Vector meson electromagnetic form factors

B. G. Lasscock, J.N. Hedditch, D. B. Leinweber, A. G. Williams

CSSM, University of Adelaide

Electromagnetic Structure in Quenched LQCD

- Charge, magnetic and quadrupole form factors calculated in Quenched LQCD.
- Charge radii and magnetic moment derived.
- Quark Models:
  - Hyperfine interaction $\frac{(\sigma_1 \cdot \sigma_2)}{(m_1 m_2)}$.
  - Magnetic moment of the ρ meson, $\mu_\rho \simeq 1.84\mu_N$ at SU(3) flavour limit.
- Lattice QCD: Oblate ρ-meson?
  Alexandrou et al. Phys. Rev. D66, 094503, 2002
- Environmental sensitivity of observables?

2pt Functions at the Hadronic Level

$$G^{\mu\nu}(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle\Omega|\chi^{\mu}(x)\chi^{\nu\dagger}(0)|\Omega\rangle \; .$$

$$G^{\mu\nu}(t,\vec{p}) = \sum_{s} e^{-E_{\rho^+}t}\langle\Omega|\chi^{\mu}(0)|\rho^+(\vec{p},s)\rangle\langle\rho^+(\vec{p},s)|\chi^{\nu\dagger}(0)|\Omega\rangle \; + \ldots$$

$$\langle\Omega|\,\chi^{\mu}(0)\,|\rho^+(\vec{p},s)\rangle = \frac{1}{\sqrt{2E}}\lambda^{i}\,\epsilon_{\mu}(p,s)$$

$$\langle\rho^+(\vec{p},s)|\,\chi^{\nu\dagger}(0)\,|\Omega\rangle = \frac{1}{\sqrt{2E}}\lambda^{j\star}\,\epsilon^{\star}_{\nu}(p,s)$$

where $p^{\mu} = (E_{\rho^+},\vec{p})$.
The transversality condition is,

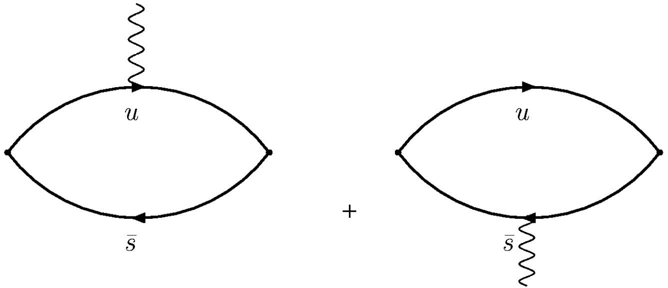
$$\sum_{s} \epsilon_{\mu}(p,s)\,\epsilon^{\star}_{\nu}(p,s) = -\left(g_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{m^2}\right)$$

$$G^{00}(t,\vec{0}) = 0$$

$$G^{kl}(t,\vec{0}) = \delta^{kl}\lambda^{k}\,\lambda^{l\star}\,e^{-m_{\rho^+}t} + \ldots$$

3pt Functions at the Hadronic Level

$\langle p', s' | J^{\alpha} | p, s \rangle$:



Form Factors, Formulas

Following,

Brodsky and Hiller, Phys. Rev. D46, 2141–2149, 1992.

For the pion and kaon,

$$\langle p' | J^\alpha | p \rangle = \frac{1}{2\sqrt{E_p E_{p'}}} [p^\alpha + p'^\alpha] F_1(Q^2)$$

For the ρ and K^*,

$$\langle p', s' | J^\mu | p, s \rangle = \frac{1}{2\sqrt{E_p E_{p'}}} \epsilon'^\star_\alpha(p', s') \epsilon_\beta(p, s) J^{\alpha\mu\beta}(p', p)$$

$$\begin{aligned} J^{\alpha\mu\beta}(p', p) &= -\{ G_1(Q^2)\, g^{\alpha\beta} [p^\mu + p'^\mu] + G_2(Q^2)[g^{\mu\beta} q^\alpha - g^{\mu\alpha} q^\beta] \\ &\quad - G_3(Q^2)\, q^\beta q^\alpha \frac{p^\mu + p'^\mu}{2M^2} \} \end{aligned}$$

Sachs Factors, Formulas

Following,

Brodsky and Hiller, Phys. Rev. D46, 2141–2149, 1992.

$$\begin{aligned} G_Q(Q^2) &= G_1(Q^2) - G_2(Q^2) + (1 + \frac{Q^2}{4m^2}) G_3(Q^2) \\ G_M(Q^2) &= G_2(Q^2) \\ G_C(Q^2) &= G_1(Q^2) + \frac{2}{3} \times \frac{Q^2}{4m^2} G_Q(Q^2) \end{aligned}$$

Where $Q^2 \simeq 0.22~\mathrm{GeV}^2$.

3pt Functions at the Hadronic Level, Cont.

$$G^{\mu\alpha\nu}(t_2, t_1, \vec{p}\,', \vec{p}) = \sum_{\vec{x_2}\vec{x_1}} e^{-i\vec{p'}.(\vec{x_2}-\vec{x_1})} e^{-i\vec{p}.\vec{x_1}} \langle\Omega|\chi^{\mu}(x_2) J^{\alpha}(x_1) \chi^{\dagger\nu}(0)|\Omega\rangle$$
$$= \sum_{i,j}\sum_{s,s'} e^{-E_i(t_2-t_1)} e^{-E_j t_1} \langle\Omega|\chi^{\mu}|p', s'\rangle\langle p', s'|J^{\alpha}|p, s\rangle\langle p, s|\chi^{\dagger\nu}|\Omega\rangle$$

To extract the form factors we take the ratios,

$$R^{\mu\alpha\nu}(p', p) = \sqrt{\frac{\left\langle G^{\mu\alpha\nu}(\vec{p'}, \vec{p}, t, t_1)\right\rangle \left\langle G^{\nu\alpha\mu}(\vec{p}, \vec{p'}, t, t_1)\right\rangle}{< G^{\mu\mu}(\vec{p'}, t) > < G^{\nu\nu}(\vec{p}, t) >}} \, .$$

3pt Functions at the Hadronic Level, Cont.

We can show that we extract form factors from the ratios,

$$G_C(Q^2) = \frac{2}{3}\frac{\sqrt{Em}}{E+m}\left(R^{101} + R^{202} + R^{303}\right)$$

$$G_M(Q^2) = \frac{\sqrt{Em}}{p_x}\left(R^{133} + R^{331}\right)$$

$$G_Q(Q^2) = \frac{m\sqrt{Em}}{p_x^2}\left(2R^{101} - R^{202} - R^{303}\right)$$

Observables

- Charge radii,

$$\langle r^2 \rangle = -6 \frac{\partial}{\partial Q^2} G_C(Q^2) \Big|_{Q^2=0}$$

- Relate to $Q^2 = 0$ through the standard assumption of a monopole form for G_C.

$$G_C(Q^2) = \left(\frac{1}{\frac{Q^2}{\Lambda^2} + 1} \right)$$

Observables, Cont.

Motivated by the scaling of $\frac{G_M(Q^2)}{G_C(Q^2)}$ for baryons, we extrapolate $G_M(Q^2)$ to $Q^2 = 0$ by assuming,

$$G_M(0) \simeq \frac{G_M(Q^2)}{G_C(Q^2)}$$

For the charge of the quark e_q and the mass of the meson M,

$$\mu_1 = \frac{e_q G_M(0)}{2M}$$

The quadrupole form factor at finite Q^2,

$$Q_1 = \frac{e_q G_Q(Q^2)}{M^2}$$

Lattice simulation parameters

- Simulations done on a $20^3 \times 40$ lattice, $a = 0.128$ fm with 380 guage field configurations.

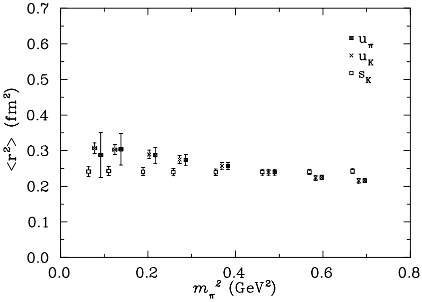
- **Martinelli et al., Nuclear Phys. B358, 212-227, 1991.**

$$
\begin{aligned}
j^\mu(x) &= \frac{1}{4}\Big(\bar{\psi}(x)(\gamma^\mu - r)U_\mu(x)\psi(x+\hat{\mu}) \\
&\quad + \bar{\psi}(x+\hat{\mu})(\gamma^\mu + r)U_\mu^\dagger(x)\psi(x)\Big) + (x \to x - \hat{\mu})) \\
&\quad + \frac{1}{2} ra \sum_\rho \partial_\rho(\bar{\psi}(x)\sigma^{\rho\mu}\psi(x))
\end{aligned}
$$

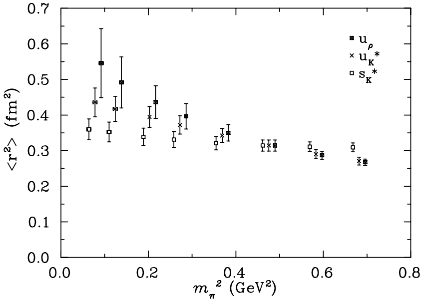
  - The current is $O(a^2)$ improved and conserved.
  - We use the FLIC fermion action with ape smearing in the irrelevant terms of the conserved current for improved scaling.

Quark Sector Contributions to the Charge Radii

Quark Sector Contributions to the Charge radii $< r^2 >$ (fm^2).



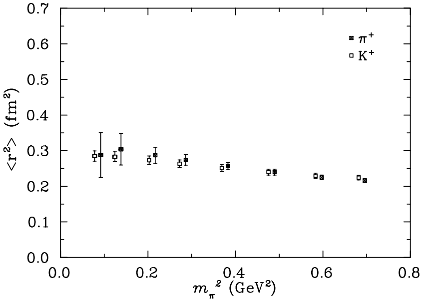
Pseudoscalar mesons



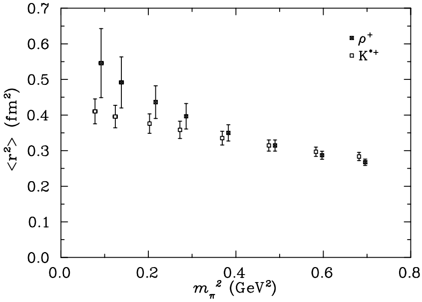
Vector mesons

Comparison of Charge Radii

Charge radii $< r^2 >$ (fm^2).



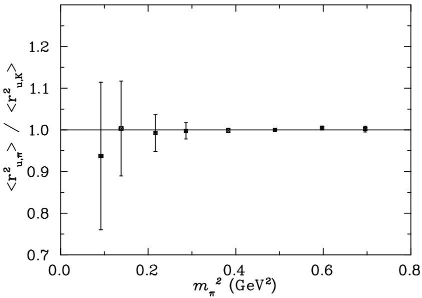
Pseudoscalar mesons



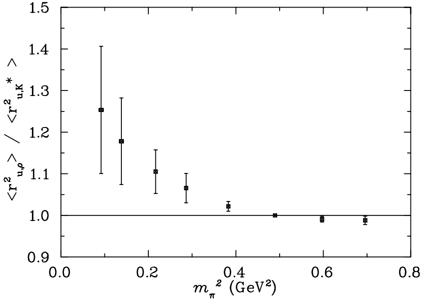
Vector mesons

Environmental Sensitivity of Charge Radii

Ratio of the up-quark contributions to $< r^2 > \ \mathrm{fm}^2$.

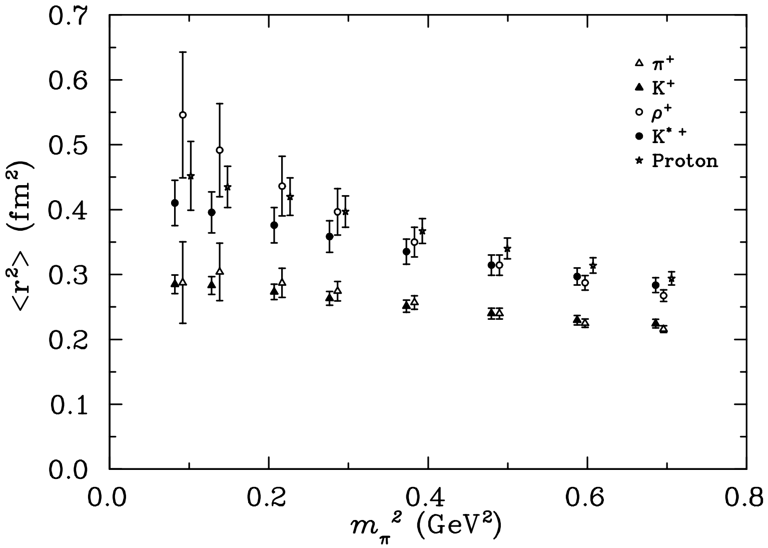


Pseudoscalar mesons

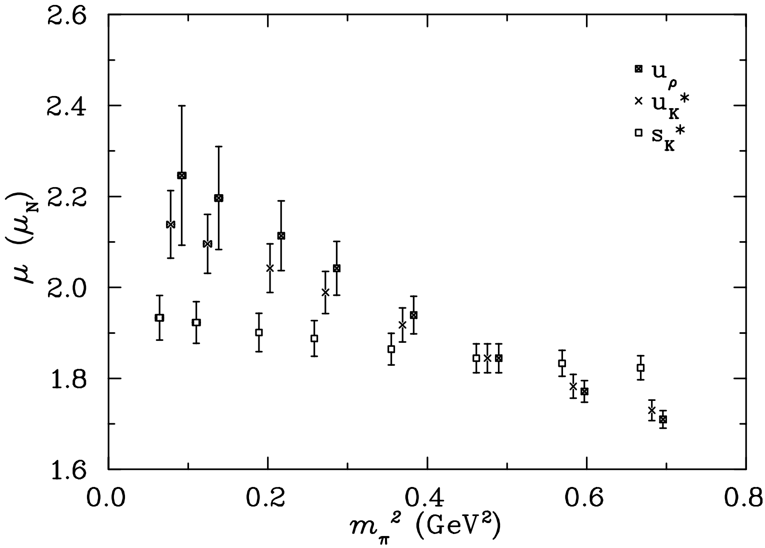


Vector mesons

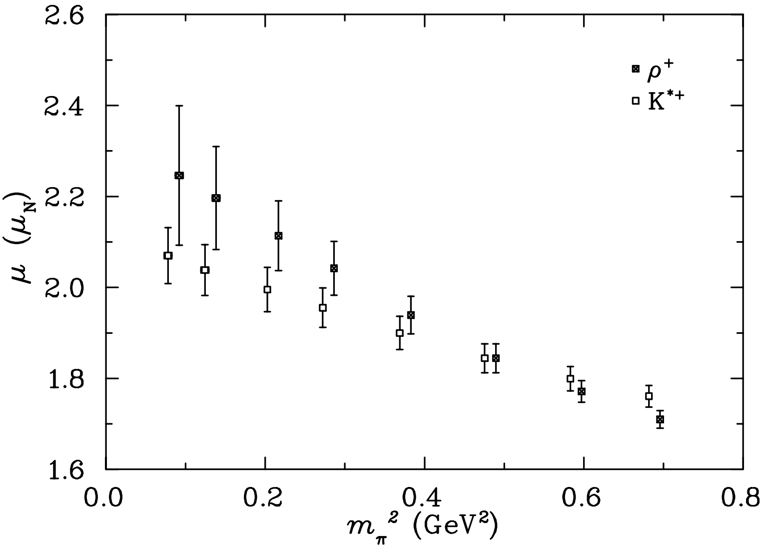
Summary of the meson and proton charge radii



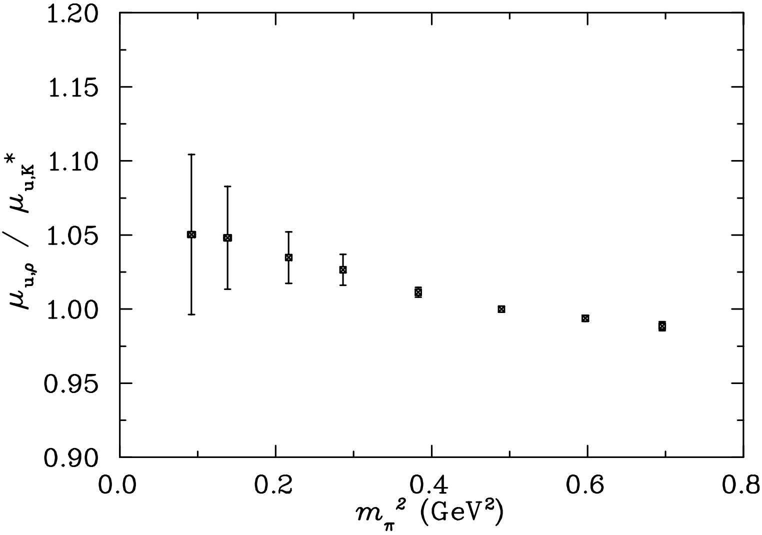
Quark Sector Contributions to the Magnetic Moment



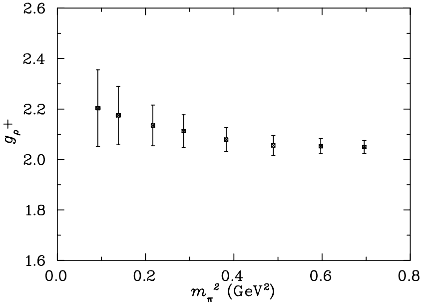
The Magnetic Moment



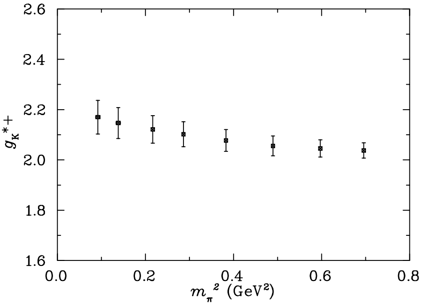
Environmental Sensitivity of the Magnetic Moment



Vector-Meson G-factor

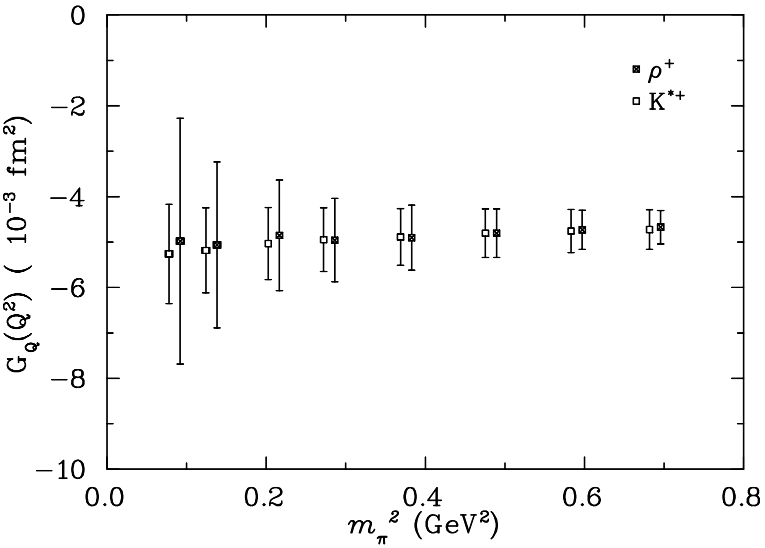


ρ-meson



K^*-meson

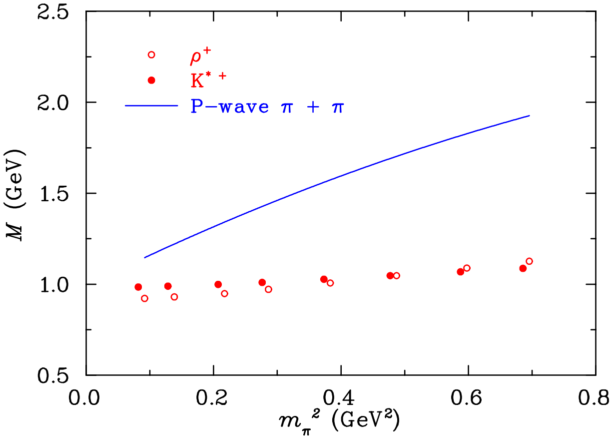
ρ-meson Quadrupole Form Factor



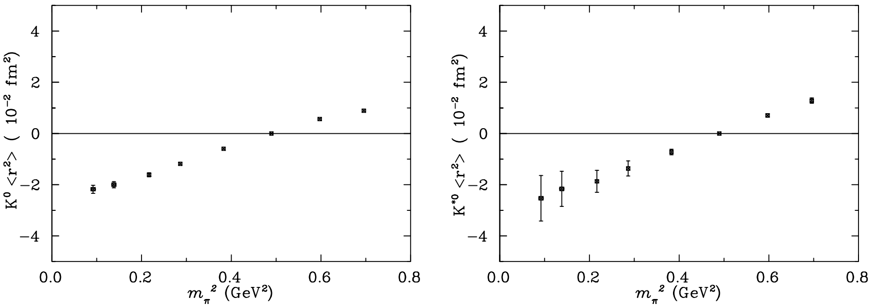
Conclusions

- Hyperfine repulsion in vector mesons in Quenched LQCD is significant.
- There is significant enviromental sensitivity in the up quark contributions to charge radii.
- The charge radius of the proton is similar to that of the ρ-meson, but consistently larger than that of the pion and kaon.
- The magnetic moment of the ρ-meson is consistent with the quark model prediction.
- Vector mesons oblate.

ρ and K^* bound in Quenched LQCD



Neutral Meson Charge Radii



Pseudoscalar mesons

Vector mesons

Neutral Meson Magnetic Moment