The first moment of the Kaon Distribution Amplitude from 2+1 flavor DWF

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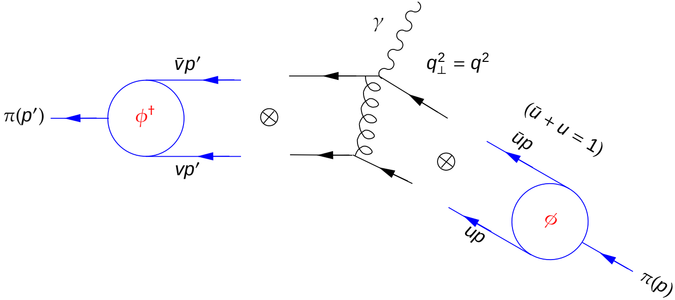
Exclusive processes at large Q^2

[Zhitnitsky and Chernyak, Brodsky and Lepage]:

$\langle\pi(p')|V_\mu|\pi(p)\rangle = F(Q^2)(p+p')_\mu$ factorizes at $Q^2 \to \infty$:

$\longrightarrow$ universal non-perturbative distribution amplitudes ϕ (overlap Hadron $\leftrightarrow$ lowest Fock state)

$\longrightarrow$ process dependent perturbative hard scattering amplitude T



$$(p+p')^\mu F(q^2) = \int_0^1 du \int_0^1 dv\ \phi^\dagger(v,q^2)T^\mu[v,u,q^2]\ \phi(u,q^2) + \text{power corrections}$$

Applications

Since distribution amplitudes ϕ_π, $\phi_K, \dots$ are universal, there are many relevant processes:

- exclusive non-leptonic decays ($B \to \pi\pi, KK$)
- (semi-leptonic decays $B \to \pi l \nu$)
- electromagnetic form factors
- ...

Here: Concentrate on Kaon distribution amplitude ϕ_K.

Definition of $\phi_K(u,Q^2)$ by non-local ME

$$\langle 0|\bar{q}(z)\gamma_\mu\gamma_5\mathcal{U}(z,-z)s(-z)|K(p)\rangle = if_K p_\mu \int_0^1 du e^{i(u-\bar{u})p\cdot z}\phi_K(u,Q^2)$$

- quarks separated by $2z$
- $\mathcal{U}(y,x) = \mathcal{P}\exp\left(ig\int_x^y d\omega^\mu A_\mu(\omega)\right)$ (Wilson line)
- enters e.g. elm. Form Factor when $(2z)^2 \sim 1/\mu^2$
- $\xi = u - \bar{u} = 2u - 1$; $(u + \bar{u} = 1)$
- $\int_0^1 du\phi(u,\mu) = 1$
- Isospin symmetry: $u \leftrightarrow (1-u)$

Moments of the distribution

- Expansion $\langle K(p)|\bar{s}(z)\gamma_\mu\gamma_5\mathcal{U}(z,-z)q(-z)|0\rangle$ around the light cone $z^2 \to 0$

  $\to$ at leading order: 1st moment $\langle u-\bar{u}\rangle = \langle\xi\rangle$

$$\langle K(p)|\bar{s}(0)\gamma_\mu\gamma_5\overleftrightarrow{D}_\nu q(0)|0\rangle = f_K(ip_\mu)(ip_\nu)\underbrace{\int_0^1 du(u-\bar{u})\phi_K(u,\mu)}_{\text{1st moment: }\langle u-\bar{u}\rangle}$$

  (vanishes for π due to Isospin symmetry ($u \leftrightarrow (1-u)$))

- Equivalently: define higher order moments (cf. previous talk)
- re-construct $\phi(u,\mu)$ from Gegenbauer moments

$$\phi(u,\mu) = \frac{3}{4}(1-\xi^2)\left(1 + \sum_{n\geq 1} a_n(\mu)C_n^{3/2}(\xi)\right)$$

  - $C_n^{3/2}$ are Gegenbauer polynomials
  - $a_1 = \frac{5}{3}\langle\xi^1\rangle$, $a_2 = \dots$
  - positive anomalous dimension, increasing with n

Results

Kaon 1st moment			a_1^K	μ/GeV	
Chernyak & Zhitnitski	(1983)	SR	0.12	1	
Khodjamirian *et al.*	(2004)	SR	0.05(2)	1	
Braun & Lenz	(2004)	SR	0.010(12)	1	
Ball & Zwicky	(2006)	SR	0.050(25)	1	
Ball & Zwicky	(2006)	SR	0.06(3)	1	
UKQCD/QCDSF	(2006)	LAT	0.0453(9)(29)	2	($N_f = 2$ impr. Wilson, prev. talk)
UKQCD	(2006)	LAT	0.055(5)	2	($N_f = 2 + 1$ DWF)

'an independent calculation on the lattice would be both timely and useful'
[Ball & Zwicky, 2006]

Lattice: Bare $\langle \xi^1 \rangle$

- $\langle K(p)|\bar{s}(0)\gamma_\mu\gamma_5\overleftrightarrow{D}_\nu q(0)|0\rangle = f_K(ip_\mu)(ip_\nu)\langle \xi^1\rangle^{\rm bare}$
- Compute bare $\langle \xi \rangle$ from double ratio

$$\frac{\sum_{\vec{x}} e^{i\vec{p}\vec{x}}\langle \bar{s}(x)\gamma_\rho\gamma_5\overleftrightarrow{D}_\mu q(x)P^\dagger(0)\rangle}{\sum_{\vec{x}} e^{i\vec{p}\vec{x}}\langle A_\nu(x)P^\dagger(0)\rangle} = \frac{\sum_{\vec{x}} e^{i\vec{p}\vec{x}}\langle O^5_{\rho\mu}(x)P^\dagger(0)\rangle}{\sum_{\vec{x}} e^{i\vec{p}\vec{x}}\langle A_\nu(x)P^\dagger(0)\rangle} \stackrel{t\to\infty}{=} \frac{(ip_\rho)(ip_\mu)}{ip_\nu}\langle \xi^1\rangle^{\rm bare}$$

  - time-dependence cancels
  - correlation effects cancel
  - decay constant (and Z_A) cancels

Classification of $O^5_{\mu\nu}$
[Martinelli, Sachrajda 1986], [Göckeler et al. 1996,1998,2005]

continuum	lattice
behaviour under C, P	behaviour under C, P
irred. under Lorentz transf.	irred. under $H(4)$

$H(4)$ less stringent bound on mixing → complicated mixing pattern

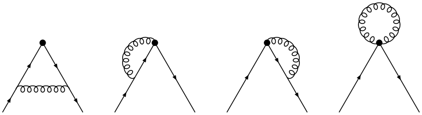
However: No mixing for choice $\frac{\sum_{\vec{x}} e^{i\vec{p}\vec{x}} \langle O^5_{\{4i\}}(x) P^\dagger(0)\rangle}{\sum_{\vec{x}} e^{i\vec{p}\vec{x}} \langle A_4(x) P^\dagger(0)\rangle} \overset{t\to\infty}{=} ip_i \langle \xi^1 \rangle^{\rm bare} \quad i = 1, 2, 3$

- average over index combinations {41}, {42}, {43}
- consider only lowest non-vanishing momentum;
  average over momenta (1,0,0), (-1,0,0), (0,1,0), ...

- NPR under way
- Perturbative Renormalisation

  compare 1-loop amputated Green's function with desired operator inserted in both schemes (lattice and $\overline{\mathrm{MS}}$)

  operator $O^5_{\mu\nu}$ ($\to V$):



  self energy ($\to \Sigma$):

Perturbative Renormalisation

Matching condition: $O^{\overline{\rm MS}}_{\rho\mu}(\mu) = Z_{O_{\rho\mu}} O^{\rm latt}_{\rho\mu}(a)$

$$Z_{O_{\rho\mu}} = \frac{1}{(1-w_0^2)Z_w}\left[1 + \frac{g^2 C_{\rm F}}{16\pi^2}\left(-\frac{8}{3}\ln(\mu^2 a^2) + \Sigma_1^{\overline{\rm MS}} - \Sigma_1 + V^{\overline{\rm MS}} - V\right)\right]$$

- Z_w DWF specific renormalization factor of domain wall height $M = 1 - w_0$
- Σ and V are contributions from wave-function and vertex graphs, resp.
- similar calculation exists (different gauge action) and we cross-checked [Capitani, 2006]
- validity if a small and μ large ...

Result: $Z_{O_{\rho\mu}} = \frac{1}{0.9082}\left[1 - \frac{g^2 C_{\rm F}}{16\pi^2} 5.2509\right]\left[1 + \frac{g^2 C_{\rm F}}{16\pi^2}\left(-\frac{8}{3}\ln(\mu^2 a^2) - 0.6713\right)\right]$

- For this project divide by Z_A since we want to renormalise $\dfrac{\sum_{\vec{x}} e^{i\vec{p}\vec{x}}\langle O^5_{\rho\mu}(x) P^\dagger(0)\rangle}{\sum_{\vec{x}} e^{i\vec{p}\vec{x}}\langle A_\nu(x) P^\dagger(0)\rangle}$
- dependent on renormalized coupling

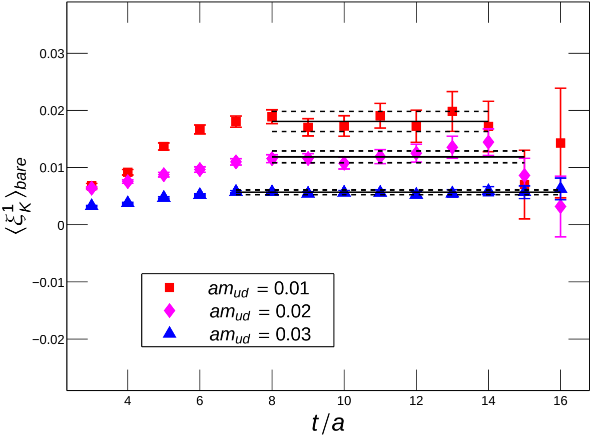
$$\frac{Z_{O_{\mu\nu}}}{Z_A}(1.6{\rm GeV}) = \left\{\begin{array}{lll} 1.2346 & \text{plaquette coupling} \\ 1.3384 & \text{continuum } \overline{\rm MS} \end{array}\right\} = 1.28 \pm 0.05$$

Simulation Parameters

configs	RBC/UKQCD
action	DWF/Iwasaki, $N_f = 2+1$, $\beta = 2.13$ [Talks by Mawhinney, Tweedie, ...]
lattice	$16^3 \times 32 \times 16$ ($24^3 \times 64 \times 16$)
quarks	$am_s = 0.04$, $am_q = 0.03; 0.02; 0.01$
meson masses	$am_K = 0.4164(10); 0.3854(10); 0.3549(14)$
lattice spacing	$a^{-1} \approx 1.60(3)$GeV ($L \approx 2fm$ ($3fm$))
statistics	300 measurements separated by 10 MC trajectories multiple sources at (0,0,0,0), (4,4,4,8), (8,8,8,16), (12,12,12,24)
smearing	Gauge invariant Jacobi smearing at the source
analysis	2 independent Jack-Knife analyses

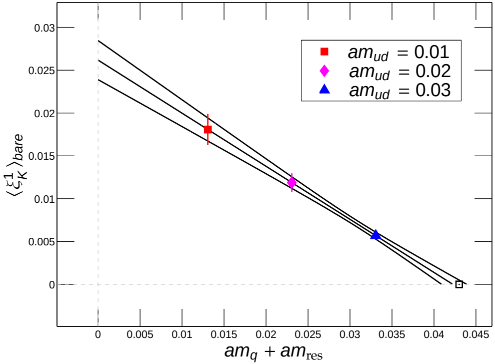
$16^3 \times 32 \times 16$, $am_{u,d} = 0.03; 0.02; 0.01$, $am_s = 0.04$,
300 measurements/source, 2, 2 and 4 pos. of the source
folded in T-direction

$$\frac{\sum_{\vec{x}} e^{i\vec{p}\vec{x}} \langle O_{4i}^5(x) P^\dagger(0) \rangle}{\sum_{\vec{x}} e^{i\vec{p}\vec{x}} \langle A_4(x) P^\dagger(0) \rangle}$$



$$\langle \xi_K^1 \rangle_{\mathrm{bare}} = 0.0057(4); 0.0119(10); 0.0181(18)$$

Bare results - χ extrapolation



- data confirms χPT: $\langle \xi_K^1 \rangle \propto (m_s - m_q)$ [Chen and Stewart, 2004, 2006]
- clear signs for partonic SU(3)-breaking effects
- χ extrapol.: $\langle \xi^1 \rangle^{\chi}_{\rm bare}$ = 0.0262(23)
- data compatible with $SU(3)$ symmetry:
  $\langle \xi^1 \rangle_{\rm bare}$ vanishes when $am_s = am_q$

Final results

$$a_1^K(2\text{GeV})|_{\overline{\text{MS}}} = \frac{5}{3}\frac{Z_{O_{\mu\nu}}}{Z_A}(2\text{GeV}) \times \langle u - \bar{u}\rangle^{\chi}_{\text{bare}} = 0.055(5)$$

(3-loop running [Larin et al., 1994] 1.6 GeV → 2.0 GeV)

Further systematics:

cut-off effects:	$O(a^2\Lambda^2_{QCD}) \simeq 2.5\%$
finte volume effects:	Check on $24^3 \times 64$-lattice is under way; preliminary results suggest that there are no noticeable FVE
renormalization:	NPR (RI-MOM) programme currently carried out

Summary and Outlook

- there are now results for $\langle\xi^1\rangle_K$ from 2 independent lattice calculations
- $\approx 10\times$ more precise than prev. sum-rule results
- confirmation of most existing sum-rule results

- compare with $24^3\times 64 \rightarrow$ check FVE
- finish up NPR

- analyse 2nd momenta $\langle\xi^2\rangle_K$, $\langle\xi^2\rangle_\pi$
- analyse 1st moment $\langle\xi^1\rangle_{K^*}$
- analyse 2nd moment $\langle\xi^2\rangle_\rho$, $\langle\xi^2\rangle_{K^*}$