One-loop Renormalisation of Lattice QCD Operators for Non-forward Matrix Elements: From Clover to Overlap Fermions

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Abstract: We consider the renormalisation of composite quark-antiquark operators with one and two lattice covariant derivatives related to the lowest moments of generalised parton distributions (GPDs) and meson distribution amplitudes (DAs). Their matrix elements are calculated in one-loop lattice perturbation theory for non-zero momentum transfer from the initial to the final state. We present the matrices of renormalisation factors for the appearing operators using clover and overlap fermions which mix between the used representations of the hypercubic group. For overlap fermions we explicitly check the absence of mixing to lower-dimensional operators of different chirality in particular representations. This favours the use of chiral fermions in future simulations related to GPDs and DAs.

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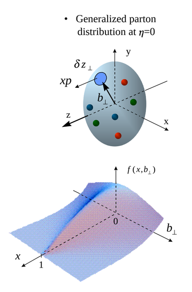
Introduction – GPDs

Generalised parton distributions (GPDs) contain more information about the hadron structure than the usual structure functions: transverse structure, orbital angular momentum carried by quarks and gluons,. . .

GPDs unify parametrisations for large class of hadronic correlators, e.g. form factors and distribution functions combine inclusive, semi-inclusive and exclusive processes



well-defined QCD objects studied in perturbation theory (Geyer, Müller, Robaschik,. . . , Ji, Radyushkin,. . .)
Limited experimental access, e.g.:
$ep \longrightarrow ep\gamma$, $ep \longrightarrow ep\pi^+\pi^-$

Need complementary information from lattice QCD (first results from QCDSF and LHPC/SESAM)

Relate lattice results to continuum by renormalisation

GPDs and non-forward matrix elements

Use local composite operators with twist 2

$(\cdots)$: index symmetrisation and trace subtraction

$$\mathcal{O}_{\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar\psi\,\gamma_{(\mu_1} \overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n)}\psi$$

Calculate non-forward matrix elements ($\Delta = p - p'$, $\overline{P} = \frac{p+p'}{2}$)

$$\langle p'|\mathcal{O}_{\mu_1\cdots\mu_n}|p\rangle = \bar\psi(p')\gamma_{(\mu_1}\psi(p)\sum_{i=0}^{\left[\frac{n-1}{2}\right]} A_{n,2i}(\Delta^2)\Delta_{\mu_2}\cdots\Delta_{\mu_{2i+1}}\overline{P}_{\mu_{2i+2}}\cdots\overline{P}_{\mu_n)}$$
$$-\frac{1}{2M}\bar\psi(p')\,i\Delta^\alpha\sigma_{\alpha(\mu_1}\psi(p)\sum_{i=0}^{\left[\frac{n-1}{2}\right]} B_{n,2i}(\Delta^2)\Delta_{\mu_2}\cdots\Delta_{\mu_{2i+1}}\overline{P}_{\mu_{2i+2}}\cdots\overline{P}_{\mu_n)}$$
$$+C_n(\Delta^2)\mathrm{Mod}(n+1,2)\frac{1}{M}\bar\psi(p')\psi(p)\Delta_{(\mu_1}\cdots\Delta_{\mu_n)}$$

Generalised form factors A, B, C related to moments of GPDs, e.g. $H(x,\xi,\Delta^2)$ ($\xi = -n\cdot\Delta$, $n\cdot\overline{p} = 1$):

$$\int_{-1}^1 dx\, x^{n-1} H(x,\xi,\Delta^2) = \sum_{i=0}^{\left[\frac{n-1}{2}\right]} A_{n,2i}(\Delta^2)(-2\xi)^{2i} + \mathrm{Mod}(n+1,2)C_n(\Delta^2)(-2\xi)^n$$

Ordinary quark distribution: $q(x) = \lim_{\xi\to 0,\Delta^2\to 0} H(x,\xi,\Delta^2)$

Renormalisation and mixing

Well known: operators in higher moments (two or more D's) mix: One-loop structure differs from Born (tree) structure
The mixing operators belong to the same irreducible representation of $H(4)$ and have identical charge conjugation parity[1]
GPDs:
First moment (one D): no mixing, renormalisation as in forward case
Second moment (two D's:) additional mixing with operators containing external ordinary derivatives ∂'s new compared to forward case

- Renormalisation matrix Z^S in scheme S:

$$\mathcal{O}_i^{\mathcal{S}}(\mu) = \sum_k Z_{ik}^S(a,\mu)\,\mathcal{O}_k(a)$$

- $Z^{\mathcal{S}}$ in MOM scheme (Λ_i amputated Green function of $\mathcal{O}_i$)

$$\frac{\sum_k Z_{ik}^{\mathrm{MOM}}(a,\mu)\,\Lambda_k}{Z_\psi^{\mathrm{MOM}}(a,\mu)}\Big|_{p^2=\mu^2} = \Lambda_i^{\mathrm{tree}} + \text{ other Dirac structures}$$

- Conversion to the $\overline{\mathrm{MS}}$ scheme:

$$Z_{ik}^{\overline{\mathrm{MS}}}(a,\mu) = Z_{ik}^{\overline{\mathrm{MS}},\mathrm{MOM}} Z_{ik}^{\mathrm{MOM}}(a,\mu) \quad \text{no summation}$$

$Z^{\overline{\mathrm{MS}},\mathrm{MOM}} \longleftarrow$ continuum perturbation theory

[1]M. Göckeler *et al.*, PR **D54** (1996) 5705

Operator choice and Feynman rules

Consider non-forward matrix elements between off-shell quark states of operators

$$\mathcal{O}^{DD}_{\mu\nu\omega} = -\frac{1}{4}\overline{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\omega\psi \qquad \mathcal{O}^{\partial\partial}_{\mu\nu\omega} = -\frac{1}{4}\partial_\nu\partial_\omega\left(\overline{\psi}\gamma_\mu\psi\right)$$
$$\mathcal{O}^{\partial D}_{\mu\nu\omega} = -\frac{1}{4}\partial_\nu\left(\overline{\psi}\gamma_\mu \overleftrightarrow{D}_\omega\psi\right) \qquad \mathcal{O}^{\partial}_{\mu\nu\omega} = -\frac{i}{2}\partial_\nu\left(\overline{\psi}\left[\gamma_\mu,\gamma_\omega\right]\psi\right)$$

In addition spin-dependent and "transversity" operators: $\gamma_\mu \longrightarrow \gamma_\mu\gamma_5$, $\gamma_\mu \longrightarrow \sigma_{\mu\tau}$
Operators $\mathcal{O}^{\partial D}$ and $\mathcal{O}^{\partial}_{\mu\nu\omega}$ absent in continuum
Lattice operators at $q \neq 0$:

$$\left(\bar\psi \overleftrightarrow{D}_\mu \psi\right)(q) = \frac{1}{2a}\sum_x \times$$
$$\left[\bar\psi(x)U_{x,\mu}\psi(x+a\hat\mu) - \bar\psi(x+a\hat\mu)U^\dagger_{x,\mu}\psi(x)\right]\begin{Bmatrix} e^{iq\cdot x} + e^{iq\cdot(x+a\hat\mu)} \\ 2e^{iq\cdot(x+a\hat\mu/2)}\end{Bmatrix}$$

Feynman rule example $O(g^0)$
$P = (p+p')/2$, $q = p' - p$

$$\mathcal{O}^{DD}_{\mu\nu\omega}(p',p) = \bar\psi(p')\gamma_\mu\psi(p)\,\frac{1}{a}\sin(aP_\nu)\,\frac{1}{a}\sin(aP_\omega)\begin{Bmatrix}\cos\frac{aq_\nu}{2}\cos\frac{aq_\omega}{2} \\ 1\end{Bmatrix}$$

Mixing for 2nd moment of GPDs (and DAs)

Define index combinations

$$\mathcal{O}_{\{\nu_1\nu_2\nu_3\}} = \frac{1}{6}\left(\mathcal{O}_{\nu_1\nu_2\nu_3} + \mathcal{O}_{\nu_1\nu_3\nu_2} + \mathcal{O}_{\nu_2\nu_1\nu_3} + \mathcal{O}_{\nu_2\nu_3\nu_1} + \mathcal{O}_{\nu_3\nu_1\nu_2} + \mathcal{O}_{\nu_3\nu_2\nu_1}\right)$$
$$\mathcal{O}_{\|\nu_1\nu_2\nu_3\|} = \mathcal{O}_{\nu_1\nu_2\nu_3} - \mathcal{O}_{\nu_1\nu_3\nu_2} + \mathcal{O}_{\nu_3\nu_1\nu_2} - \mathcal{O}_{\nu_3\nu_2\nu_1} - 2\,\mathcal{O}_{\nu_2\nu_3\nu_1} + 2\,\mathcal{O}_{\nu_2\nu_1\nu_3}$$
$$\mathcal{O}_{\langle\langle\nu_1\nu_2\nu_3\rangle\rangle} = \mathcal{O}_{\nu_1\nu_2\nu_3} + \mathcal{O}_{\nu_1\nu_3\nu_2} - \mathcal{O}_{\nu_3\nu_1\nu_2} - \mathcal{O}_{\nu_3\nu_2\nu_1}$$

Classify operators according to irrep $\tau_k^{(l)}$ and given charge conjugation parity C (l dimension, k labels inequivalent irreps of same l)

$\tau_2^{(4)}$, $C = -1$

$$\mathcal{O}^{DD}_{\{124\}} \quad \mathcal{O}^{\partial\partial}_{\{124\}}$$

$\tau_1^{(8)}$, $C = -1$

$$\mathcal{O}_1 = \mathcal{O}^{DD}_{\{114\}} - \frac{1}{2}\left(\mathcal{O}^{DD}_{\{224\}} + \mathcal{O}^{DD}_{\{334\}}\right) \quad \mathcal{O}_2 = \mathcal{O}^{\partial\partial}_{\{114\}} - \frac{1}{2}\left(\mathcal{O}^{\partial\partial}_{\{224\}} + \mathcal{O}^{\partial\partial}_{\{334\}}\right)$$
$$\mathcal{O}_3 = \mathcal{O}^{DD}_{\langle\langle 114\rangle\rangle} - \frac{1}{2}\left(\mathcal{O}^{DD}_{\langle\langle 224\rangle\rangle} + \mathcal{O}^{DD}_{\langle\langle 334\rangle\rangle}\right) \quad \mathcal{O}_4 = \mathcal{O}^{\partial\partial}_{\langle\langle 114\rangle\rangle} - \frac{1}{2}\left(\mathcal{O}^{\partial\partial}_{\langle\langle 224\rangle\rangle} + \mathcal{O}^{\partial\partial}_{\langle\langle 334\rangle\rangle}\right)$$
$$\mathcal{O}_5 = \mathcal{O}^{\partial D,5}_{||213||} \qquad \mathcal{O}_6 = \mathcal{O}^{\partial D,5}_{\langle\langle 213\rangle\rangle}$$

An additional operator is zero in one-loop
Operator of lower dimension with different chirality:

$$\mathcal{O}_8 = \mathcal{O}^{\partial}_{114} - \frac{1}{2}\left(\mathcal{O}^{\partial}_{224} + \mathcal{O}^{\partial}_{334}\right)$$

One-loop results I

Form of renormalisation matrix

$$Z_{ik} = \delta_{ik} - \frac{g_R^2 C_F}{16\pi^2}\left(\gamma_{ik}\ln(a^2\mu^2) + B_{ik}\right)$$

$\tau_2^{(4)}$, $C = -1$

$$\gamma = \begin{pmatrix} \frac{25}{6} & -\frac{5}{6} \\ 0 & 0 \end{pmatrix}$$

$$B^{\mathrm{clover}} = \begin{pmatrix} -11.563 + 2.898c_{sw} - 0.984c_{sw}^2 & 0.024 - 0.255c_{sw} - 0.016c_{sw}^2 \\ 0 & 20.618 + 4.746c_{sw} - 0.543c_{sw}^2 \end{pmatrix}$$

$$B^{\mathrm{overlap}} = \begin{pmatrix} -47.4441 & -0.95719 \\ 0 & -17.418 \end{pmatrix}$$

$\tau_1^{(8)}$, $C = -1$

$$\gamma = \begin{pmatrix} \frac{25}{6} & -\frac{5}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{7}{6} & -\frac{5}{6} & 1 & -\frac{3}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 0 & -\frac{2}{3} & \frac{2}{3} \end{pmatrix}$$

One-loop results II

$$B^{\mathrm{clover}} = B^{(0)} + c_{sw}\,B^{(1)} + c_{sw}^2\,B^{(2)}$$

$$B^{(0)} = \begin{pmatrix} -12.127 & \begin{pmatrix} 1.491 \\ -2.737 \end{pmatrix} & 0.368 & \begin{pmatrix} 0.015 \\ 0.994 \end{pmatrix} & 0.016 & 0.150 \\ 0 & 20.618 & 0 & 0 & 0 & 0 \\ 3.306 & \begin{pmatrix} -8.015 \\ 18.184 \end{pmatrix} & -14.852 & \begin{pmatrix} 4.431 \\ -4.302 \end{pmatrix} & -0.928 & 0.738 \\ 0 & 0 & 0 & 20.618 & 0 & 0 \\ 0 & 3.264 & 0 & 0 & 0.350 & 0.015 \\ 0 & 3.264 & 0 & 0 & 0.005 & 0.360 \end{pmatrix}$$

$$B^{(1)} = \begin{pmatrix} 2.922 & \begin{pmatrix} -0.213 \\ -0.686 \end{pmatrix} & -0.033 & \begin{pmatrix} 0.015 \\ 0.173 \end{pmatrix} & -0.019 & 0.057 \\ 0 & 4.746 & 0 & 0 & 0 & 0 \\ 0.333 & \begin{pmatrix} -0.766 \\ -0.055 \end{pmatrix} & 2.152 & \begin{pmatrix} 1.206 \\ 0.970 \end{pmatrix} & -1.758 & 2.298 \\ 0 & 0 & 0 & 4.746 & 0 & 0 \\ 0 & -1.441 & 0 & 0 & 1.648 & 0.866 \\ 0 & -1.441 & 0 & 0 & 0.289 & 2.225 \end{pmatrix}$$

$$B^{(2)} = \begin{pmatrix} -0.982 & \begin{pmatrix} -0.078 \\ -0.101 \end{pmatrix} & -0.029 & \begin{pmatrix} 0.035 \\ 0.042 \end{pmatrix} & -0.001 & 0.007 \\ 0 & -0.543 & 0 & 0 & 0 & 0 \\ 0.371 & \begin{pmatrix} -0.551 \\ 0.215 \end{pmatrix} & -1.707 & \begin{pmatrix} 0.371 \\ 0.116 \end{pmatrix} & -0.443 & 0.103 \\ 0 & 0 & 0 & -0.543 & 0 & 0 \\ 0 & 1.416 & 0 & 0 & -1.703 & 0.568 \\ 0 & 1.416 & 0 & 0 & 0.189 & -1.325 \end{pmatrix}$$

One-loop results III

B^{overlap} (one realisation of the non-forward derivative)

$$\begin{pmatrix} -48.1089 & -3.43393 & 0.53795 & 1.33526 & 0.03132 & 0.64459 \\ 0 & -17.4180 & 0 & 0 & 0 & 0 \\ 3.31602 & 20.2671 & -46.8416 & -10.9597 & -3.52389 & 1.79504 \\ 0 & 0 & 0 & -17.41796 & 0 & 0 \\ 0 & -12.2181 & 0 & 0 & -34.1678 & 1.91865 \\ 0 & -12.2181 & 0 & 0 & 0.6396 & -32.8888 \end{pmatrix}$$

$\{\mathcal{O}_1, \mathcal{O}_8\}$ mixing problem

In one-loop $1/a$ contributions to the matrix element of operator $\mathcal{O}^{DD}_{\mu\nu\omega}$
$\tau_2^{(4)}$:
$1/a$ mixing absent for clover and overlap fermions
$\tau_1^{(8)}$:
Clover fermion: Mixing of operator $\mathcal{O}_1$ with the lower dimensional operator of opposite chirality $\mathcal{O}_8$
One-loop result (multiplicative mixing)

$$\mathcal{O}_1|_{1/a-\mathrm{part}} = \frac{g_R^2 C_F}{16\pi^2}(-0.518 + 0.0832\,c_{sw} - 0.00983\,c_{sw}^2)\frac{1}{a}\mathcal{O}_8^{\mathrm{tree}}$$

Need nonperturbative subtraction from matrix element of $\mathcal{O}_1$

Overlap fermions: $1/a$ mixing absent!

Summary & outlook

- First steps made to renormalise GPDs and DAs in a systematic way
- Complete results for Wilson fermions, see[2]
- 1-link operators: results as in forward case
- 2-link operators: One-loop Z-factors obtained for clover[3] and overlap[4] fermions
  Results concerning mixing are valid in general
  Tadpole/mean field improvement possible (not discussed here)
- Mixing is more complicated than for forward matrix elements
- Small mixing for $\tau_2^{(4)}$ (three different indices)
  No contributions from lower dimensional operators in one loop
- Mixing sizeable for $\tau_1^{(8)}$ (two indices equal)
  Clover: Additional mixing with a lower dimensional operator, breakdown of perturbation theory
  Overlap: No mixing
  favours the use of chiral fermions in future simulations related to GPDs and DAs
- Adding of improved gauge actions to overlap fermions
- Is the 3rd moment in the nonforward case feasible having completely free Lorentz indices?

[2]M. Göckeler *et al.*, NP B717 (2005) 304
[3]M. Göckeler *et al.*, hep-lat/0605002
[4]M. Göckeler *et al.*, in preparation