

Nucleon structure with dynamical domain-wall fermions

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July 25, 2006, Lattice 2006, Tucson, AZ, USA

RBC and UKQCD collaborations have been producing some dynamical DWF ensembles:

- RBC 2-flavor: DBW2 and DWF, $a^{-1} = 1.7$ GeV, $m_{\text{res}} = 0.00137(5)$,
 - $m_{\text{sea}} = 0.04, 0.03, \text{ and } 0.02$,
 - single volume, $16^3 \times 32 \times 12$,
- RBC/UKQCD (2+1)-flavor, Iwasaki and DWF, $a^{-1} = 1.6$ GeV, $m_{\text{res}} = 0.00308(3)$,
 - $m_{\text{strange}} = 0.04$ and $m_{\text{up,down}} = 0.03, 0.02, \text{ and } 0.01$,
 - two volumes, $16^3 \times 32 \times 16$ and $24^3 \times 64 \times 16$.

Here we report some nucleon form factors and moments of structure functions.

- With domain wall fermions (DWF) which preserves almost exact chiral symmetry on the lattice, the non-perturbative renormalization of the relevant currents is simple.

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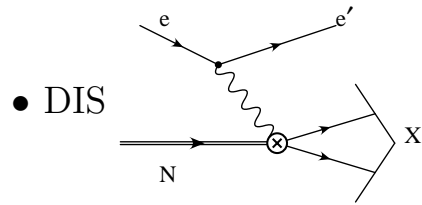
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The easiest are the vector and axial charges, known from neutron β decay, $g_V = G_F \cos \theta_c$ and $g_A/g_V = 1.2695(29)$ ¹:

- $g_V \propto \lim_{q^2 \rightarrow 0} g_V(q^2)$ with $\langle n | V_\mu^-(x) | p \rangle = i \bar{u}_n [\gamma_\mu g_V(q^2) + q_\lambda \sigma_{\lambda\mu} g_M(q^2)] u_p e^{-iqx}$,
- $g_A \propto \lim_{q^2 \rightarrow 0} g_A(q^2)$ with $\langle n | A_\mu^-(x) | p \rangle = i \bar{u}_n \gamma_5 [\gamma_\mu g_A(q^2) + q_\mu g_P(q^2)] u_p e^{-iqx}$.

Structure functions: measured in deep inelastic scatterings (and RHIC/Spin):



$$\left| \frac{\mathcal{A}}{4\pi} \right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu}$$

$$W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$$

$$W^{\{\mu\nu\}}(x, Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1(x, Q^2) + \left(P^\mu - \frac{\nu}{q^2} q^\mu \right) \left(P^\nu - \frac{\nu}{q^2} q^\nu \right) \frac{F_2(x, Q^2)}{\nu}$$

$$W^{[\mu\nu]}(x, Q^2) = i \epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu} (g_1(x, Q^2) + g_2(x, Q^2)) - \frac{q \cdot S P_\sigma}{\nu^2} g_2(x, Q^2) \right)$$

with $\nu = q \cdot P$, $S^2 = -M^2$, $x = Q^2/2\nu$.

- Unpolarized: $F_1(x, Q^2)$, $F_2(x, Q^2)$,
- Polarized: $g_1(x, Q^2)$, $g_2(x, Q^2)$.

- The same structure functions appear in RHIC/Spin (which also provides $h_1(x, Q^2)$).

¹The Particle Data Group.

Moments of the structure functions are accessible on the lattice:

$$\begin{aligned}
2 \int_0^1 dx x^{n-1} F_1(x, Q^2) &= \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_q(\mu) + \mathcal{O}(1/Q^2), \\
\int_0^1 dx x^{n-2} F_2(x, Q^2) &= \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_q(\mu) + \mathcal{O}(1/Q^2), \\
2 \int_0^1 dx x^n g_1(x, Q^2) &= \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_{\Delta q}(\mu) + \mathcal{O}(1/Q^2), \\
2 \int_0^1 dx x^n g_2(x, Q^2) &= \frac{1}{2n+1} \sum_{q=u,d} [e_{2,n}^q(\mu^2/Q^2, g(\mu)) d_n^q(\mu) - 2e_{1,n}^q(\mu^2/Q^2, g(\mu)) \langle x^n \rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2)
\end{aligned}$$

- c_1 , c_2 , e_1 , and e_2 are the **Wilson coefficients** (perturbative),
- $\langle x^n \rangle_q(\mu)$, $\langle x^n \rangle_{\Delta q}(\mu)$ and d_n are forward nucleon matrix elements of certain local operators.

Lattice operators:

- Unpolarized (F_1/F_2):

$$\frac{1}{2} \sum_s \langle P, S | \mathcal{O}_{\{\mu_1 \mu_2 \dots \mu_n\}}^q | P, S \rangle = 2 \langle x^{n-1} \rangle_q(\mu) [P_{\mu_1} P_{\mu_2} \dots P_{\mu_n} + \dots - (\text{trace})]$$

$$\mathcal{O}_{\mu_1 \mu_2 \dots \mu_n}^q = \bar{q} \left[\left(\frac{i}{2} \right)^{n-1} \gamma_{\mu_1} \overleftrightarrow{D}_{\mu_2} \dots \overleftrightarrow{D}_{\mu_n} - (\text{trace}) \right] q$$

On the lattice we can measure: $\langle x \rangle_q$, $\langle x^2 \rangle_q$ and $\langle x^3 \rangle_q$.

- Higher moment operators mix with lower dimensional ones: operators belonging in irreducible representations of $O(4)$ transform reducibly under the lattice Hyper-cubic group.
- Only $\langle x \rangle_q$ can be measured with $\vec{P} = 0$.

- Polarized (g_1/g_2):

$$-\langle P, S | \mathcal{O}_{\{\sigma\mu_1\mu_2\cdots\mu_n\}}^{5q} | P, S \rangle = \frac{2}{n+1} \langle x^n \rangle_{\Delta q}(\mu) [S_\sigma P_{\mu_1} P_{\mu_2} \cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}_{\sigma\mu_1\mu_2\cdots\mu_n}^{5q} = \bar{q} \left[\left(\frac{i}{2} \right)^n \gamma_5 \gamma_\sigma \vec{D}_{\mu_1} \cdots \vec{D}_{\mu_n} - (\text{traces}) \right] q$$

$$\langle P, S | \mathcal{O}_{[\sigma\{\mu_1\}\mu_2\cdots\mu_n]}^{[5]q} | P, S \rangle = \frac{1}{n+1} d_n^q(\mu) [(S_\sigma P_{\mu_1} - S_{\mu_1} P_\sigma) P_{\mu_2} \cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}_{[\sigma\mu_1]\mu_2\cdots\mu_n}^{[5]q} = \bar{q} \left[\left(\frac{i}{2} \right)^n \gamma_5 \gamma_{[\sigma} \vec{D}_{\mu_1]} \cdots \vec{D}_{\mu_n} - (\text{traces}) \right] q$$

and transversity (h_1):

$$\langle P, S | \mathcal{O}_{\rho\nu\{\mu_1\mu_2\cdots\mu_n\}}^{\sigma q} | P, S \rangle = \frac{2}{m_N} \langle x^n \rangle_{\delta q} [(S_\rho P_\nu - S_\nu P_\rho) P_{\mu_1} P_{\mu_2} \cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}_{\rho\nu\mu_1\mu_2\cdots\mu_n}^{\sigma q} = \bar{q} \left[\left(\frac{i}{2} \right)^n \gamma_5 \sigma_{\rho\nu} \vec{D}_{\mu_1} \cdots \vec{D}_{\mu_n} - (\text{traces}) \right] q$$

- On the lattice we can measure: $\langle 1 \rangle_{\Delta q}$ (g_A), $\langle x \rangle_{\Delta q}$, $\langle x^2 \rangle_{\Delta q}$, d_1 , d_2 , $\langle 1 \rangle_{\delta q}$ and $\langle x \rangle_{\delta q}$.
- Only $\langle 1 \rangle_{\Delta q}$, $\langle x \rangle_{\Delta q}$, d_1 , and $\langle 1 \rangle_{\delta q}$ can be measured with $\vec{P} = 0$.

On the lattice, in general, we calculate the relevant matrix elements of these currents

- with a lattice cutoff, $a^{-1} \sim 1\text{-}2 \text{ GeV}$,
- and extrapolate to the continuum, $a \rightarrow 0$,

introducing lattice renormalization, e.g.: $g_{V,A}^{\text{renormalized}} = Z_{V,A} g_{V,A}^{\text{lattice}}$.

Also, unwanted lattice artefact may result in unphysical mixing of chirally distinct operators.

DWF makes g_A/g_V particularly easy, because:

- the chiral symmetry is almost exact, and
- maintains $Z_A = Z_V$, so that $g_A^{\text{lattice}}/g_V^{\text{lattice}}$ directly yields the renormalized value.

And other non-perturbative renormalizations are feasible too, with often negligible unwanted mixings.

So DWF is a powerful tool for calculating nucleon matrix elements on the lattice: RBC had demonstrated this.

Our formulation follows the standard one,

- Two-point function: $G_N(t) = \text{Tr}[(1 + \gamma_t) \sum_{\vec{x}} \langle T B_1(x) B_1(0) \rangle]$, using $B_1 = \epsilon_{abc} (u_a^T C \gamma_5 d_b) u_c$ for proton,
- Three-point functions,
 - vector: $G_V^{u,d}(t, t') = \text{Tr}[(1 + \gamma_t) \sum_{\vec{x}'} \sum_{\vec{x}} \langle T B_1(x') V_t^{u,d}(x) B_1(0) \rangle]$,
 - axial: $G_A^{u,d}(t, t') = \frac{1}{3} \sum_{i=x,y,z} \text{Tr}[(1 + \gamma_t) \gamma_i \gamma_5 \sum_{\vec{x}'} \sum_{\vec{x}} \langle T B_1(x') A_i^{u,d}(x) B_1(0) \rangle]$.

with fixed $t' = t_{\text{source}} - t_{\text{sink}}$ and $t < t'$.

- From the lattice estimate

$$g_\Gamma^{\text{lattice}} = \frac{G_\Gamma^u(t, t') - G_\Gamma^d(t, t')}{G_N(t)},$$

with $\Gamma = V$ or A , the renormalized value

$$g_\Gamma^{\text{ren}} = Z_\Gamma g_\Gamma^{\text{lattice}},$$

is obtained.

- Non-perturbative renormalizations, defined by

$$[\bar{u}\Gamma d]_{\text{ren}} = Z_\Gamma [\bar{u}\Gamma d]_0,$$

satisfies $Z_A = Z_V$ well, so that

$$\left(\frac{g_A}{g_V} \right)^{\text{ren}} = \left(\frac{G_A^u(t, t') - G_A^d(t, t')}{G_V^u(t, t') - G_V^d(t, t')} \right)^{\text{lattice}}.$$

g_A is also described as $\Delta u - \Delta d$.

Renormalization: $\mathcal{O}^{\text{ren}} = Z_{\mathcal{O}}(a\mu)\mathcal{O}^{\text{lat}}(a)$,

- lattice complications: operator mixing from broken Lorentz or chiral symmetry,
- NPR is required when mixing with lower dimensional operator occurs.

We calculate $Z_{\mathcal{O}}(a\mu)$ non-perturbatively in RI/MOM scheme² with perturbative matching to $\overline{\text{MS}}$.

- compute off-shell matrix element of the operator, \mathcal{O} , in Landau gauge,
- impose a MOM scheme condition $\text{Tr } V_{\mathcal{O}}(p^2)\Gamma|_{p^2=\mu^2} \frac{Z_{\mathcal{O}}}{Z_q} = 1$,
 - $V_{\mathcal{O}}(p^2)$ is the relevant amputated vertex,
 - Γ is an appropriate projector,
- extrapolate to the chiral limit, defining the RI scheme,
- in an appropriate window, $\Lambda_{\text{QCD}} \ll \mu^2 \ll a^{-1}$, a scale invariant

$$Z_{\text{rgi}} = \frac{Z(\mu^2)}{C(\mu^2)}$$

is obtained, with the operator running $C(\mu^2)$ in the continuum perturbation theory.

- Now we can perturbatively match to e.g. $\overline{\text{MS}}$.

Works nicely with DWF.

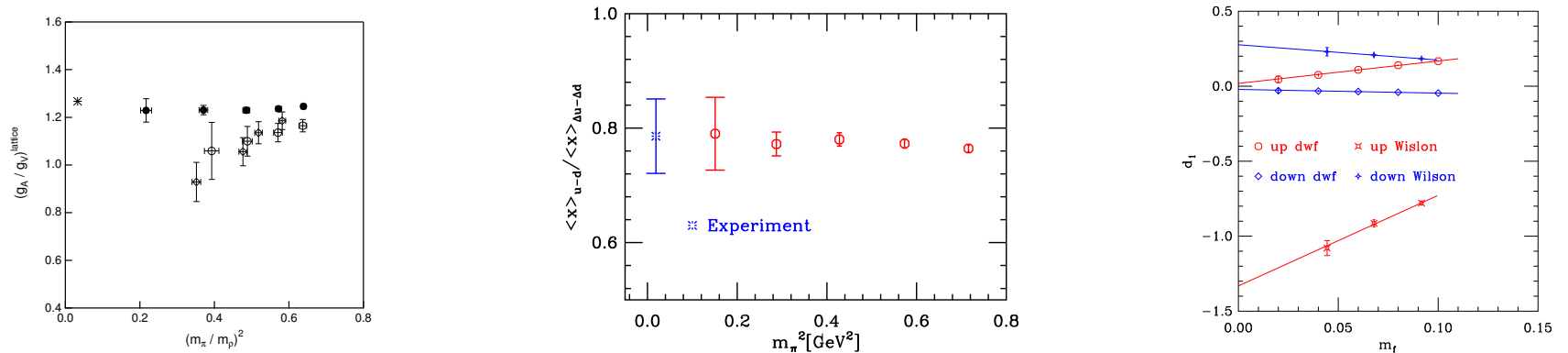
²Martinelli et. al, Nucl. Phys. B455, 81 (1995).

Rectangular gauge actions such as Iwasaki and DBW2, $S_G = \beta[c_0 \Sigma W_{1,1} + c_1 \Sigma W_{1,2}]$, with $c_0 + 8c_1 = 1$, help both:

- good chiral behavior, *i.e.* close enough to the continuum, and
- sufficiently large volume to contain a nucleon.

With quenched DBW2 calculations RBC had demonstrated³:

- $(g_A/g_V)^{\text{lattice}} = (g_A/g_V)^{\text{ren}}$ is strongly volume-dependent.
 - It is not appreciably quark-mass dependent once the volume is sufficiently large,
 - $g_A/g_V = 1.212(27)_{\text{stat.}}(24)_{\text{syst.}}$, compared with the experiment of 1.2695(29).



- NPR for structure function moments is well-behaved.
- Both $\langle x \rangle_{u-d}$ and $\langle x \rangle_{\Delta u - \Delta d}$ overshoot the experiments by about 50%,
 - but their ratio, again naturally renormalized, agree well with the experiment
 - and again without much appreciable quark-mass dependence.
- $\langle 1 \rangle_{\delta u - \delta d} = 1.193(30)$, $\overline{\text{MS}}$ (2 GeV) 2-loop running.
- d_1 , though not renormalized yet, appears small in the chiral limit.

³S. Sasaki, K. Orginos, SO, and T. Blum, Phys.Rev.D68:054509, 2003 (hep-lat/0306007); K. Orginos, T. Blum and SO, Phys.Rev.D73:094503, 2006 (hep-lat/0505024).

Here we report on two dynamical DWF ensembles.

1. RBC 2-flavor DBW2+DWF dynamical calculations:

- $a^{-1} = 1.7$ GeV, $m_{\text{res}} = 0.00137(5)$
- $m_{\text{sea}} = 0.04, 0.03,$ and $0.02,$
- $m_{\pi} = 700, 610,$ and 490 MeV; $m_N = 1.5, 1.4,$ and 1.3 GeV (a few % errors),
- single volume, $16^3 \times 32 \times 12,$
- about 220 configurations at each m_{sea} value,

Analysis is almost complete except NPR.

2. RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical calculations:

- $a^{-1} = 1.6$ GeV, $m_{\text{res}} = 0.00308(3),$
- $m_{\text{strange}} = 0.04$ and $m_{\text{up,down}} = 0.03, 0.02,$ and $0.01,$
- $m_{\pi} = 620, 520$ and 390 MeV; $m_N = 1.4, 1.3,$ and 1.2 GeV (10-20 % errors),
- two volumes,
 - $16^3 \times 32 \times 16,$ 2 fm across, ensemble production complete, UKQCD analyses ongoing, and
 - $24^3 \times 64 \times 16,$ 3 fm across, ensemble production ongoing, RBC analyses ongoing (24-30 configurations).

Preliminary analysis for the larger volume.

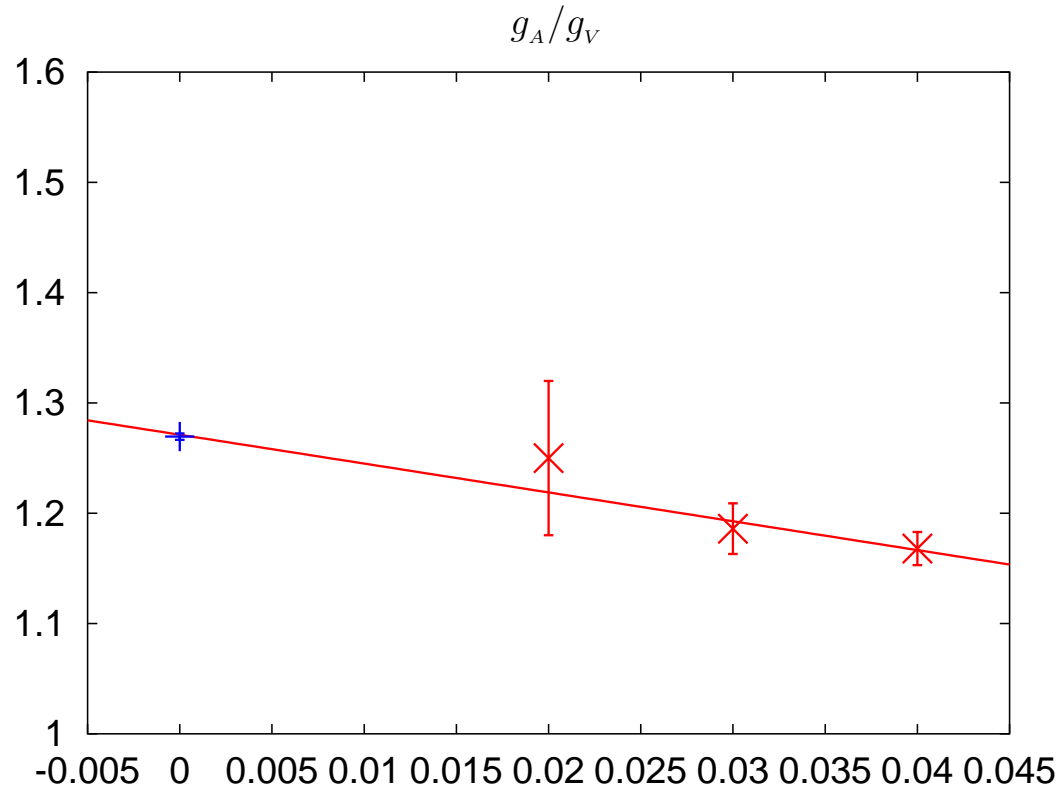
Observables: those not requiring finite momentum transfer

- ratios g_A/g_V and $\langle x \rangle_{u-d}/\langle x \rangle_{\Delta u-\Delta d},$ naturally renormalized,
- $d_1,$ yet to be renormalized but still interesting.

bare values of the others have been calculated but are waiting for renormalization.

RBC 2-flavor DBW2+DWF dynamical: $a^{-1} = 1.7$ GeV, $16^3 \times 32 \times 12$, $m_{\text{res}} = 0.00137(5)$, about 220 configurations,

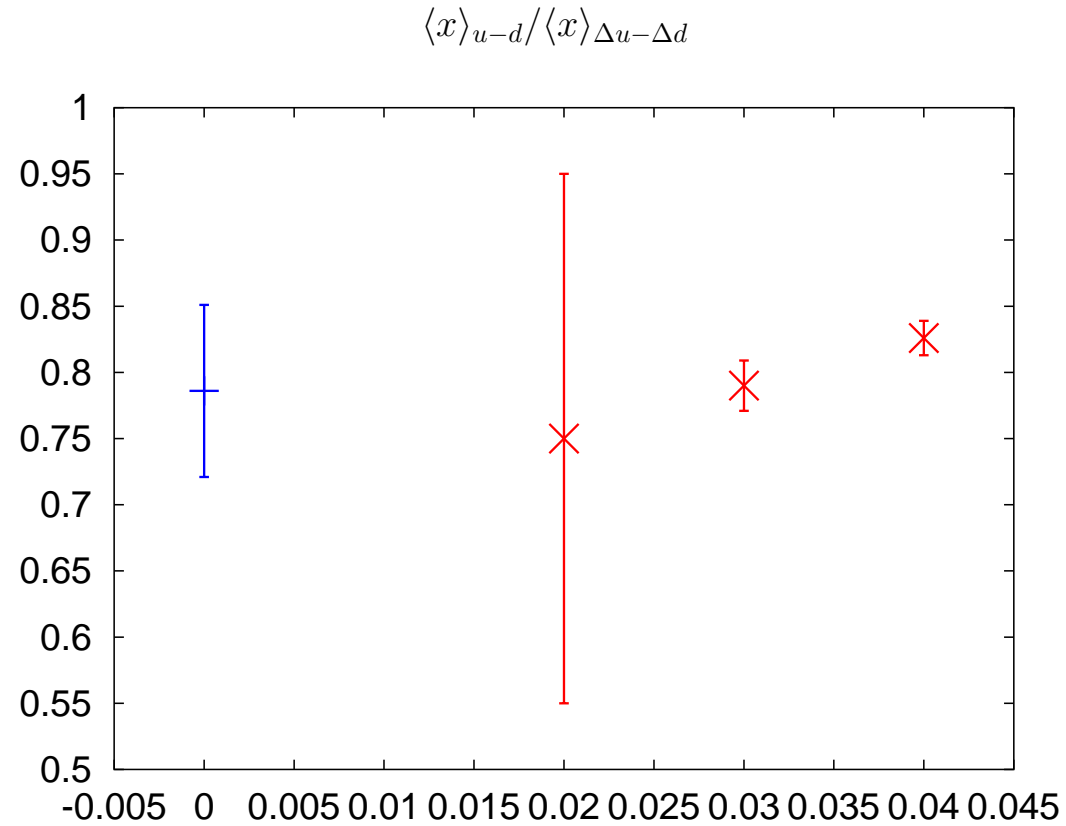
- $m_{\text{sea}} = 0.04, 0.03, \text{ and } 0.02$; $m_{\pi} = 700, 610, \text{ and } 490$ MeV; $m_N = 1.5, 1.4, \text{ and } 1.3$ GeV (a few % errors),



in agreement with experiment,
 $g_A/g_V = 1.27(5)$, despite rather small volume,
 mild quark-mass dependence.

RBC 2-flavor DBW2+DWF dynamical: $a^{-1} = 1.7$ GeV, $16^3 \times 32 \times 12$, $m_{\text{res}} = 0.00137(5)$, about 220 configurations,

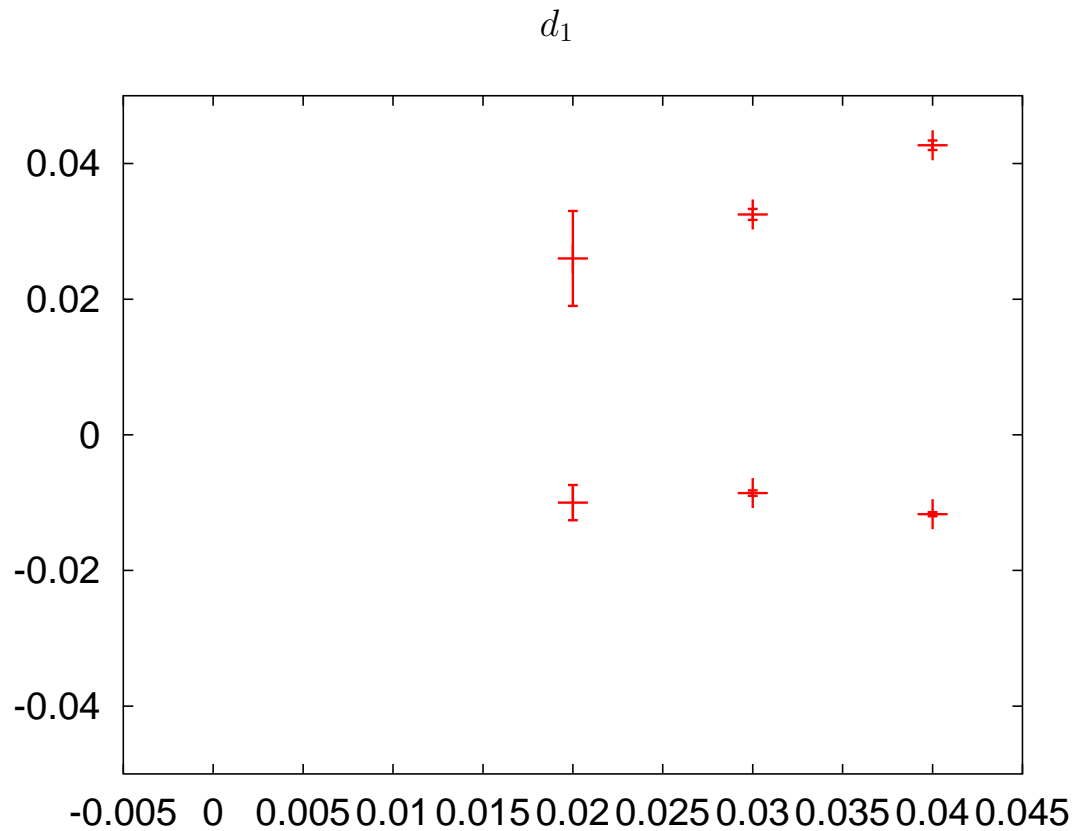
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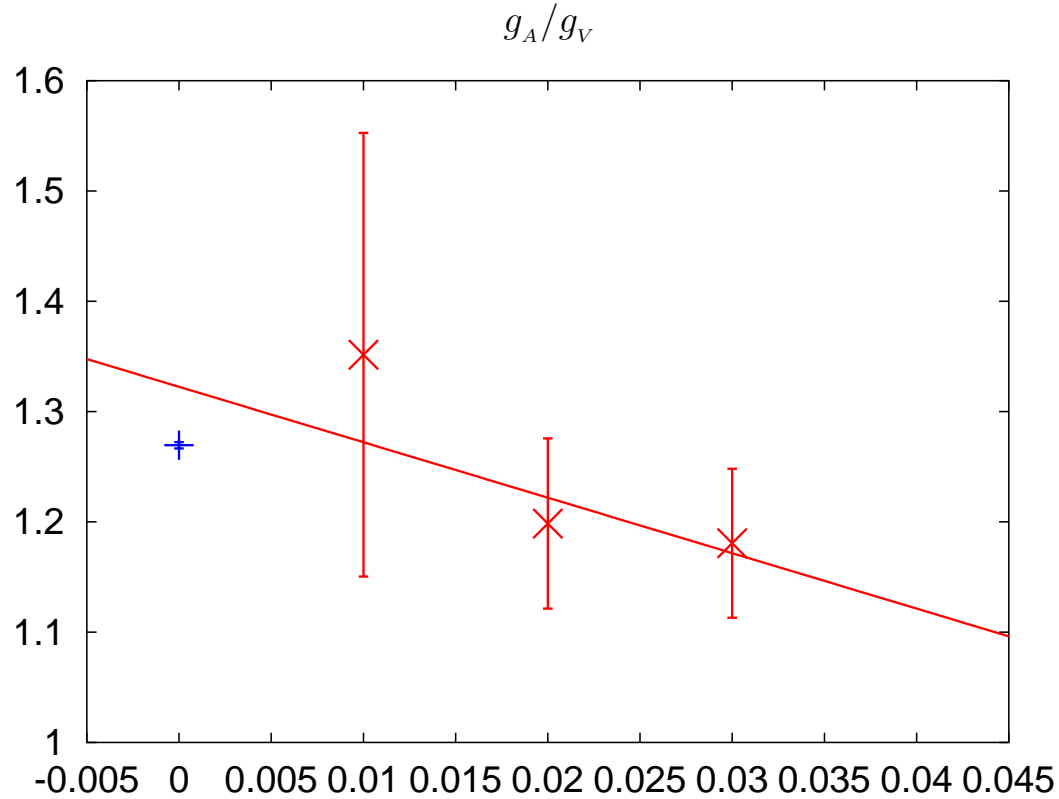
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appears small, though not renormalized.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.6$ GeV, $m_{\text{res}} = 0.00308(3)$, $m_{\text{strange}} = 0.04$,

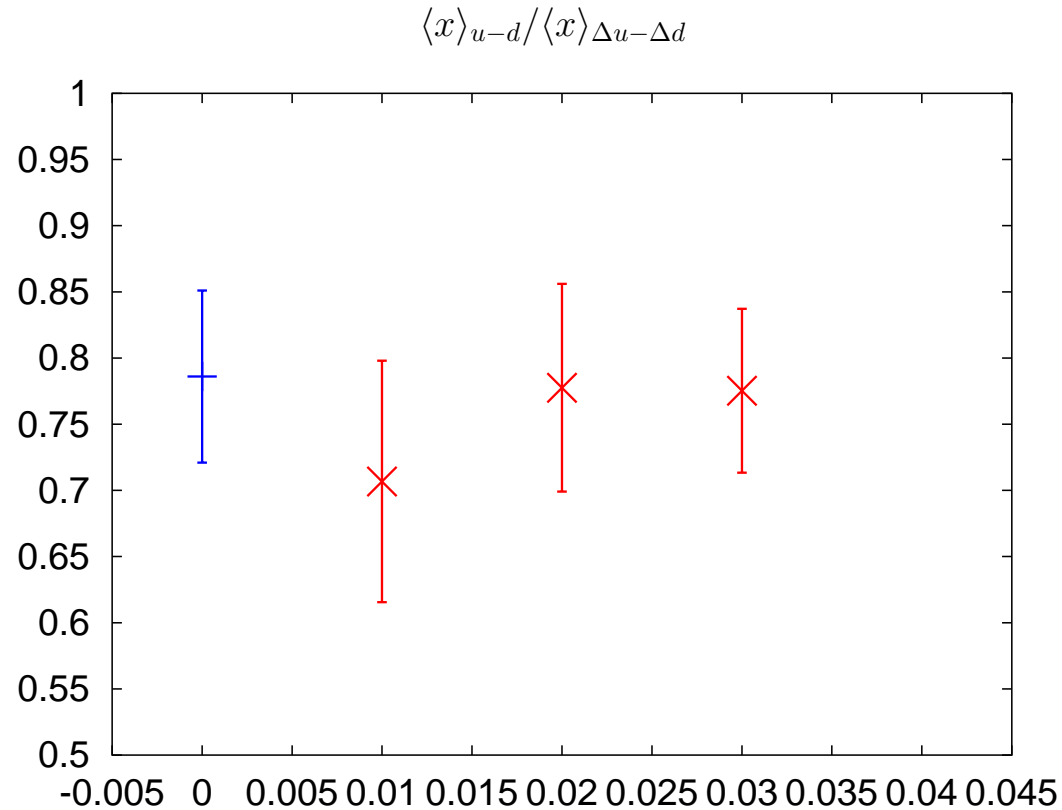
- $m_{\text{up,down}} = 0.03, 0.02, \text{ and } 0.01$; $m_{\pi} = 620, 520 \text{ and } 390$ MeV; $m_N = 1.4, 1.3, \text{ and } 1.2$ GeV (10-20 % errors),
- larger of the two volumes, $24^3 \times 64 \times 16$, 3 fm across, ongoing, preliminary analyses with 25-30 configurations,



consistent with experiment,
 $g_A/g_V = 1.32(11)$, despite rather small statistics,
 mild quark-mass dependence.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.6$ GeV, $m_{\text{res}} = 0.00308(3)$, $m_{\text{strange}} = 0.04$,

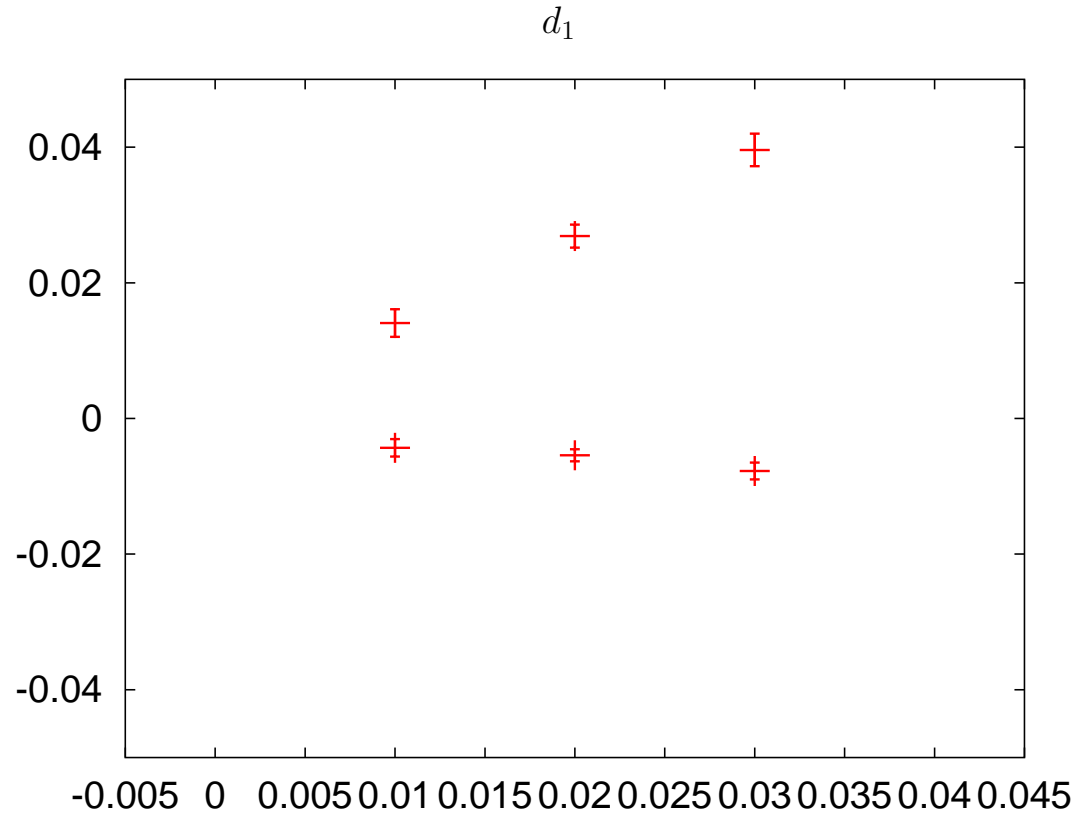
- $m_{\text{up,down}} = 0.03, 0.02, \text{ and } 0.01$; $m_{\pi} = 620, 520 \text{ and } 390$ MeV; $m_N = 1.4, 1.3, \text{ and } 1.2$ GeV (10-20 % errors),
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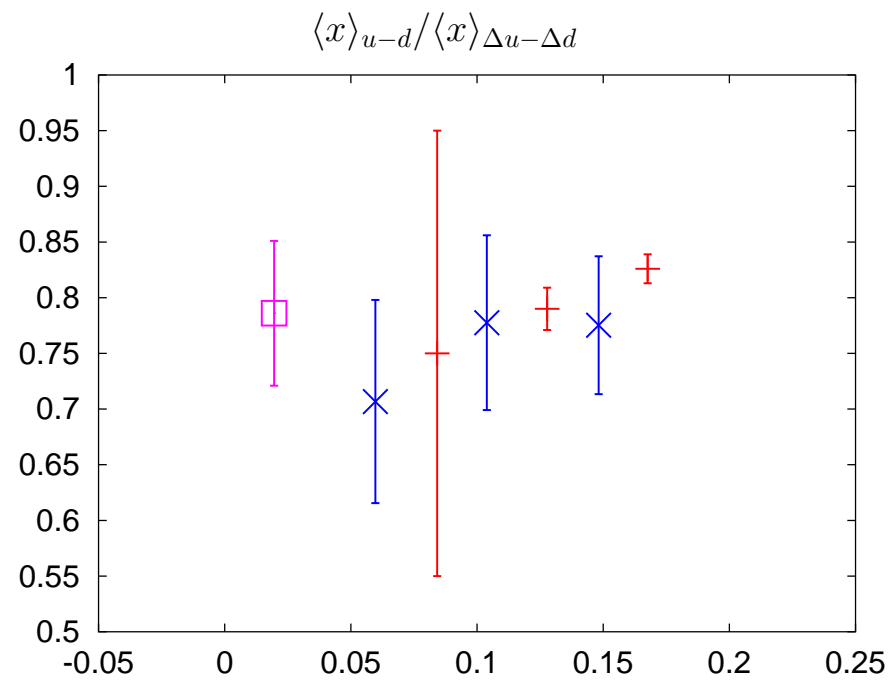
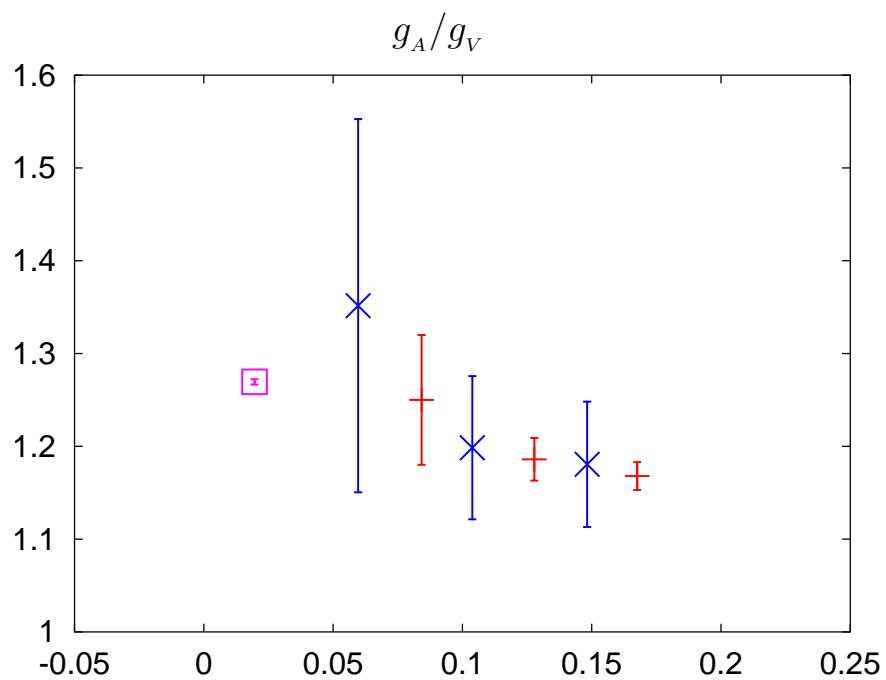
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- $m_{\text{up,down}} = 0.03, 0.02, \text{ and } 0.01$; $m_{\pi} = 620, 520 \text{ and } 390$ MeV; $m_N = 1.4, 1.3, \text{ and } 1.2$ GeV (10-20 % errors),
- larger of the two volumes, $24^3 \times 64 \times 16$, 3 fm across, ongoing, preliminary analyses with 25-30 configurations,



appears small, though not renormalized.

We can combine the 2-flavor and (2+1)-flavor ratios in m_π^2 (GeV²) scale:



Conclusions:

1. RBC 2-flavor DBW2+DWF dynamical calculations are almost complete:

- $a^{-1} = 1.7$ GeV, $m_{\text{res}} = 0.00137(5)$
 - $m_{\text{sea}} = 0.04, 0.03,$ and $0.02,$
 - NPR on the way.
- Ratios, g_A/g_V and $\langle x \rangle_{u-d}/\langle x \rangle_{\Delta u-\Delta d}$, in agreement with experiment, despite rather small volume:
 - $g_A/g_V = 1.27(5),$
 - mild quark-mass dependence.
- d_1 appears small, though not renormalized.

2. RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical calculations are ongoing,

- $a^{-1} = 1.6$ GeV, $m_{\text{res}} = 0.00308(3),$
 - $m_{\text{strange}} = 0.04$ and $m_{\text{up,down}} = 0.03, 0.02,$ and $0.01,$
 - the larger, 3-fm across, of the two volumes reported here,
 - preliminary.
- Ratios, g_A/g_V and $\langle x \rangle_{u-d}/\langle x \rangle_{\Delta u-\Delta d}$, consistent with experiment, despite rather small statistics:
 - $g_A/g_V = 1.32(11),$
 - mild quark-mass dependence.
- d_1 appears small, though not renormalized.