Distribution Amplitudes of Pseudoscalar Mesons

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on behalf of the QCDSF Collaboration

Lattice 2006, Tucson, Arizona, USA

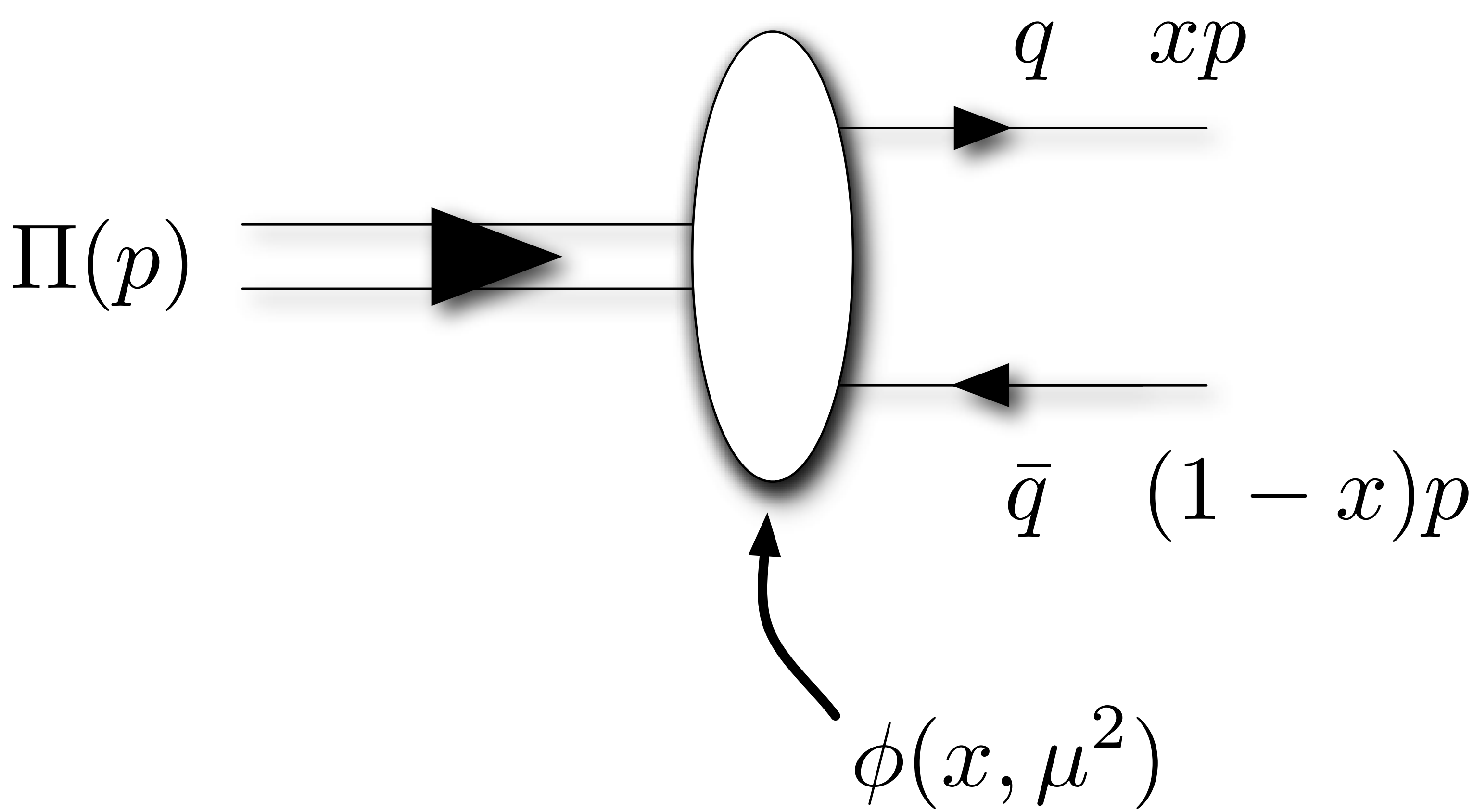
Distribution Amplitudes

- Exclusive B decays, weak radiative decays
- B physics with SCET
- QCD factorization
  exclusive & semi-exclusive processes
  - Form factors, diffraction, meson production in e^+e^- & $\gamma\gamma$ annihilation
- Light-cone sum rules
- Relativistic heavy-ion collisions

Phenomenology

- Surprising uncertainty in literature, model dependence
- For π: most recently from CLEO&CELLO data, Bakulev et.al., PLB578:91(2004)
- For K: sum rules, Ball et.al., hep-ph/0510338
- Two recent lattice investigations:
  hep-lat/0606012 - this talk
  hep-lat/0607018 - next talk by Jüttner

Leading twist meson DA



Scale dependence

$$\phi(x,\mu^2) \sim \int_{k_\perp^2<\mu^2} d^2k_\perp \phi(x,k_\perp)$$

- Process-independent, carries info on meson
- Scale dependence: ERBL-evolution

  Bethe-Salpeter equation

  => Eigenfunctions are Gegenbauer polynomials $C_n^{3/2}(2x-1)$

$$\phi(x,\mu^2) = 6x(1-x)\sum_{n=0}^{\infty} a_n(\mu^2)C_n^{3/2}(2x-1)$$

- For π, ρ, η, η' and Φ:

  G parity => $a_{odd} = 0$

  => mirror symmetry

- K, K*: $a_{odd} \neq 0$

$$\phi(x,\mu^2) = 6x(1-x)\sum^{\infty} a_n(\mu^2)C_n^{3/2}(2x-1)$$

- For π

  G pa

  => m

Goal:
Compute lowest moments
$a_1(\mu^2), a_2(\mu^2)$
for kaon and
$a_2(\mu^2)$
for pion

- K, K*: $a_{odd} \neq 0$

Lattice calculations

$$\langle\Omega|\mathcal{O}_{\{\mu_0\ldots\mu_n\}}|\Pi(\vec{p})\rangle = \mathrm{i}\, f_\Pi\, p_{\mu_0}\ldots p_{\mu_n}\langle\xi^n\rangle$$

$$x = \frac{1}{2}(1+\xi)$$

- Local operator via light-cone OPE
- Yields moments of DAs w.r.t. *x*
- Matrix elements from ratios of two-point functions

Choice of operators

$$\mathcal{O}^a_{41} = \mathcal{O}_{\{41\}}, \qquad \vec{p} = (2\pi/L, 0, 0)$$
$$\mathcal{O}^b_{44} = (\mathcal{O}_{44} - 1/3(\mathcal{O}_{ii}))\,, \qquad \vec{p} = \vec{0}$$

$$\mathcal{O}^a_{412} = \mathcal{O}_{\{412\}}, \qquad \vec{p} = (2\pi/L, 2\pi/L, 0)$$
$$\mathcal{O}^b_{411} = (\mathcal{O}_{411} - 1/2(\mathcal{O}_{422} + \mathcal{O}_{433}))\,, \qquad \vec{p} = (2\pi/L, 0, 0)$$

Working points

β	κ_{sea}	Volume	r_0/a	am_π
5.20	0.13420	$16^3 \times 32$	4.077(70)	0.5847(12)
5.20	0.13500	$16^3 \times 32$	4.754(45)	0.4148(13)
5.20	0.13550	$16^3 \times 32$	5.041(53)	0.2907(15)
5.25	0.13460	$16^3 \times 32$	4.737(50)	0.4932(10)
5.25	0.13520	$16^3 \times 32$	5.138(55)	0.3821(13)
5.25	0.13575	$24^3 \times 48$	5.532(40)	0.25556(55)
5.29	0.13400	$16^3 \times 32$	4.813(82)	0.5767(11)
5.29	0.13500	$16^3 \times 32$	5.227(75)	0.42057(92)
5.29	0.13550	$24^3 \times 48$	5.566(64)	0.32696(64)
5.29	0.13590	$24^3 \times 48$	5.840(70)	0.23956(71)
5.40	0.13500	$24^3 \times 48$	6.092(67)	0.40301(43)
5.40	0.13560	$24^3 \times 48$	6.381(53)	0.31232(67)
5.40	0.13610	$24^3 \times 48$	6.714(64)	0.22081(72)

Dynamical Clover, n_f=2

Mass-degenerate quarks

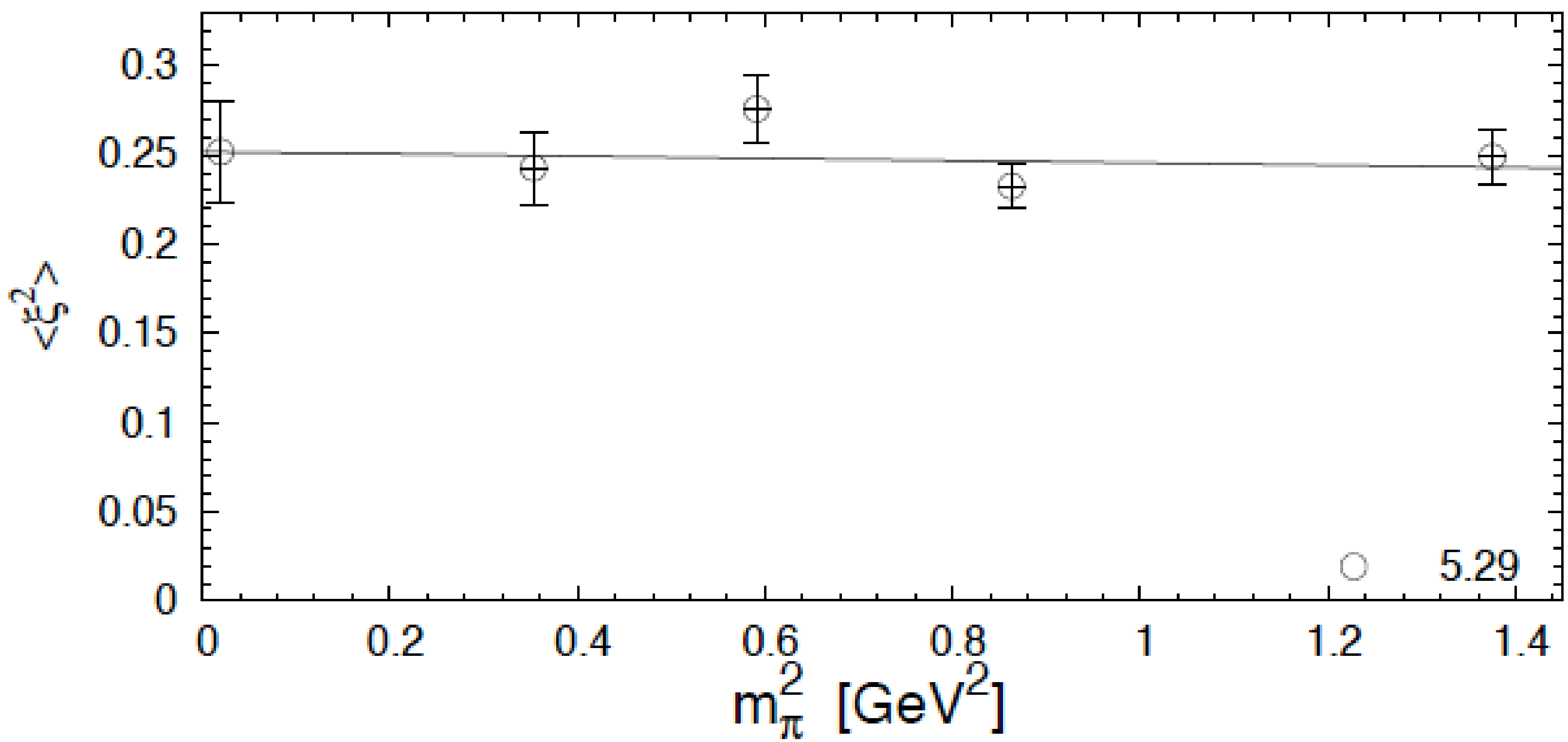


FIG. 4: Chiral extrapolation of $\langle \xi^2 \rangle_\pi$ at constant β for $\beta = 5.40$ (top) and $\beta = 5.29$ (bottom) for $\mathcal{O}^a_{412}$ from Eq. (22) in the $\overline{\text{MS}}$ scheme at $\mu^2 = 4\,\text{GeV}^2$.

χPT: Chen et.al., PRL92:202001(2004)

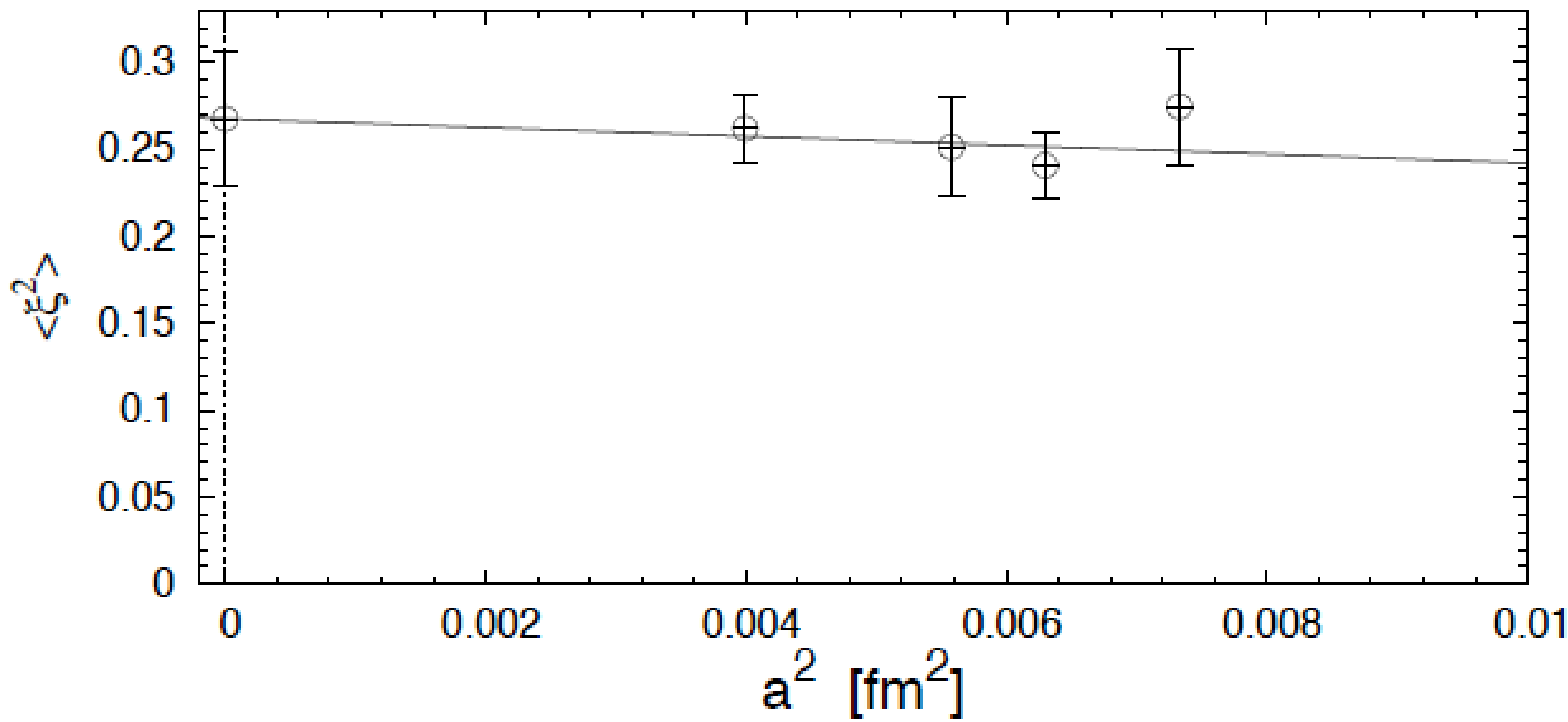


FIG. 6: Results for $\langle \xi^2 \rangle_\pi$ for each value of β at the physical pion mass as a function of a^2 for $\mathcal{O}^a_{412}$ from Eq. (22) in the $\overline{\mathrm{MS}}$ scheme at $\mu^2 = 4\,\mathrm{GeV}^2$.

Our result:

$$\langle \xi^2 \rangle_\pi(\mu^2 = 4\,\mathrm{GeV}^2) = 0.269(39)$$

$$a_2^\pi(4\,\mathrm{GeV}^2) = 0.201(114)$$

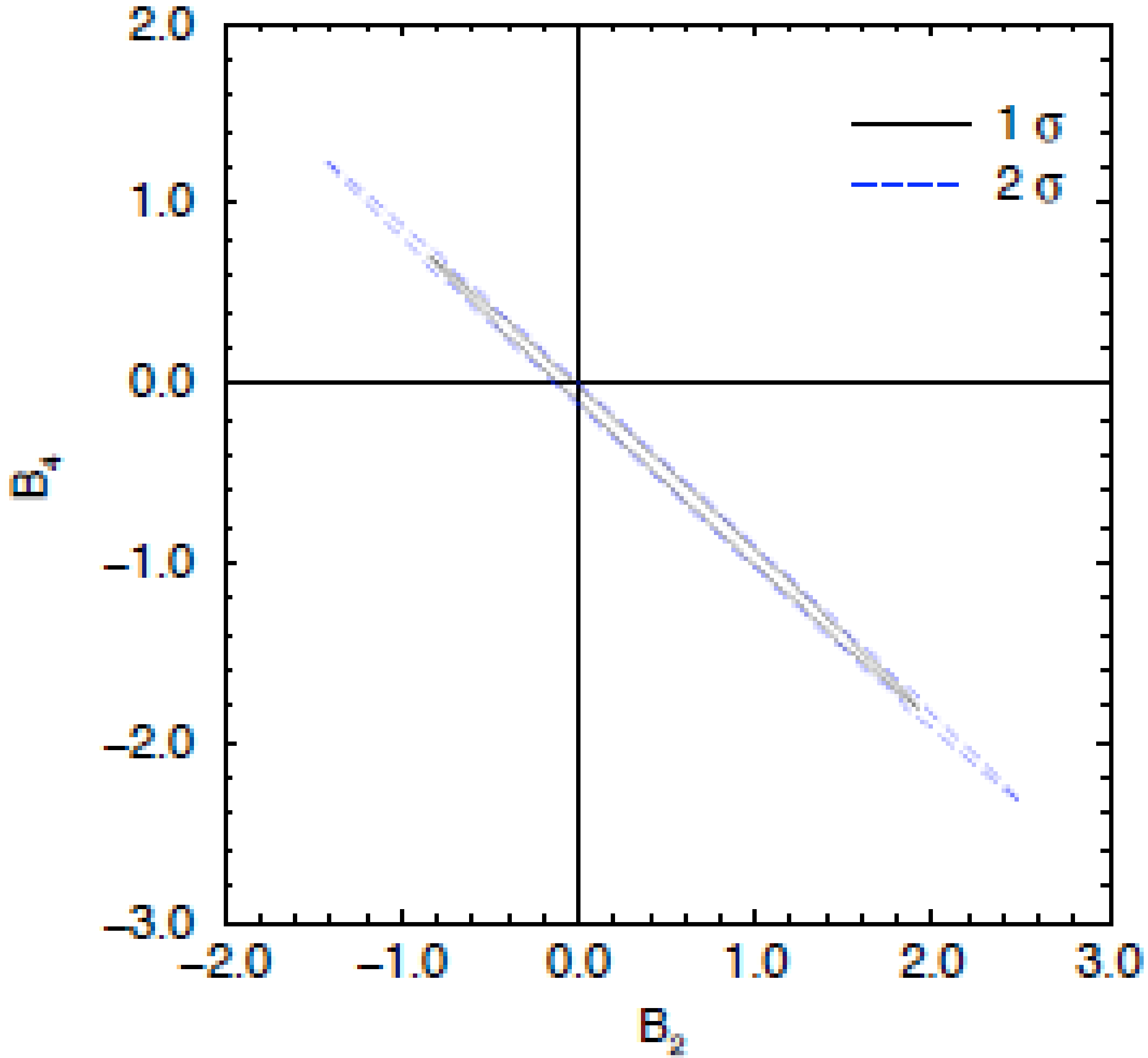
Compare to Del Debbio et.al., NPPS119:416(2003):

$$\langle \xi^2 \rangle_\pi(\mu^2 = 4\,\mathrm{GeV}^2) = 0.286(49)^{+0.030}_{-0.013}$$

Larger than asymptotic value:

$$\langle \xi^2 \rangle_\pi(\mu^2 \to \infty) = 0.2$$

$F_{\pi 0 \to \gamma^* \gamma}$ at leading twist



Diehl et.al., EPJC22:439(2001)

$F_{\pi 0 \to \gamma^* \gamma}$ various models

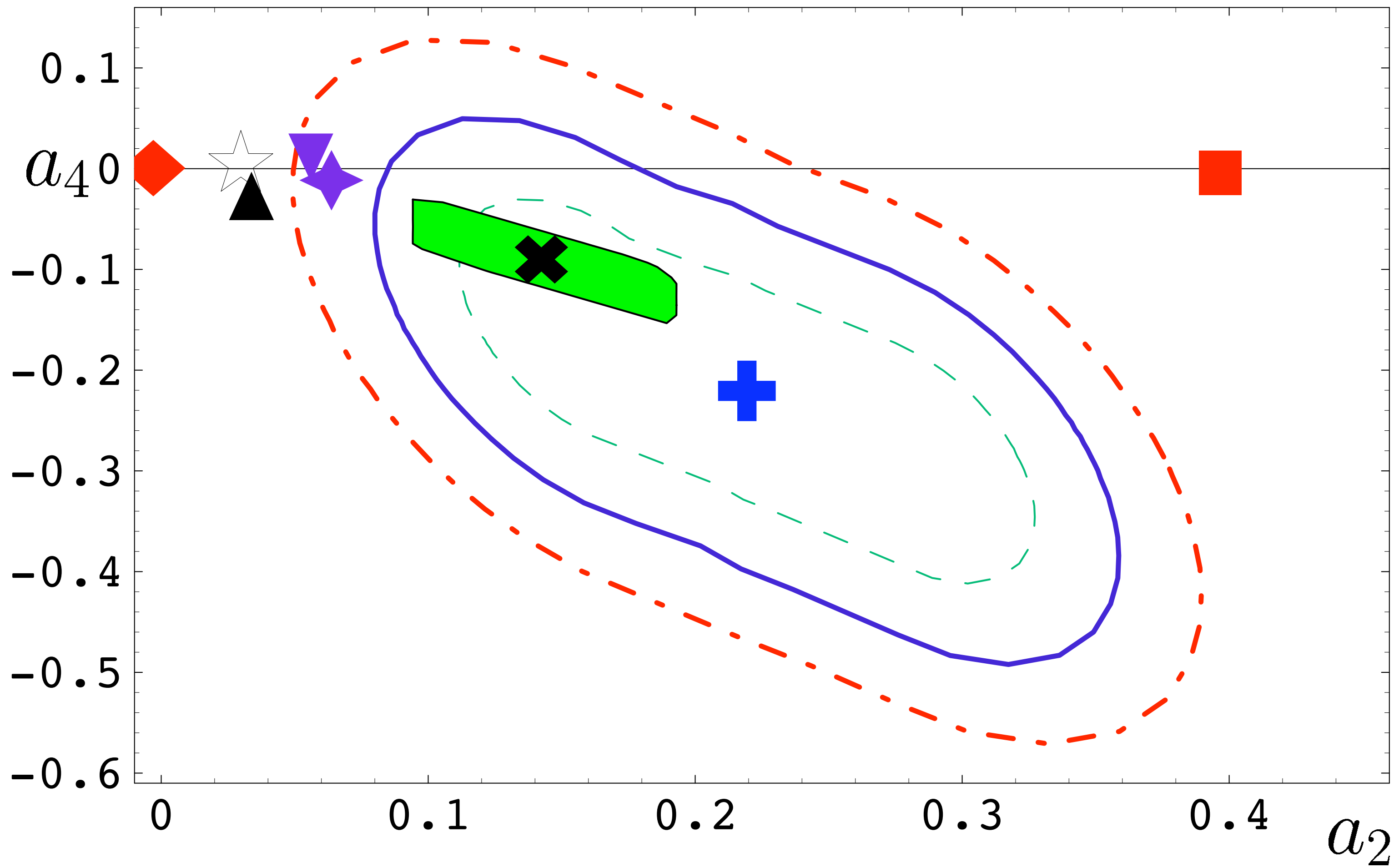


Figure from Bakulev et.al., PLB578:91(2004)

$F_{\pi 0\to\gamma^*\gamma}$ various models

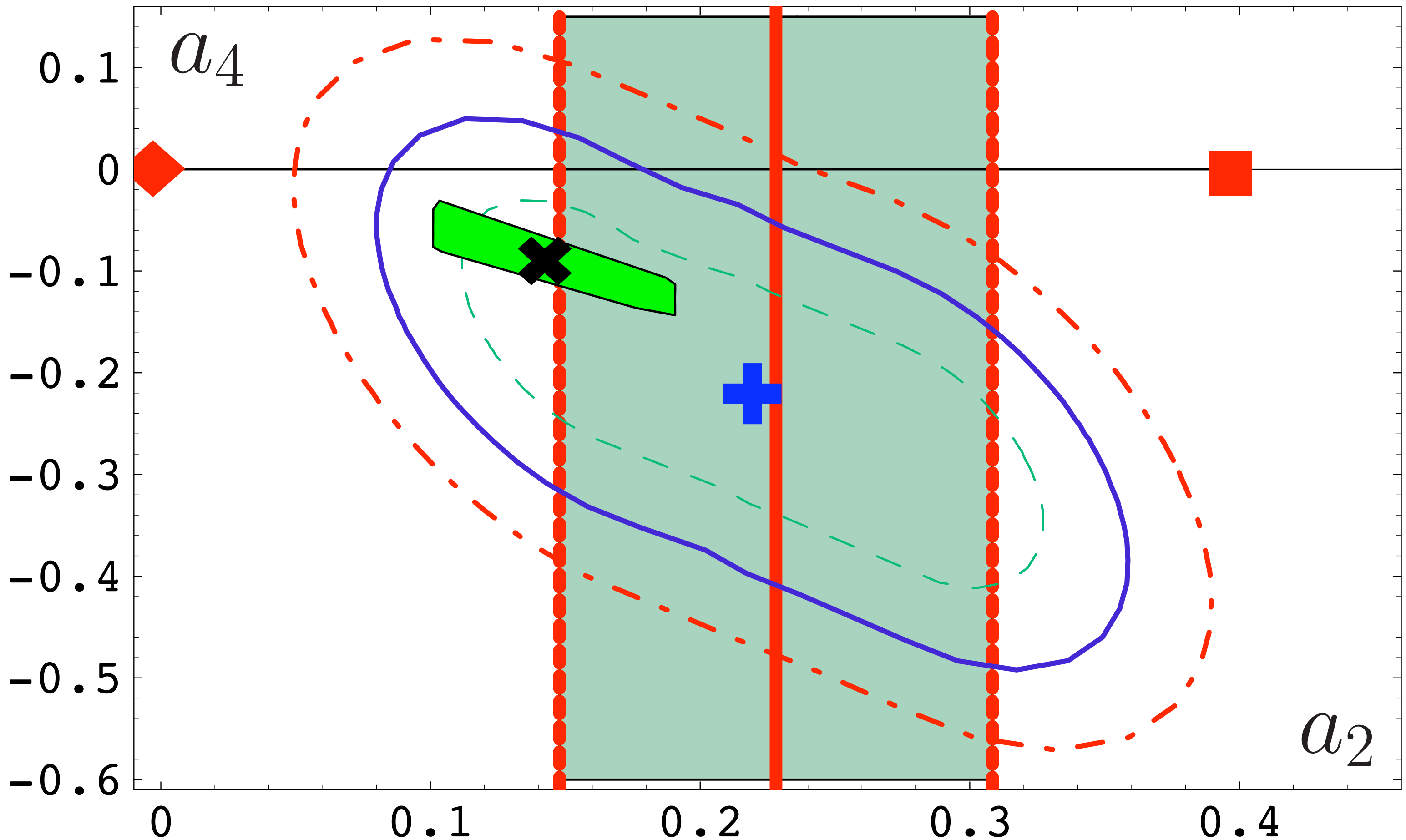


Figure from Bakulev et.al., PLB578:91(2004)

$F_{\pi 0 \to \gamma^* \gamma}$ various models

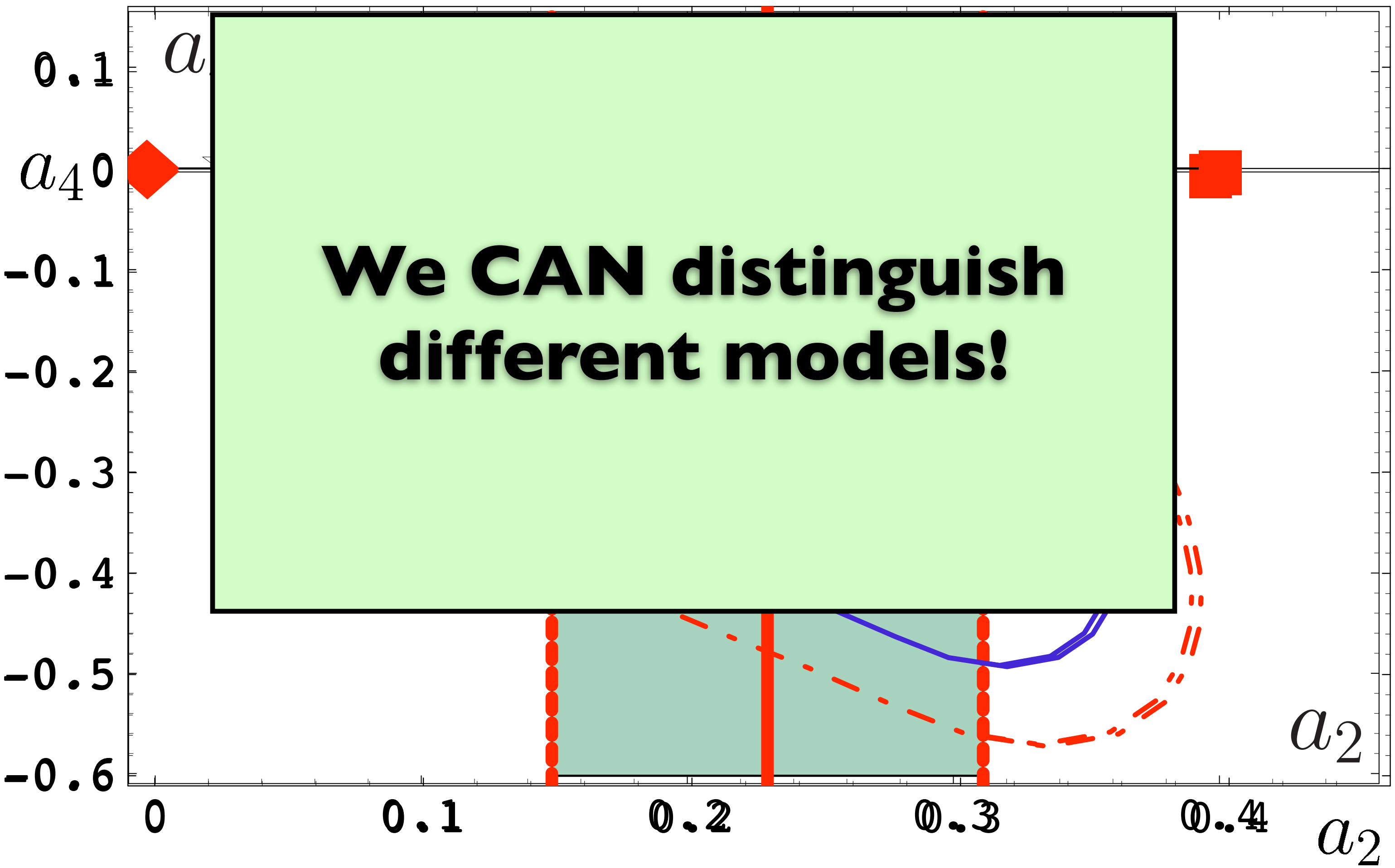


Figure from Bakulev et.al., PLB578:91(2004)

Mass non-deg. quarks

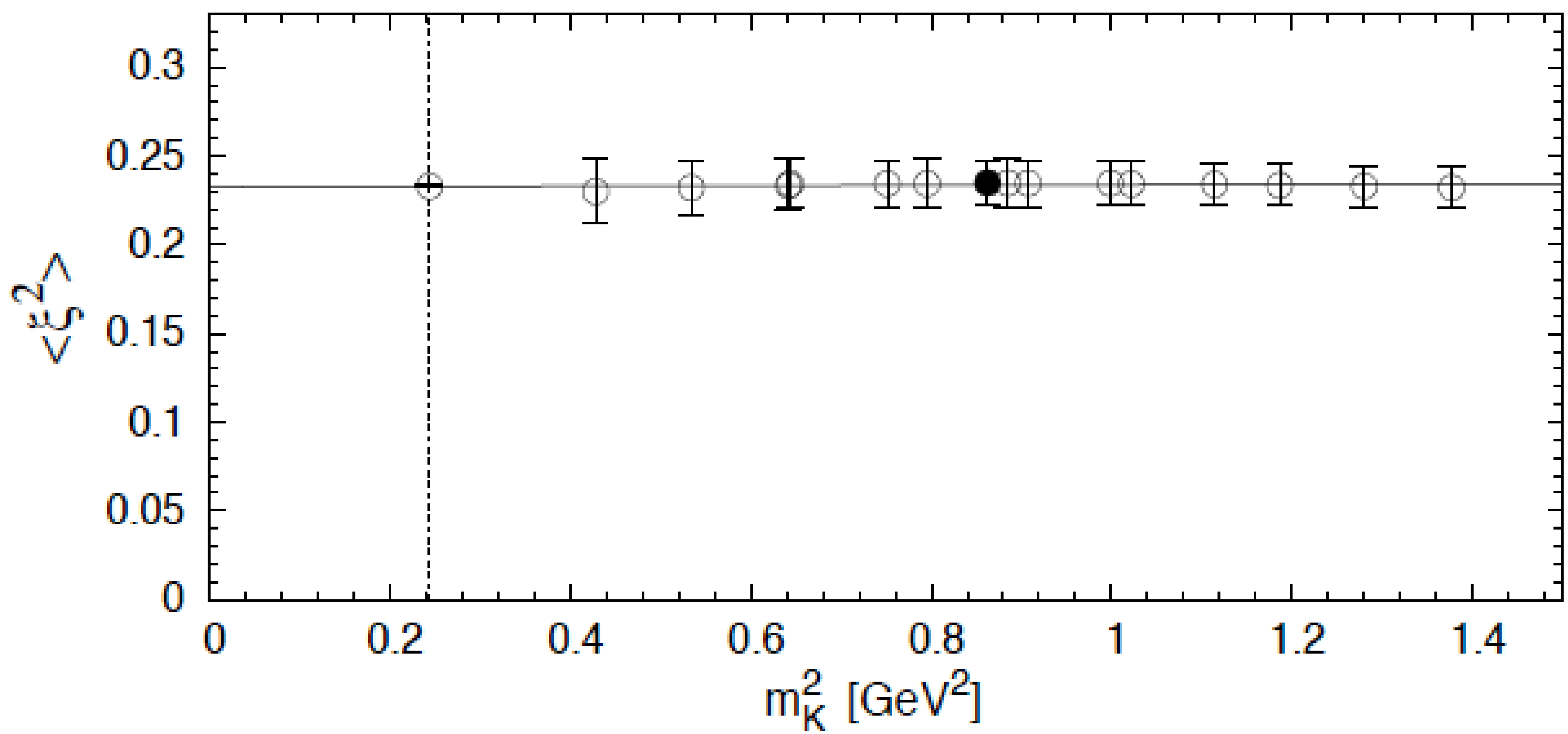


FIG. 7: $\langle \xi^2 \rangle_K$, extracted from Eq. (28) at the working point, $\beta = 5.29$, $\kappa_{\rm sea} = 0.13500$, as a function of the squared Kaon mass, m_K^2, for various choices of the valence quark masses. Results are quoted in the $\overline{\rm MS}$ scheme at $\mu^2 = 4\,{\rm GeV}^2$. The vertical dotted line corresponds to the physical Kaon mass.

Only β=5.29, est. syst. error from cont. extrap.

Averaging over 4 values of κ_{sea}:

$$\langle \xi^2 \rangle_K (\mu^2 = 4\,\mathrm{GeV}^2) = 0.260(6)$$

$$\langle \xi^2 \rangle_K / \langle \xi^2 \rangle_\pi \simeq 1$$

Chernyak&Zhitnisky:

$$0.59(4)$$

↔

Ball et.al.
Khodjamirian et.al.:

$$\simeq 1$$

Averaging over 4 values of κ_{sea}:

$$\langle \xi^2 \rangle_K(\mu^2 = 4\,\mathrm{GeV}^2) = 0.260(6)$$

We CAN distinguish different models!

Chernya

t.al.

jan et.al.:

0.

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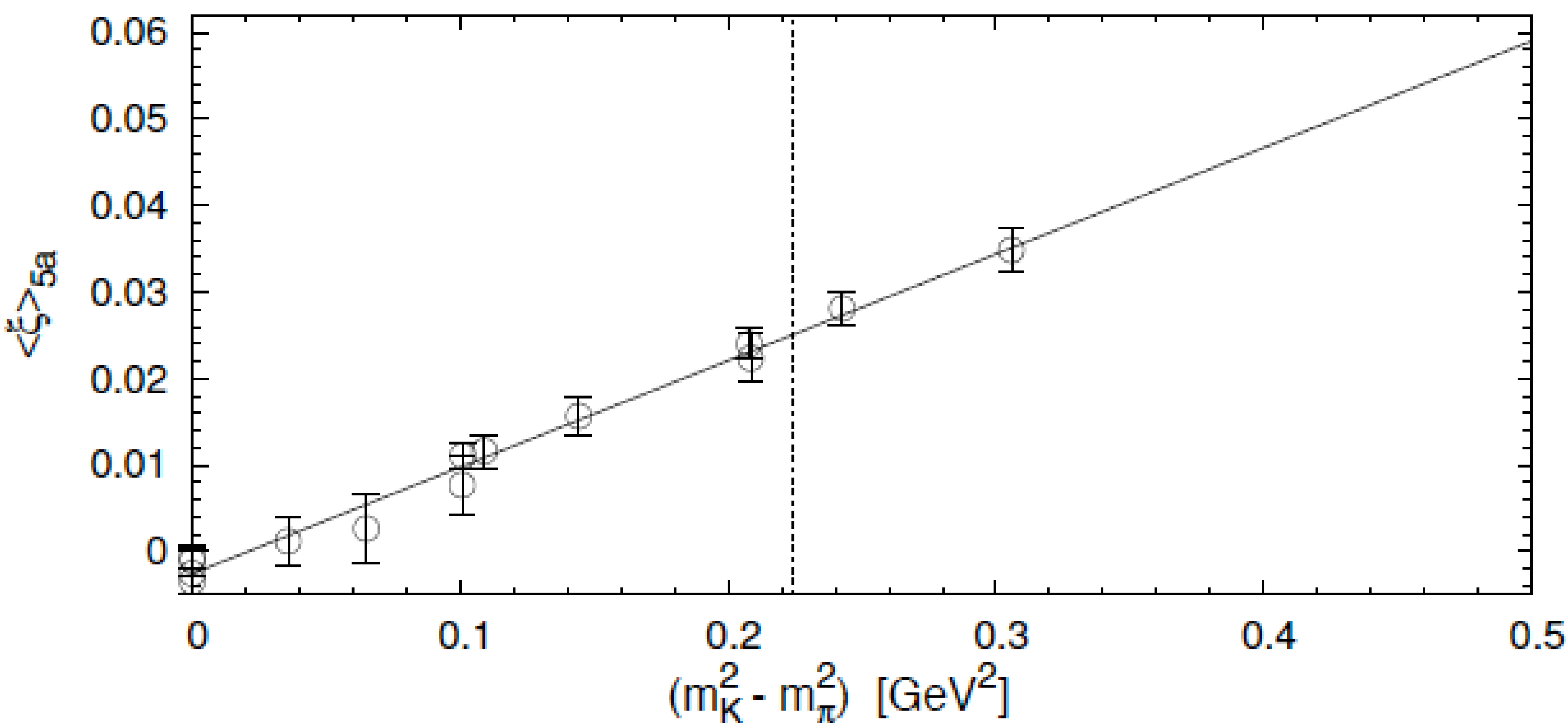

FIG. 11: Result for $\langle\xi\rangle_a^5$ and $\langle\xi\rangle_b^5$ for $\beta = 5.29$, $\kappa_{\rm sea} = 0.13590$, in the $\overline{\rm MS}$ scheme at $\mu^2 = 4\,{\rm GeV}^2$. The vertical dotted line corresponds to the physical $m_K^2 - m_\pi^2$ mass difference.

Averaging over 4 values of κ_{sea}:

$$\langle \xi \rangle_K(\mu^2 = 4\,\mathrm{GeV}^2) = 0.0272(5)$$

$$a_1^K(4\,\mathrm{GeV}^2) = 0.0453(9)(29)$$

Recent controversy in literature, see Ball et.al., hep-lat/0603063:

$$a_1^K(4\,\mathrm{GeV}^2) = 0.05(25)$$

Averaging over 4 values of κ_{sea}:

$$\langle \xi \rangle_K (\mu^2 = 4\,\mathrm{GeV}^2) = 0.0272(5)$$

Established solution of phenomenlogical puzzle!

Re … see

Summary

- $a_2^\pi(4\,\mathrm{GeV}^2) = 0.201(114)$: larger than asymptotic values, distinguishes models
- $a_2^K(4\,\mathrm{GeV}^2) = 0.175(18)(47)$: about the same as a_2^π, also distinguishes models
- $a_1^K(4\,\mathrm{GeV}^2) = 0.0453(9)(29)$: confirms sum-rule estimate, settles debate

Outlook

- Lower pion masses (300 MeV and below)
- Improved chiral perturbation theory (J.W. Chen, private communication)
- DAs for the nucleon
  - phenomenologically different applications
  - strong impact for models