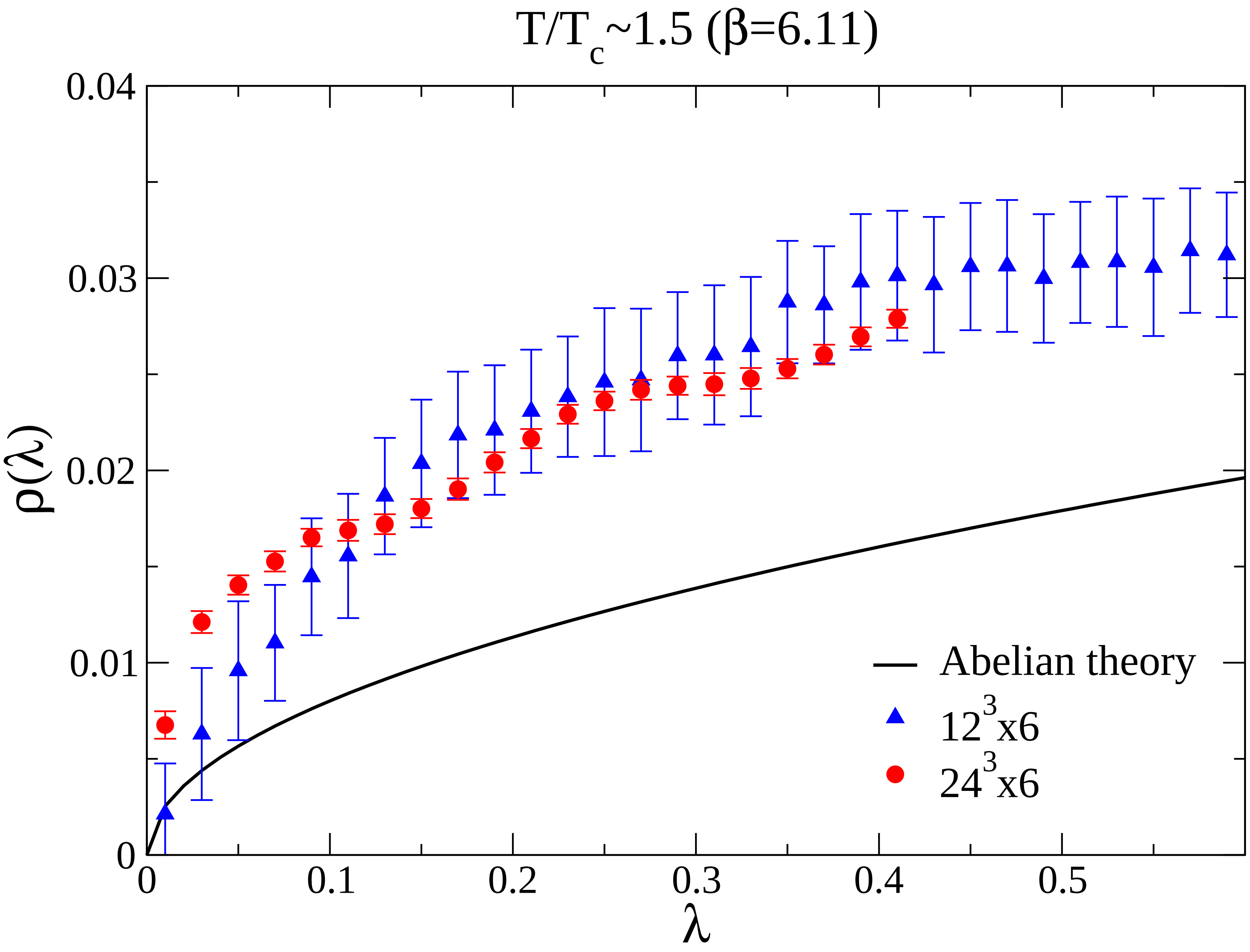

Figure 7: $\rho(\lambda)$ in the deconfinement phase. The behavior of $\rho(\lambda)$ in the deconfinement phase is qualitatively the same as in the confinement phase and the FP eigenvalue density does not show a critical behavior.

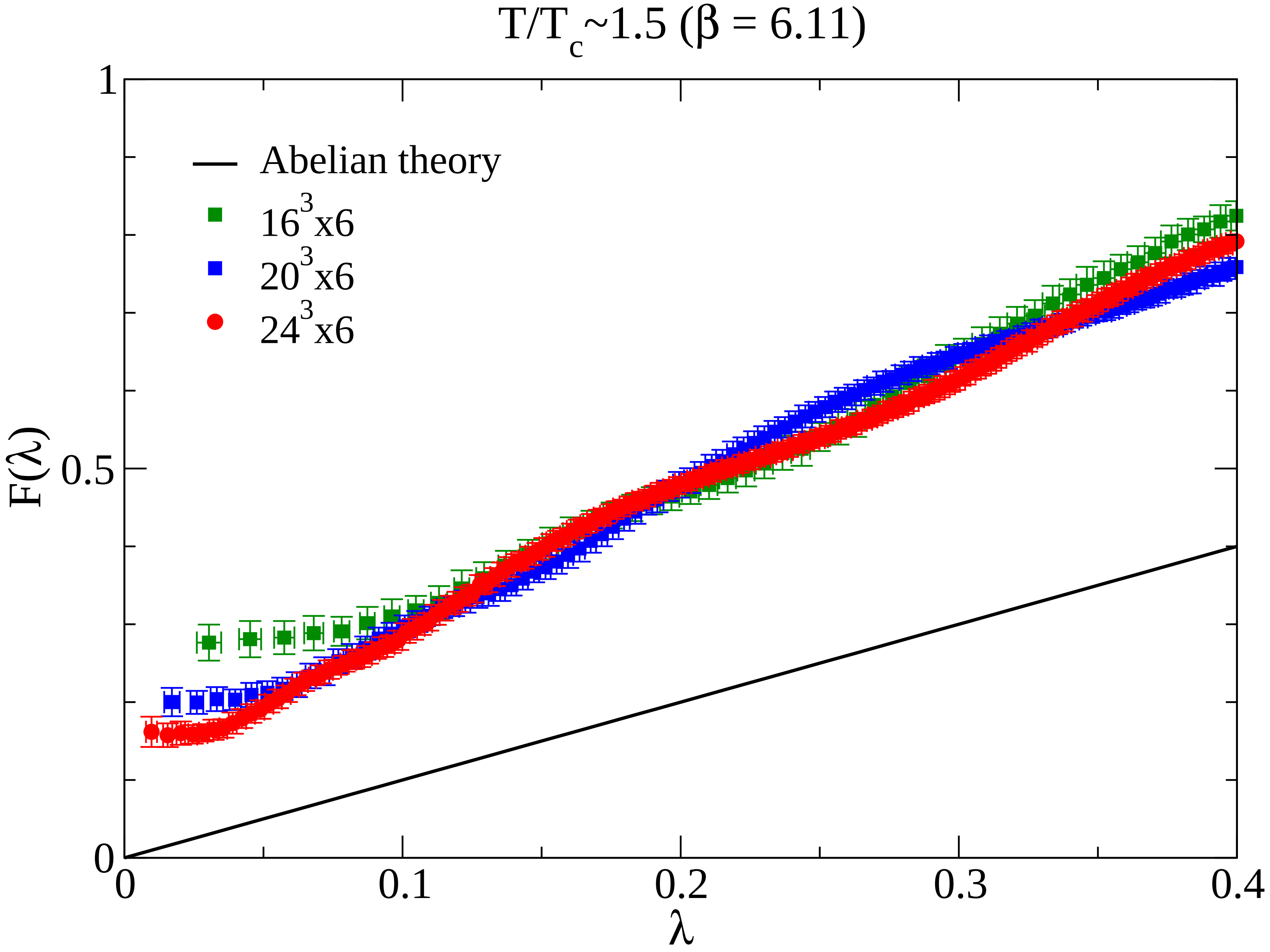

Figure 8: $\boldsymbol{F(\lambda)}$ in the deconfinement phase. $\boldsymbol{F(\lambda)}$ shows a similar behavior as in the confinement phase.

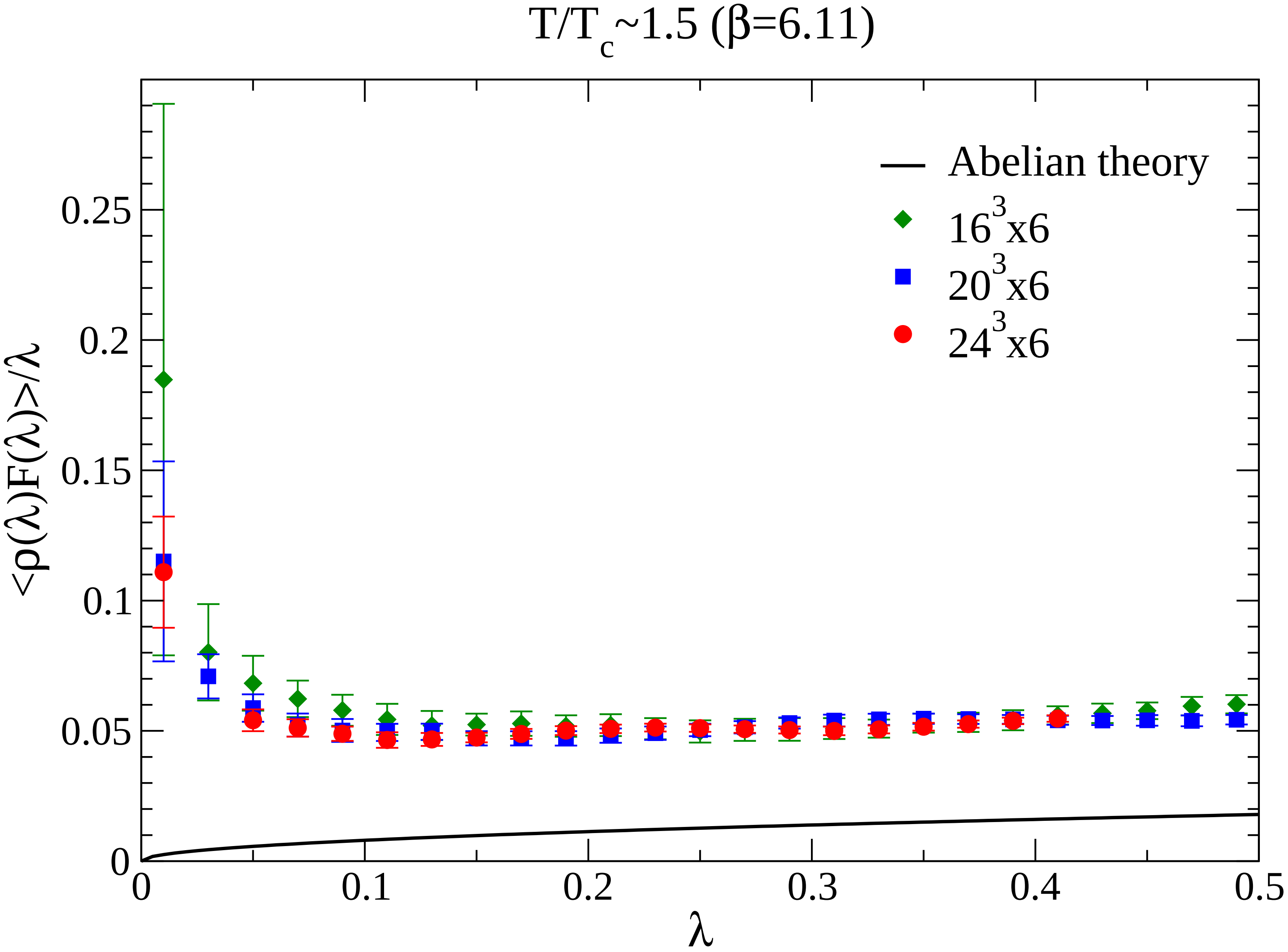

Figure 9: $\rho(\lambda)F(\lambda)/\lambda$ in the deconfinement phase. The necessary condition $\lim_{\lambda\to 0}\rho(\lambda)F(\lambda)/\lambda > 0$ is satisfied even in the deconfinement phase.

4 Summary

Summary

▶ We investigated the eigenvalue distribution of the FP operator and it is revealed that...

- The FP eigenvalue density of near-zero modes in SU(3) Yang-Mills theory is greater than that of the abelian theory and the necessary condition is satisfied in the confinement phase. Therefore, the color-Coulomb potential is more singular than the Coulomb potential in the infrared region.

- The near-zero modes of the FP operator survive above the critical temperature and the necessary condition is satisfied even in the deconfinement phase. This is the reason why the color-Coulomb potential is a confining potential in the deconfinement phase and does not show a critical behavior.