IMPROVED PERTURBATION THEORY FOR IMPROVED LATTICE ACTIONS

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ABSTRACT

We present a systematic improvement of perturbation theory for gauge fields on the lattice [1]; the improvement entails resumming, to all orders in the coupling constant, a dominant subclass of tadpole diagrams.

This method, originally proposed for the Wilson gluon action [2], is extended here to encompass all possible gluon actions made of closed Wilson loops; any fermion action can be employed as well.

The effect of resummation is to replace various parameters in the action (coupling constant, Symanzik/clover coefficient) by "dressed" values; the latter are solutions to certain coupled integral equations, which are easy to solve numerically.

Positive features of method:

- Gauge invariant
- Systematically applied at any given order in perturbation theory
- Absorbs in the dressed parameters the bulk of tadpole contributions

Applications:

- Additive renormalization of fermion masses
- Multiplicative renormalization Z_V (Z_A) of vector (axial) current

⋆ Improved perturbative results compare favorably with available non-perturbative estimates

I. INTRODUCTION

Since the earliest studies of quantum field theories on a lattice, it was recognized that quantities measured through numerical simulation are characterized by significant renormalization effects, which must be properly taken into account before meaningful comparisons to corresponding physical observables can be made.

As has been rigorously demonstrated [3], the renormalization procedure can be formally carried out in a systematic way to any given order in perturbation theory. However, calculations are notoriously difficult, as compared to continuum regularization schemes; furthermore, the convergence rate of the resulting asymptotic series is often unsatisfactory.

A number of approaches have been pursued in order to improve the behaviour of perturbation theory, among them Refs. [4,5]. These approaches share in common the aim to reorganize perturbative series in terms of an expansion coefficient which would be more suitable than the bare coupling constant g_0; the definition of such a "renormalized" coupling constant is not unique, but can depend on the observables under study and on an energy scale. It is expected that such a definition will reabsorb a large part of the tadpole contibutions which are known to dominate lattice perturbation theory.

Some years ago, a method was proposed to sum up a whole subclass of tadpole diagrams, dubbed **"cactus"** diagrams, to all orders in perturbation theory [2,6]; this procedure has a number of desirable features: It is gauge invariant, it can be systematically applied to improve (to all orders) results obtained at any given order in perturbation theory, and it does indeed absorb the bulk of tadpole contributions into an intricate redefinition of the coupling constant; in cases where non-perturbative estimates of renormalization coefficients are also available for comparison, the agreement with cactus improved perturbative results is significantly better as compared to results from bare perturbation theory.

In the present work [1] we extend the improved perturbation theory method of Refs. [2,6], to encompass the large class of actions which are used nowadays in simulations of QCD. This class includes Symanzik improved gluon actions with any arbitrary combination of closed Wilson loops, combined with any fermionic action.

In Section II we present our calculation, deriving expressions for a dressed gluon propagator, as well as for dressed gluon and fermion vertices, as a result of the summation of cactus diagrams to all orders. We show how these dressed constituents are employed to improve 1-loop and 2-loop Feynman diagrams coming from bare perturbation theory. In Section III we apply our improved renormalizaton procedure to a number of test cases involving Symanzik gluons and Wilson/clover/overlap fermions.

Clearly, all resummation procedures, whether in the continuum or on the lattice, bear a caveat: A one-sided resummation could ruin desirable partial cancellations which might exist among those diagrams which are resummed and others which are not; what is worse, the end result might depend on the gauge. As we shall see, no partial cancellations will be ruined in our procedure, due to the distinct N-dependence of the resummed diagrams (N is the number of colors); furthermore, our results will be gauge independent.

II. THE METHOD

In this Section, we start illustrating our method by showing how the gluon propagator is dressed by the inclusion of cactus diagrams. We will then dress gluon and fermion vertices as well. Finally, we will explain how this procedure is applied to Feynman diagrams at a given order in bare perturbation theory, concentrating on the 1- and 2-loop case.

Dressing the propagator

We consider the Symanzik improved gluon action (involving Wilson loops with up to 6 links) :

$$\begin{aligned} S_G = \frac{2}{g_0^2} \Big[& c_0 \sum_{\text{plaquette}} \mathrm{Re\,Tr}\,(1 - U_{\textbf{plaquette}}) \\ & + c_1 \sum_{\text{rectangle}} \mathrm{Re\,Tr}\,(1 - U_{\textbf{rectangle}}) \\ & + c_2 \sum_{\text{chair}} \mathrm{Re\,Tr}\,(1 - U_{\textbf{chair}}) \\ & + c_3 \sum_{\text{parallelogram}} \mathrm{Re\,Tr}\,(1 - U_{\textbf{parallelogram}}) \Big] \end{aligned} \tag{1}$$

The coefficients c_i satisfy the normalization condition

$$c_0 + 8c_1 + 16c_2 + 8c_3 = 1 \tag{2}$$

which ensures the correct classical continuum limit of the action.

U_i ($i = \mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}$) are products of link variables $U_{x,\mu}$ around the perimeter of the closed loop.

The links are parametrized in terms of the continuum gauge fields $A_\mu(x)$:

$$U_{x,\mu} = \exp\left(i\, g_0\, a\, A_\mu(x + a\hat{\mu}/2)\right)$$

$$A_\mu(x) = A^a_\mu(x)\, T^a,\ \mathrm{Tr}(T^a\, T^b) = \frac{1}{2}\,\delta^{ab}$$

Using the Baker-Campbell-Hausdorff (BCH) formula, U_i takes the form:

Definitions:

a : lattice spacing, (set to 1)

$\hat{\mu}$: unit vector in direction μ

T^a : $SU(N)$ generator

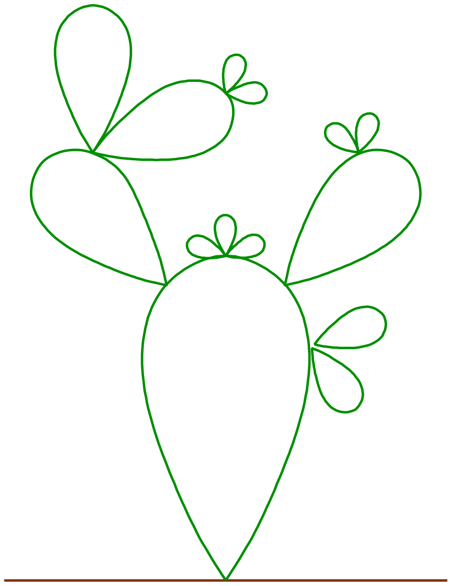
$F_i^{(1)}$: sum of gauge fields on the links of loop i ,

$F_i^{(j)}$ $(j > 1)$: j-th degree polynomials in the gauge fields, constructed from nested commutators

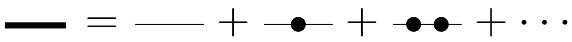
$$U_i = \exp\left(ig_0\, F_i^{(1)} + ig_0^2 F_i^{(2)} + ig_0^3 F_i^{(3)} + \mathcal{O}(g_0^4)\right)$$

Cactus Diagrams:

- tadpole diagrams which become disconnected if any one of their vertices is removed
- each vertex is constructed solely from the $F_i^{(1)}$ parts of the action.
- they dress the gluon propagator

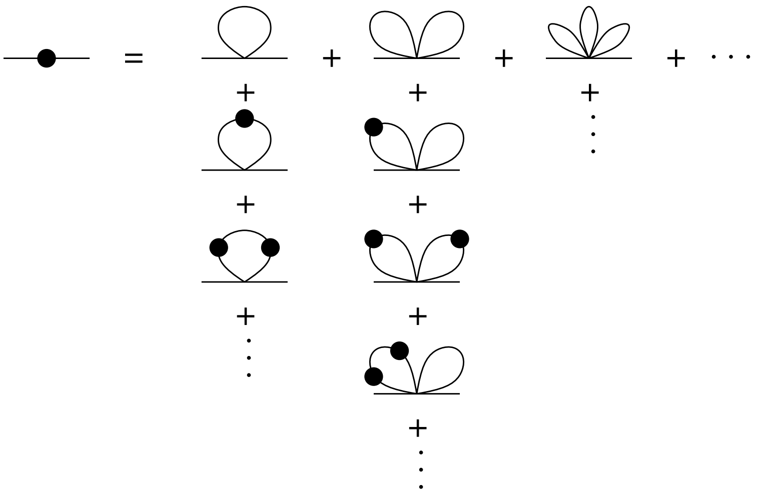


A diagrammatic equation for the dressed gluon propagator (——) in terms of the bare propagator (——) and 1-particle irreducible (1PI) vertices (●) reads:



(3)

The 1PI vertex obeys the following recursive equation:



(4)

The bare inverse gluon propagator D^{-1} results from the total gluon action,

$$D^{-1}_{\mu\nu}(k) = \sum_{\rho}\left(\hat{k}^2_\rho \delta_{\mu\nu} - \hat{k}_\mu \hat{k}_\rho \delta_{\rho\nu}\right) d_{\mu\rho} + \frac{1}{1-\xi}\,\hat{k}_\mu \hat{k}_\nu$$

$$\equiv \mathbf{c_0} G^{(0)}_{\mu\nu}(k) + \mathbf{c_1}\, G^{(1)}_{\mu\nu}(k)$$

$$+ \mathbf{c_2}\, G^{(2)}_{\mu\nu}(k) + \mathbf{c_3}\, G^{(3)}_{\mu\nu}(k) + \frac{1}{1-\xi}\,\hat{k}_\mu \hat{k}_\nu$$

<u>Definitions:</u>

$d_{\mu\nu} = (1-\delta_{\mu\nu})\left[C_0 - C_1\hat{k}^2 - C_2(\hat{k}^2_\mu + \hat{k}^2_\nu)\right]$, $\quad \hat{k}_\mu = 2\sin\frac{k_\mu}{2}$, $\quad \hat{k}^2 = \sum_\mu \hat{k}^2_\mu$

$C_0 = c_0 + 8c_1 + 16c_2 + 8c_3$, $\quad C_1 = c_2 + c_3$, $\quad C_2 = c_1 - c_2 - c_3$

ξ: gauge fixing parameter

$G^{(i)}(k)$ are symmetric and transverse matrices ($\sum_\nu G^{(i)}_{\mu\nu}(k)\,\hat{k}_\nu = 0$)

$G^{(i)}(k)$ originate from $\mathrm{Tr}(F^{(1)}_i\, F^{(1)}_i)$ term of the gluon action

⇒ diagrams on the r.h.s. of Eq. (4) are a linear combination of $G^{(i)}(k)$

This implies that the 1PI vertex $G^{\mathrm{1PI}}(k)$ can be written as:

$$\boxed{G^{\mathrm{1PI}}(k) = \alpha_0\, G^{(0)}(k) + \alpha_1\, G^{(1)}(k) + \alpha_2\, G^{(2)}(k) + \alpha_3\, G^{(3)}(k)}$$

$\alpha_i = \alpha_i(N, g_o, c_0, c_1, c_2, c_3)$, independent of the momentum

Eq. (3) leads to the dressed propagator $D^{\mathrm{dr}}(k)$:

$$D^{\mathrm{dr}} = D + D\, G^{\mathrm{1PI}}\, D + D\, G^{\mathrm{1PI}}\, D\, G^{\mathrm{1PI}}\, D + \cdots = D\left(\frac{\mathbb{1}}{\mathbb{1} - G^{\mathrm{1PI}}\, D}\right)$$

$$\Longrightarrow (D^{\mathrm{dr}})^{-1} = \tilde{\mathbf{c}}_0\ G^{(0)} + \tilde{\mathbf{c}}_1\ G^{(1)} + \tilde{\mathbf{c}}_2\ G^{(2)} + \tilde{\mathbf{c}}_3\ G^{(3)} + \frac{1}{1-\xi}\hat{k}_\mu \hat{k}_\nu$$

$$(\tilde{\mathbf{c}}_\mathbf{i} \equiv \mathbf{c}_\mathbf{i} - \alpha_\mathbf{i})$$

Dressing replaces the bare coefficients c_i with improved ones $\tilde{c}_i$, and leaves the longitudinal part intact (ensures gauge invariance)

In terms of the dressed propagator, Eq. (4) can be *drawn* as:

(5)

A typical diagram on the r.h.s. of Eq. (5):

- is a sum of 4 terms
- has $(n-2)/2$ 1-loop integrals (contraction of two powers of $F_i^{(1)}$ via a dressed propagator) and will contribute one power of $\beta_i(\tilde{c}_0, \tilde{c}_1, \tilde{c}_2, \tilde{c}_3)$, where:

$$\beta_0 = \int_{-\pi}^{\pi} \frac{d^4q}{(2\pi)^4} \left(2\,\hat{q}_\mu^2\, D^{\mathrm{dr}}_{\nu\nu}(q) - 2\,\hat{q}_\mu\,\hat{q}_\nu\, D^{\mathrm{dr}}_{\mu\nu}(q)\right) \tag{6}$$

$$\beta_1 = \int_{-\pi}^{\pi} \frac{d^4q}{(2\pi)^4} \Big((8 - \hat{q}_\nu^2 - \hat{q}_\mu^2)\hat{q}_\mu^2 D^{\mathrm{dr}}_{\nu\nu}(q) - 2\hat{q}_\mu \hat{q}_\nu (4 - \hat{q}_\nu^2) D^{\mathrm{dr}}_{\mu\nu}(q)\Big)$$

$$\beta_2 = \int_{-\pi}^{\pi} \frac{d^4q}{(2\pi)^4} \left(\hat{q}_\mu^2 (8 - \hat{q}_\nu^2)\, D^{\mathrm{dr}}_{\rho\rho}(q)/2 - \hat{q}_\mu\,\hat{q}_\rho (8 - \hat{q}_\nu^2)\, D^{\mathrm{dr}}_{\mu\rho}(q)/2\right)$$

$$\beta_3 = \int_{-\pi}^{\pi} \frac{d^4q}{(2\pi)^4} \left(3\,\hat{q}_\mu^2 (4 - \hat{q}_\nu^2)\, D^{\mathrm{dr}}_{\rho\rho}(q)/2 - 3\,\hat{q}_\mu\,\hat{q}_\nu\,(4 - \hat{q}_\rho^2)\, D^{\mathrm{dr}}_{\mu\nu}(q)/2\right)$$

- μ, ν, ρ assume distinct values; no summation implied
- β_i: gauge independent.

We define:

$$F(n;N) = \sum_{\substack{\text{complete pairwise}\\ \text{contractions}}} \mathrm{Tr}\{T^{a_1}T^{a_2}\ldots T^{a_n}\}$$

$$= \frac{1}{2^{n/2}(n/2)!}\sum_{P\in S_n}\delta_{a_1a_2}\delta_{a_3a_4}\ldots\delta_{a_{n-1}a_n}\,\mathrm{Tr}\left\{T^{P(a_1)}T^{P(a_2)}\ldots T^{P(a_n)}\right\}$$

Generating function $G(z;N)$:

$$G(z;N) \equiv \sum_{n=0}^{\infty}\frac{z^n}{n!}F(n;N) = e^{z^2(N-1)/(4N)}\,L^1_{N-1}(-z^2/2) \tag{7}$$

Upon contraction, an n-leg diagram in Eq. (5), with its vertex coming from the term U_i of the Langrangian, will merely result in the following multiple of $G^{(i)}$:

Additional Information

$F(2n+1;N) \equiv 0$

$L^{\alpha}_{\beta}(x)$: Laguerre polynomials

$F(n;N) = \frac{d^n}{dz^n}G(z;N)|_{z=0}$

$$\frac{c_i}{g_0^2}\,\frac{(i\,g_0)^n}{n!}\,\frac{n\,F(n;N)}{2(N^2-1)}\,4\,\beta_i^{(n-2)/2}\,G^{(i)}$$

Mathematical form of Eq. (5):

$$\boxed{\begin{aligned}&\alpha_0\;G^{(0)}+\alpha_1\;G^{(1)}+\alpha_2\;G^{(2)}+\alpha_3\;G^{(3)} =\\ &\qquad\sum_{i=0}^{3}\;\sum_{n=4,6,8,\ldots}^{\infty}\frac{c_i}{g_0^2}\,\frac{(i\,g_0)^n}{n!}\,\frac{n\,F(n;N)}{2(N^2-1)}4\beta_i^{(n-2)/2}\,G^{(i)} \qquad (8)\end{aligned}}$$

$G^{(i)}$: independent functions (for $c_2 = 0$) of the external momentum

$\Rightarrow$ Eq. (8) amounts to 4 equations for the 4 coefficients α_i

⋆ All combinatorial weights are correctly incorporated in Eq. (8)

Unknown quantities: Coefficients α_i appearing on l.h.s. of Eq. (8) and inside the integrals β_i

Combining: 4 Eqs. (8) and Eq. (7):

$$\frac{c_i-\alpha_i}{c_i}\,(N^2-1) = e^{-\beta_i\, g_0^2\,(N-1)/(4N)}\,\cdot$$
$$\left(\frac{N-1}{N}\,L^1_{N-1}(g_0^2\,\beta_i/2) + 2\,L^2_{N-2}(g_0^2\,\beta_i/2)\right) \qquad (9)$$

In solving Eqs. (9), each choice of values for $(c_i\,,\,g_0\,,\,N)$ leads to a set of values for $\tilde{c}_i \equiv c_i - \alpha_i$.

- **Problem:** $\tilde{c}_i$ do not satisfy Eq. (2) (no longer normalized)
- **Solution:** express results in terms of a normalized set of improved coefficients $\tilde{c}_i/\tilde{C}_0$ and an improved $\tilde{g}_0^2 = g_0^2/\tilde{C}_0$.

The rescaled quantities $\tilde{\gamma}_i$ must now satisfy the coupled equations:

$$\tilde{\gamma}_i = \frac{\gamma_i e^{-\tilde{\beta}_i(N-1)/(4N)}}{N^2-1}\left(\frac{N-1}{N}L^1_{N-1}(\tilde{\beta}_i/2) + 2L^2_{N-2}(\tilde{\beta}_i/2)\right) \qquad (10)$$

For gauge groups $SU(2)$ and $SU(3)$,

$$(N=2): \tilde{\gamma}_i = \gamma_i e^{-\tilde{\beta}_i/8}\left(1-\frac{\tilde{\beta}_i}{12}\right)$$

$$(N=3): \tilde{\gamma}_i = \gamma_i e^{-\tilde{\beta}_i/6}\left(1-\frac{\tilde{\beta}_i}{4}+\frac{\tilde{\beta}_i^2}{96}\right)$$

Definitions:

$\tilde{C}_0 = \tilde{c}_0 + 8\tilde{c}_1 + 16\tilde{c}_2 + 8\tilde{c}_3$

rescaled quantities:

$\gamma_i \equiv \frac{c_i}{g_0^2}$

$\tilde{\gamma}_i \equiv \frac{\tilde{c}_i}{g_0^2}$

$\tilde{\beta}_i(\tilde{c}_0,\tilde{c}_1,\tilde{c}_2,\tilde{c}_3) \equiv g_0^2\beta_i(\tilde{c}_0,\tilde{c}_1,\tilde{c}_2,\tilde{c}_3)$

A solution for $\tilde{\gamma}_i$ always exists for all physically interesting values of c_i, and for all values of g_0 well inside the strong coupling region.

Numerical solutions of Eq. (10) can be found using a fixed point procedure, applicable to equations of the type $x = f(x)$:

$$\tilde{\gamma}_i = f_i(\tilde{\gamma}_i) \;\Rightarrow\; \tilde{\gamma}_i = \lim_{m\to\infty} \tilde{\gamma}_i^{(m)},\ [\tilde{\gamma}_i^{(0)} = \gamma_i,\ \tilde{\gamma}_i^{(m+1)} = f_i(\tilde{\gamma}_i^{(m)})]$$

Convergence has been verified in a number of extreme cases.

Numerical values of improved coefficients

Plaquette action ($c_0=1,\ c_1=c_2=c_3=0$):

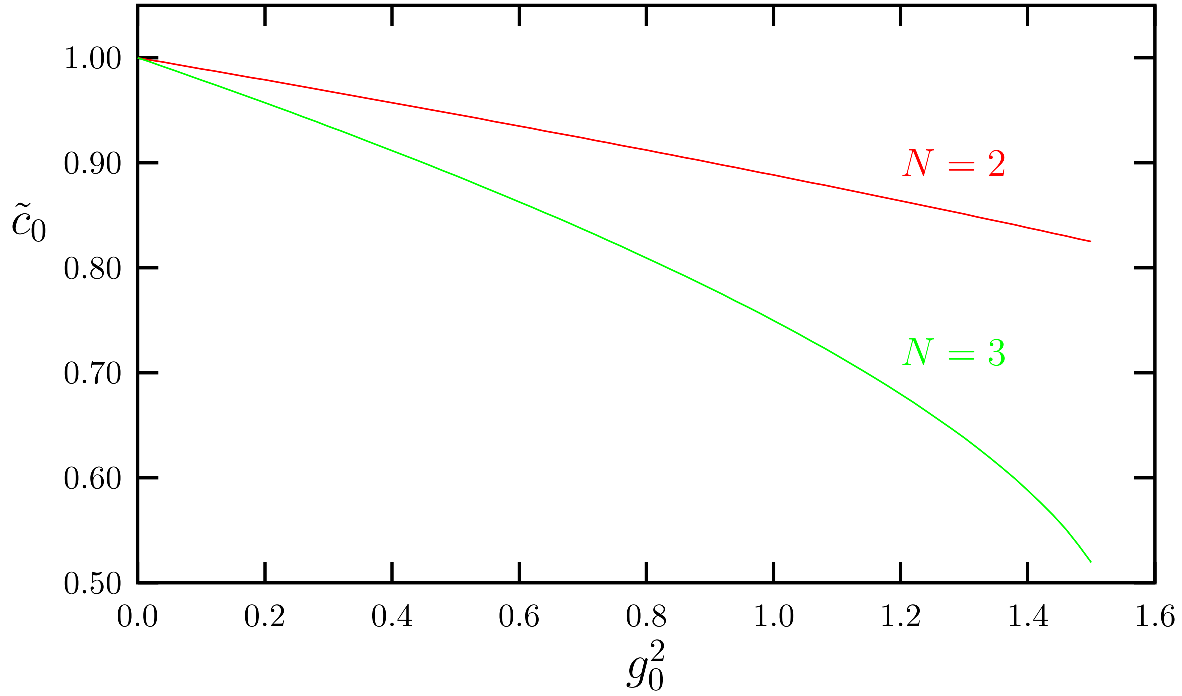


Fig.1: Improved coefficient $\tilde{c}_0$ for N=2 and N=3 (plaquette action)

Tree-level Symanzik action ($c_0=5/3,\ c_1=-1/12, c_2=c_3=0$):

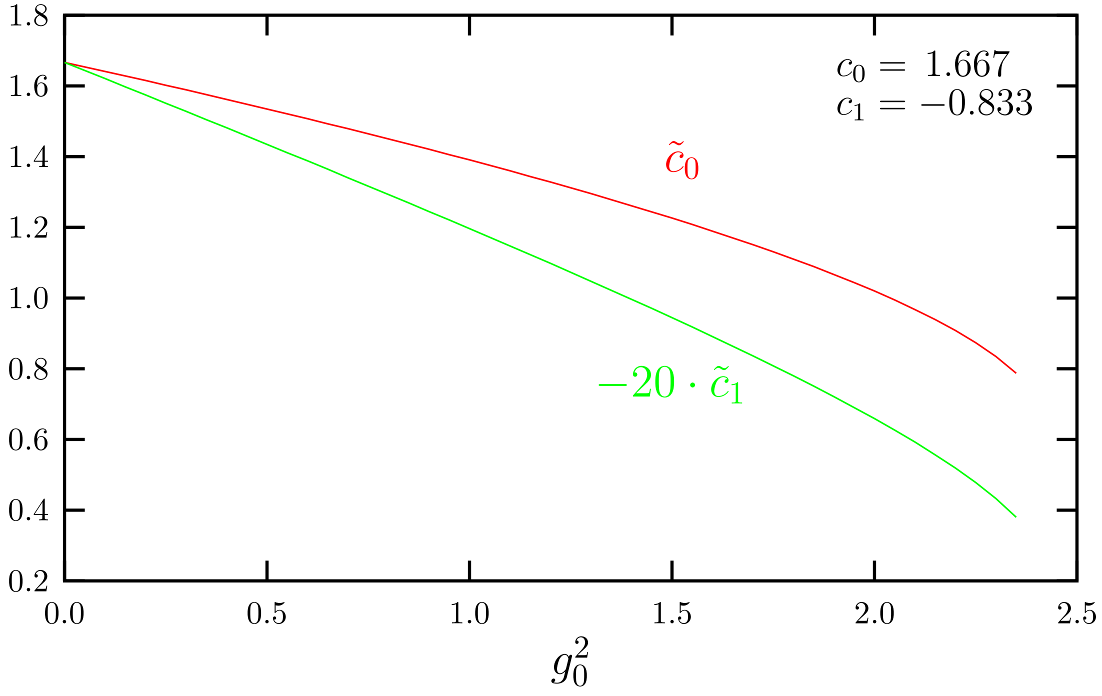


Fig.2: Improved coefficients $\tilde{c}_0$ and $\tilde{c}_1$ (tree-level Symanzik improved action, $N=3$)

Iwasaki action (c_0=3.648, c_1= − 0.331, c_2=c_3=0):

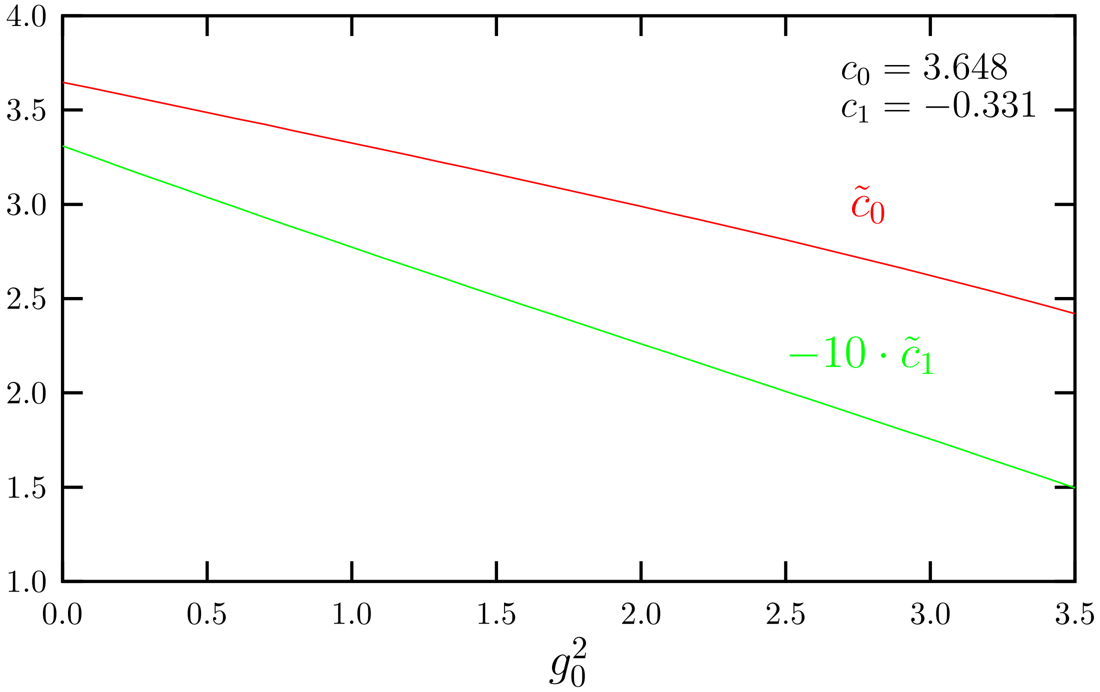


Fig.3: Improved coefficients $\tilde{c}_0$ and $\tilde{c}_1$ (Iwasaki action, $N = 3$)

Tadpole improved Lüscher-Weisz (TILW) actions (c_2=0):

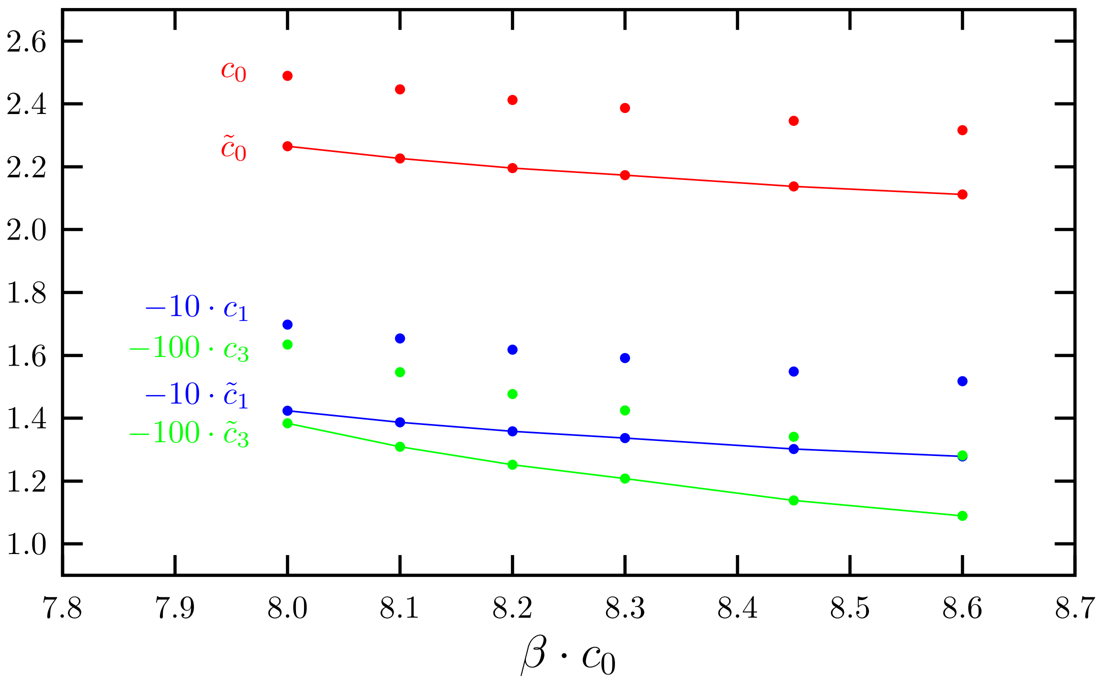


Fig.4: Coefficients c_0 , c_1 , c_3 and their dressed counterparts $\tilde{c}_0$, $\tilde{c}_1$, $\tilde{c}_3$ for different values of $\beta\, c_0 = 6\, c_0/g_0^2$ (TILW actions, $N = 3$)

DBW2 gluon action ($c_0{=}c_0(\beta),\ c_1{=}c_1(\beta),\ c_2{=}c_3{=}0$):

Some standard values for c_0 and c_1 (obtained *starting* from $\beta\, c_0 = 6.0$ and 6.3), as well as $\tilde{c}_0$ and $\tilde{c}_1$ are shown in the Table below.

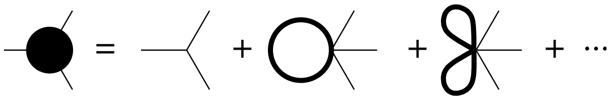
β	c_0	c_1	$\tilde{c}_0$	$\tilde{c}_1$
1.1636	5.29078	-0.53635	3.39826	-0.22528
0.6508	12.2688	-1.4086	8.8070	-0.7313

TABLE I. Improved coefficients $\tilde{c}_0$ and $\tilde{c}_1$ in the DBW2 action, for $\beta = 6/g_0^2 = 1.1636$ and 0.6508

Dressing vertices

3-gluon vertex
results from a Taylor expansion of U_i (coming from action Eq. (1)) to 3rd order in g_0. Only terms of the form $\mathrm{Tr}(F_i^{(1)}\, F_i^{(2)})$ will appear, since $\mathrm{Tr}(F_i^{(3)})$ and $\mathrm{Tr}((F_i^{(1)})^3)$ will vanish.

By analogy with Eq. (5), the dressed 3-gluon vertex equals:



(11)

A similar procedure leads from the bare 3-gluon vertex:

$$V_3 = \mathbf{c_0}\, V_3^{(0)} + \mathbf{c_1}\, V_3^{(1)} + \mathbf{c_2}\, V_3^{(2)} + \mathbf{c_3}\, V_3^{(3)}$$

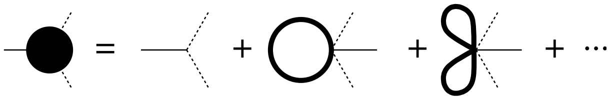
to the dressed one:

$$\begin{aligned} V_3^{\mathrm{dr}} &= \sum_{i=0}^{3} c_i \left(\sum_{l=0}^{\infty} \frac{(ig_0)^{2l+1}}{(2l{+}1)!} \frac{2}{(N^2{-}1)} F(2l{+}2; N) \beta_i^l \right) (ig_0)^{-1} V_3^{(i)} \\ &= \tilde{\mathbf{c}}_\mathbf{0}\, V_3^{(0)} + \tilde{\mathbf{c}}_\mathbf{1}\, V_3^{(1)} + \tilde{\mathbf{c}}_\mathbf{2}\, V_3^{(2)} + \tilde{\mathbf{c}}_\mathbf{3}\, V_3^{(3)} \end{aligned} \qquad (12)$$

3-point fermion-antifermion-gluon vertex

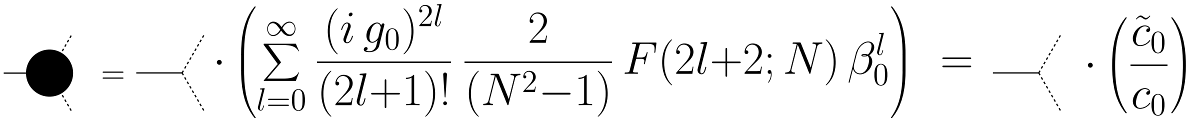
For Wilson and Overlap fermion actions (no Wilson loops) these vertices remain unaffected.

Clover action is amenable to improvement ⇒



(13)

(dotted line: fermions). Just as in Eq. (12), we find:



$$= \cdot \left(\sum_{l=0}^{\infty} \frac{(i\, g_0)^{2l}}{(2l+1)!} \, \frac{2}{(N^2-1)} \, F(2l+2; N)\, \beta_0^l \right) = \cdot \left(\frac{\tilde{c}_0}{c_0} \right)$$

4-gluon vertex

The BCH expansion of $\mathrm{Tr}(U_i)$ contributes to this vertex in the form:

$$\mathrm{Tr}((F_i^{(1)})^4),\ \mathrm{Tr}((F_i^{(2)})^2) \text{ and } \mathrm{Tr}(F_i^{(1)}\, F_i^{(3)})$$

(may dress differently from each other)

The dressed vertex produced from $\mathrm{Tr}((F_i^{(1)})^4)$ is *NOT* a multiple of its bare counterpart.This poses no problem: Such terms must be omitted while dressing 1- and 2-loop diagrams in typical cases, since their contribution is already included in dressing diagrams with one less loop.

Dressing the remaining terms, $\mathrm{Tr}((F_i^{(2)})^2)$ and $\mathrm{Tr}(F_i^{(1)}\, F_i^{(3)})$, amounts to replacing c_i by $\tilde{c}_i$.

⋆ The same considerations as above apply to all higher vertices from both the gluon and fermion actions as well.

The improvement procedure in a nutshell

The steps involved in the resummation of cactus diagrams can now be described quite succinctly:

- Substitute gluon propagators in Feynman diagrams by their dressed counterparts. The latter are obtained by the replacement $c_i \longrightarrow \tilde{c}_i = g_0^2\,\tilde{\gamma}_i$, where $\tilde{\gamma}_i$ are the solutions of Eqs. (10).
- Perform the same replacement, $c_i \longrightarrow \tilde{c}_i$, on the 3-gluon vertex.
- Account for dressing of the 3-point vertex from the clover action by adjusting the clover coefficient $c_{\rm SW}$: $c_{\rm SW} \longrightarrow c_{\rm SW} \cdot (\tilde{c}_0/c_0)$
- In dressing subleading-order diagrams, avoid double counting, i.e., subtract terms which were included in dressing leading-order diagrams. These are very easy to identify and subtract: Writing a general subleading-order result (aside from an overall prefactor) as: $a/N^2 + b + c\,N_f/N$ (N_f : number of fermion flavors), the quantity to subtract will include all of a/N^2 (because terms with BCH commutators are higher order in N), and it will be a multiple of $(2N^2-3)$; thus subtraction boils down to the substitution:

  $$a/N^2 + b + c\,N_f/N \longrightarrow \left(b + \frac{2}{3}\,a\right) + c\,N_f/N$$

  (see Refs. [2,7–9] for different applications of this). The remaining subleading vertices dress exactly as the propagators and 3-point vertices above.

III. APPLICATIONS

We turn now to two different applications of cactus improvement: The additive mass renormalization for clover fermions and the 1-loop renormalization of the axial and vector currents using the overlap action. Both cases employ Symanzik improved gluons; hence, our results are presented for various sets of Symanzik coefficients.

Critical mass of clover fermions

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An ultra-local discretization of the fermion action without doubling breaks chirality → we must demand a zero renormalized fermion mass, to ensure chiral symmetry while approaching the continuum limit. The bare mass is additively renormalized from its zero tree-level value to a critical value dm.

Calculations for dm presented here:

- employ clover fermions and Symanzik improved gluons
- 1-loop perturbation theory to obtain($dm_{1-\mathrm{loop}}$)
- the result is then dressed with cactus diagrams to arrive at the improved value $dm^{\mathrm{dr}}_{1-\mathrm{loop}}$.

⋆ The result can be written as a polynomial in the clover parameter

⋆ 1-loop results are independent of N_f (number of fermion flavors)

⋆ The improvement procedure requires us to choose definite values of g_0 and N

Numerical values of dm $(N = 3)$:

Plaquette action at $\beta = 6.0$

$$\begin{aligned} dm_{1-\mathrm{loop}} &= -0.434285489(1) + 0.1159547570(3)c_{\mathrm{SW}} \\ &+ 0.0482553833(1)c_{\mathrm{SW}}^2 \\ dm_{1-\mathrm{loop}}^{\mathrm{dr}} &= -0.579221119(2) + 0.1159547570(3)c_{\mathrm{SW}} \\ &+ 0.03618067788(9)c_{\mathrm{SW}}^2 \end{aligned} \tag{14}$$

in agreement with Ref. [8]

Iwasaki action at $\beta = 1.95$

$$\begin{aligned} dm_{1-\mathrm{loop}} &= -0.6773690760(3) + 0.2342165224(9)c_{\mathrm{SW}} \\ &+ 0.0806966864(3)c_{\mathrm{SW}}^2 \\ dm_{1-\mathrm{loop}}^{\mathrm{dr}} &= -0.757856451(1) + 0.1671007819(8)c_{\mathrm{SW}} \\ &+ 0.044746728234(1)c_{\mathrm{SW}}^2 \end{aligned} \tag{15}$$

dm can be used to evaluate the critical hopping parameter κ_{cr}:

$$\kappa_{\mathrm{cr}} \equiv \frac{1}{2\,dm + 8\,r} \qquad (r\text{: Wilson parameter})$$

Perturbative (unimproved and dressed) and non-perturbative results are listed in Table II for specific values of c_{SW}. It is clear that cactus dressing leads to results for κ_{cr} which are much closer to values obtained from simulations.

Action	N_f	β	c_{SW}	$\kappa_{\mathrm{cr,1-loop}}$	$\kappa_{\mathrm{cr,1-loop}}^{\mathrm{dr}}$	$\kappa_{\mathrm{cr}}^{\mathrm{non-pert}}$
Plaquette	0	6.00	1.479	0.1301	0.1362	0.1392 [10,11]
Plaquette	0	6.00	1.769	0.1275	0.1337	0.1353 [10,11]
Plaquette	2	5.29	1.9192	0.1262	0.1353	0.1373 [12,13]
Iwasaki	2	1.95	1.53	0.1292	0.1388	0.1421 [14]

TABLE II. 1-loop results and non-perturbative values for κ_{cr}

One-loop renormalization of fermionic currents

We investigate the renormalization constant Z_V (Z_A) of the flavor non-singlet vector (axial) current.

Calculations for Z_V, Z_A presented here:

- employ Overlap fermions and Symanzik improved gluons
- 1-loop perturbation theory
- bare results are dressed using cactus improvement method

For the overlap action $Z_V = Z_A$ [15] and in the $\overline{MS}$ scheme:

$$Z_{V,A}(a,p) = 1 - g_0^2\, z_{1V,A}$$

Using cactus improvement

$$Z^{\rm dr}_{V,A}(a,p) = 1 - g_0^2\; z^{\rm dr}_{1V,A}$$

To compute of $z^{dr}_{1V,A}$ one must dress the Symanzik coefficients and the propagators as described in Section II.

Action	$\beta{=}6/g_0^2$	$Z_{V,A}(\rho{=}1.0)$	$Z^{\rm dr}_{V,A}(\rho{=}1.0)$	$Z_{V,A}(\rho{=}1.4)$	$Z^{\rm dr}_{V,A}(\rho{=}1.4)$
Plaquette	6.00	1.26427	1.35247	1.14707	1.19615
Symanzik	5.00	1.24502	1.29231	1.13574	1.16207
Symanzik	5.07	1.24164	1.28735	1.13386	1.15932
Symanzik	6.00	1.20418	1.23484	1.11311	1.13019
TILW	3.7120	1.27581	1.31941	1.15259	1.17690
TILW	3.6018	1.28223	1.32764	1.15613	1.18146
TILW	3.4772	1.28946	1.33677	1.16012	1.18651
TILW	3.3985	1.29434	1.34298	1.16282	1.18995
TILW	3.3107	1.29973	1.34972	1.16579	1.19369
TILW	3.2139	1.30569	1.35705	1.16908	1.19774
Iwasaki	1.95	1.39343	1.44921	1.21724	1.24847
Iwasaki	2.20	1.34872	1.38773	1.19256	1.21440
Iwasaki	2.60	1.29507	1.31940	1.16293	1.17656
DBW2	0.6508	1.49631	1.45362	1.27543	1.25057

TABLE III. Results for $Z_{V,A}, Z^{\rm dr}_{V,A}$, using $\rho{=}1.0, \rho{=}1.4$ (ρ: overlap parameter)

Improvement is more apparent for the case of the plaquette action, while the effect of dressing is smaller for improved gluon actions. This was to be expected, since the latter actions were constructed in a way as to reduce lattice artifacts, in the first place.

The dependence of $Z_{V,A}$ and $Z^{\rm dr}_{V,A}$ on the overlap parameter ρ is more clearly shown in Fig.5, where we plot our results for three actions: Plaquette, Iwasaki and TILW.

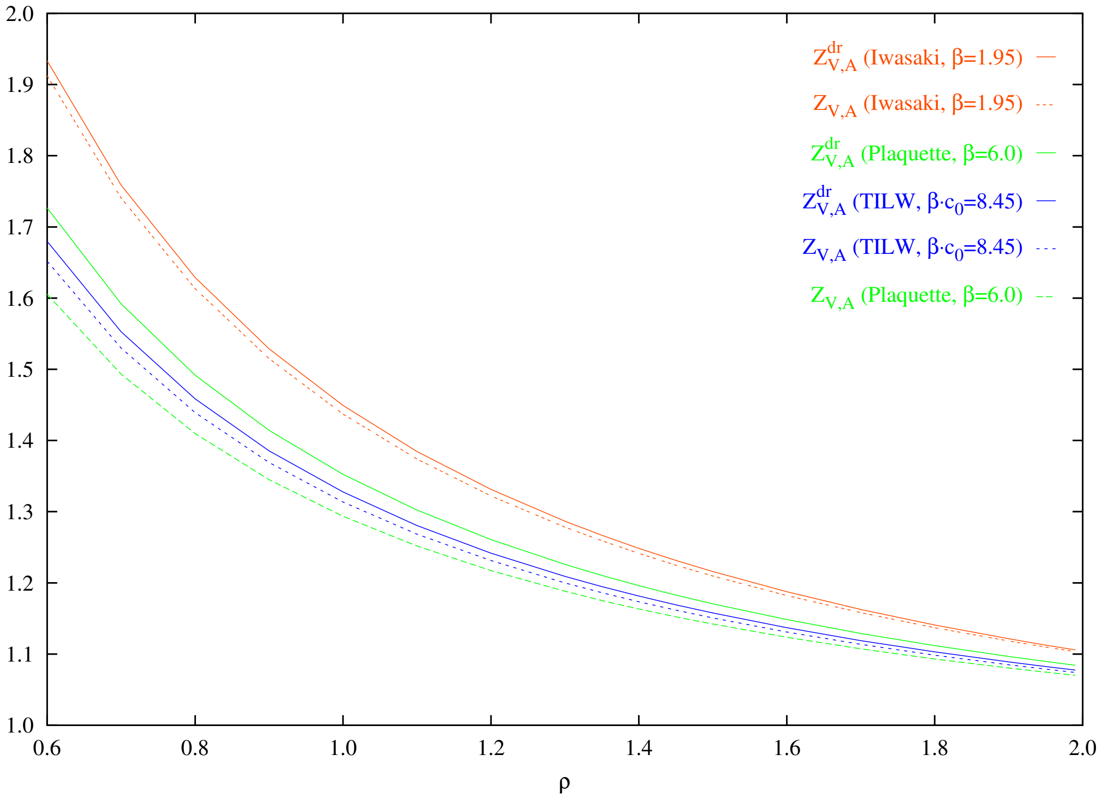

Fig.5: Plots of $Z_{V,A}$ and $Z^{\rm dr}_{V,A}$ for the plaquette, Iwasaki and TILW actions.

A comparison between our improved $Z_{V,A}$ values and some non-perturbative estimates [16], shows that improvement moves in the right direction. Cactus dressing had already been tested using clover fermions [6], and it turns out to be as good as standard tadpole improvement [5], but still not very close to non-perturbative results.

IV. CONCLUSIONS

- Resummation of cactus diagrams is readily applicable to any observable in lattice gauge theories
- This procedure for improving bare perturbation theory is gauge invariant
- It can be applied in a systematic fashion to improve (to all orders) results obtained at any given order in perturbation theory.

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