Update on the Physics of Light Pseudoscalar Mesons

THE MILC COLLABORATION

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1. Motivation

This poster presents an update to our previously published work [1, 2, 3] on the masses and decay constants of the π-K system with improved staggered quarks [4].

Such calculations make possible:

- A sensitive check of algorithms and methods — including the $\sqrt[4]{\mathrm{Det}}$ trick for dynamical staggered quarks — by comparing lattice results for f_π to the well-determined experimental value.
- A precise extraction of the CKM matrix element V_{us} from f_K or f_K/f_π, competitive with the world-average from alternative methods.
- A determination of the light quark masses and their ratios with high precision.
- A calculation of the physical coefficients of the $\mathcal{O}(p^4)$ chiral Lagrangian (the Gasser-Leutwyler L_i).
- A test of the applicability of staggered chiral perturbation theory (SχPT) [5, 6, 7] for describing lattice data.
- A determination of the extra, unphysical parameters that enter SχPT. This allows, *e.g.*, use of heavy-light SχPT [8] for computations of decay constants and form factors in D and B systems [9] — without introducing large additional uncertainties.

2. MILC Gauge Configurations

- The MILC Collaboration has generated an extensive set of gauge configurations with three flavors of improved staggered quarks: $m'_u = m'_d \equiv \hat{m}'$, and m'_s. (Primes indicate simulation values; corresponding masses without primes are the physical values. Masses of valence quarks are denoted by m_x and m_y.)
- Configurations are referred to as:
  - $a \approx 0.15$ fm = "coarser"
  - $a \approx 0.12$ fm = "coarse"
  - $a \approx 0.09$ fm = "fine"
  - $a \approx 0.06$ fm = "super-fine" [in progress]
  - There are also "extra coarse" configurations: $a \approx 0.18$ fm, but discretization errors are too large for them to be used in the π-K analysis.
- The simulation strange quark masses m'_s are in the range $0.70 m_s \lesssim m'_s \lesssim 1.2 m_s$.
- The lowest pion mass on the coarser, coarse and fine lattices is $m_\pi \approx 240\,\mathrm{MeV}$ ($m_{u,d} \sim 11\,\mathrm{MeV}$). A run on the super-fine lattices with $m_\pi \approx 430\,\mathrm{MeV}$ is half-finished, and lighter-mass runs are starting.
- The physical volumes of the lattices range from $\approx (2.4\,\mathrm{fm})^3$ to $\approx (3.4\,\mathrm{fm})^3$, with the large volumes being used for runs with the lightest quark masses.
- The lattice spacing is kept approximately fixed within each ensemble (coarser, coarse, fine and super-fine) as the light quark mass is varied using the length r_1 [10, 11] from the static quark potential to set the scale. The absolute scale is set from the Υ $2S$-$1S$ splitting determined by the HPQCD Collaboration [12, 13] on most of our lattices $\Rightarrow r_1 = 0.318(7)$ fm.

3. Data Subsets

- To get good fits to SχPT forms, we need to place upper limits on quark masses. We consider two data subsets:
  - Subset I:
    - Valence Goldstone pion masses $\lesssim 350\mathrm{MeV}$.
    - Other pion masses $\lesssim 550\mathrm{MeV}$ (coarse), $\lesssim 460\mathrm{MeV}$ (fine), $\lesssim 400\mathrm{MeV}$ (super-fine)
    - The largest sea quark masses and coarser runs are eliminated.
    - 122 data points are used.
    - This set is used to determine the L_i (and systematic errors on other quantities).
  - Subset II:
    - Valence Goldstone masses $\lesssim 570\mathrm{MeV}$.
    - Other pion masses $\lesssim 780\mathrm{MeV}$ (coarser), $\lesssim 725\mathrm{MeV}$ (coarse), $\lesssim 630\mathrm{MeV}$ (fine), $\lesssim 590\mathrm{MeV}$ (super-fine)
    - All sea quark masses and coarser runs are included.
    - 978 data points are used.
    - This data set is not chiral, but is used to interpolate around physical m_s, with LO and NLO params fixed from Subset I fits.
    - This subset is used for central values of decay constants and quark masses.

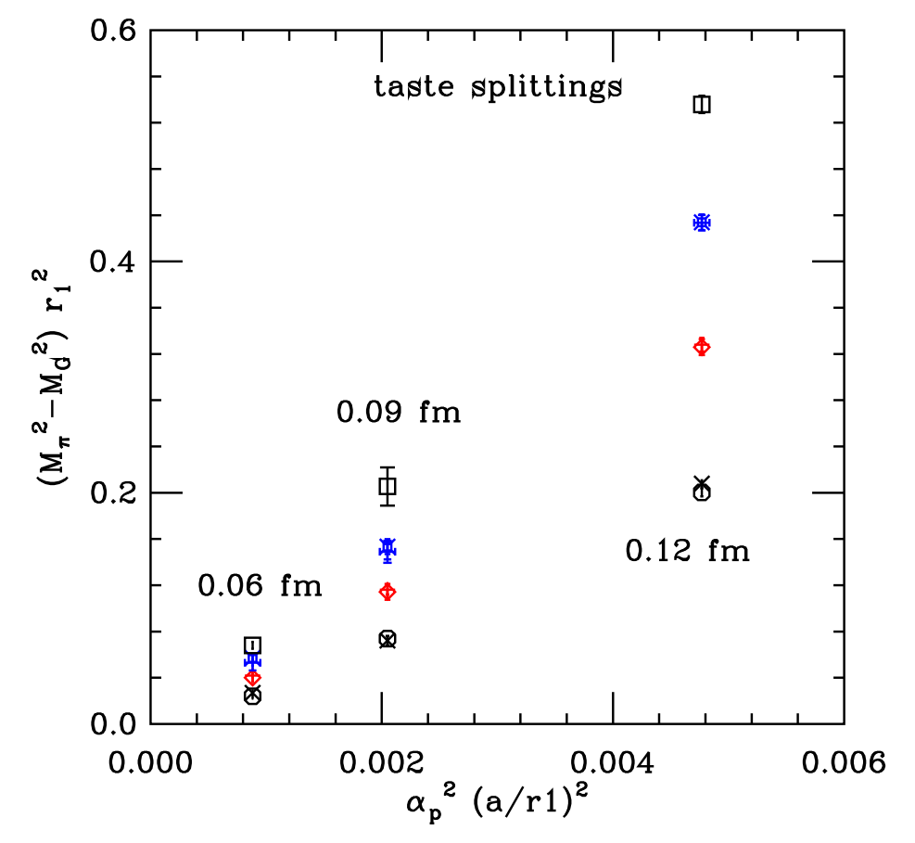
4. Chiral Fits

- For Goldstone masses and decay constants, we have extensive partially quenched data, typically all combinations of 8 or 9 valence quark masses between $0.1 m'_s$ and m'_s. Goldstone quantities have the smallest statistical errors, so we concentrate on them.
- Statistical errors are very small: 0.1% to 0.8% (squared masses); 0.1% to 1.4% (decay constants).
- We fit decay constants and masses together, and we fit different lattices spacings together. The confidence level of the joint fit is 0.99
- Subset I.
  - We expect errors of NLO SχPT to be of order 0.8%, implying that NNLO terms are needed.
  - NNLO SχPT logs are unknown. However, for higher masses, where NNLO terms are important, such logs should be smoothly varying, and well approximated by NNLO analytic terms.
  - The NNLO fit has 20 unconstrained parameters:
  - An additional 6 tightly constrained parameters allow for variation of physical LO and NLO params with a ($\sim \alpha_S a^2 \Lambda^2_{\mathrm{QCD}} \approx 2\%$), giving a total of 26 parameters.
- Subset II
  - Even NNLO fits break down. We add 18 NNNLO analytic terms and 10 tightly constrained variations of NNLO parameters with a.
  - We fix (within errors) LO and NLO terms from the fit to subset I. We are then left with 28 unconstrained parameters and 26 tightly constrained ones.

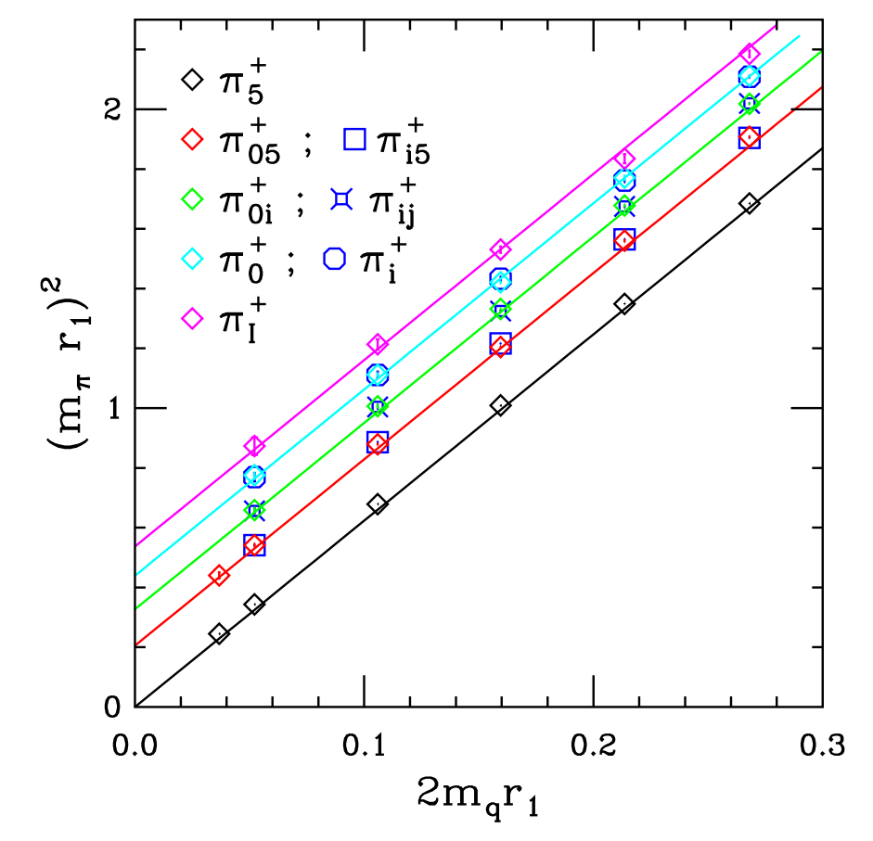
5. Fitting Parameters

- High order χPT fits are necessary because the statistical errors are so small.
- If we did not care about the confidence level, we could perform NLO fits with a much smaller number of parameters.
- LO results (decay constants, quark masses) are not much affected:
  - NLO SχPT fit has 12 parameters,
    - It changes f_π by 4%; f_K by 1%; m_s by 3% (on subset I).
    - But $\chi^2/d.o.f. = 9.5$ for 110 $d.o.f.$.
  - The NLO SχPT fit with taste-violating parameters input and a dependence of physical parameters set to 0 has 6 parameters.
    - It changes f_π by 2%; f_K by 1%; m_s by 0.5% (subset I)
    - $\chi^2/d.o.f. = 40.5$ for 110 $d.o.f.$.
  - Note: Such fits can change LECs quite significantly (by 3 or 4σ).
- If the physics isn't right, ~ 40 parameters WILL NOT enable you to fit the data:
  - Comparable fits to the continuum form (all taste-violating terms set to 0) has 36 parameters, but $\chi^2/d.o.f. = 8.8$ for 204 $d.o.f.$; $\mathrm{CL} < 10^{-250}$.
  - Comparable fits with all chiral logs and finite volume corrections omitted from the fit function (*i.e.*, analytic function only) are poor ⇒ Good evidence for chiral logarithms:
    - Removing finite volume effects from data first (*cf*. Becirevic & Villadoro) gives 38 params, but $\chi^2/d.o.f. = 3.1$ for 202 $d.o.f.$; $\mathrm{CL} < 10^{-43}$.
    - Retaining finite volume effects in the data leaves 38 parameters, but $\chi^2/d.o.f. = 7.4$ for 202 $d.o.f.$; $\mathrm{CL} < 10^{-194}$.
  - We also tried separate linear fits of m_π^2 or f_π *vs.* quark mass:
    - For m_π^2 there are 6 params, and a $\chi^2/d.o.f. \approx 20$ for 234 $d.o.f.$.
    - For f_π there are 10 params, and a $\chi^2/d.o.f. \approx 25$ for 230 $d.o.f.$.

6. Taste Splitting in the Pion Masses

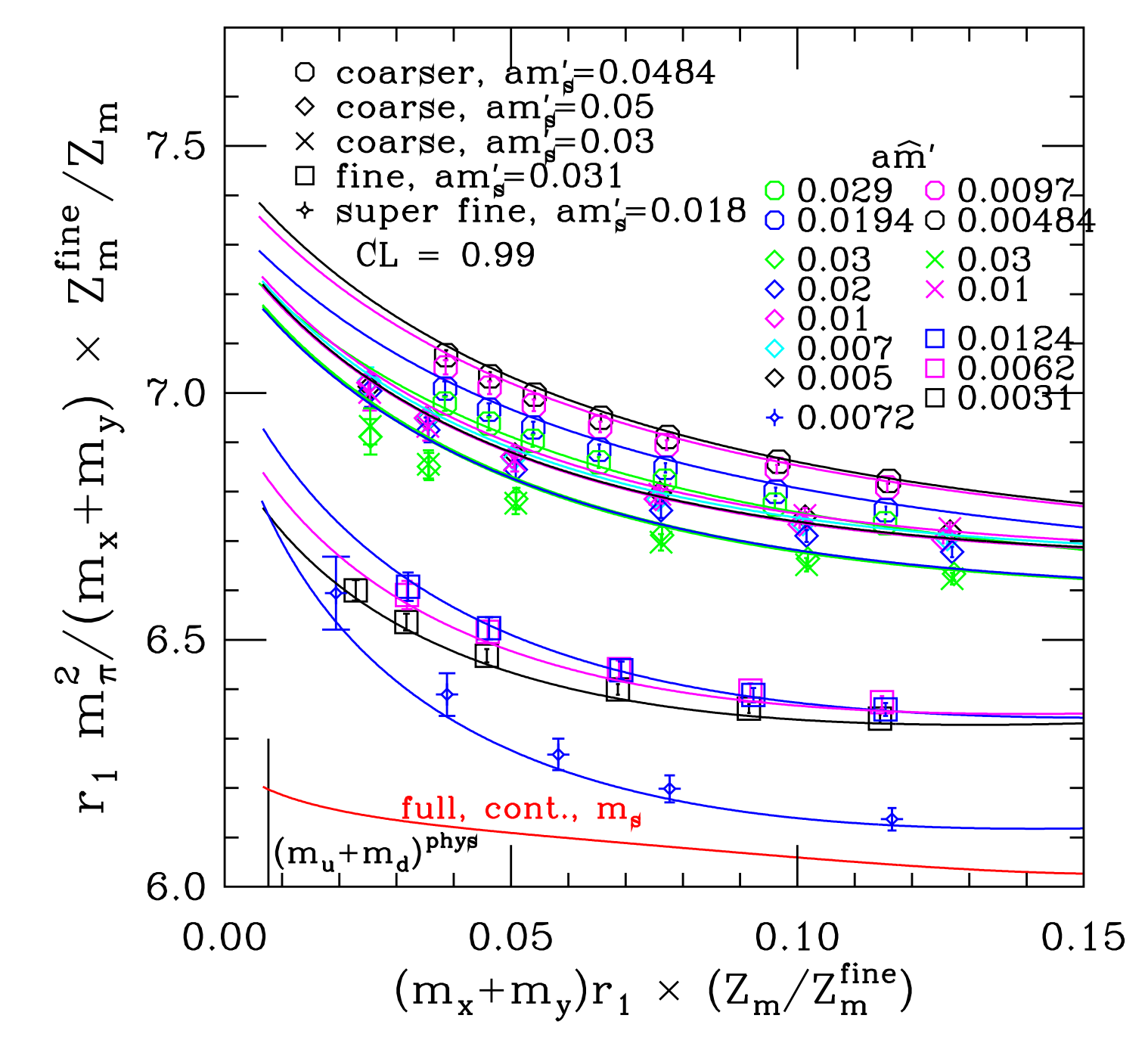


Taste splitting among the pions for lattice spacings 0.12, 0.09 and 0.06 fm with $\hat{m}' = 0.4\, m'_s$. The splittings are plotted as a function of $a^2\alpha_s^2$, the variable in which they are expected to be linear at small lattices spacings.



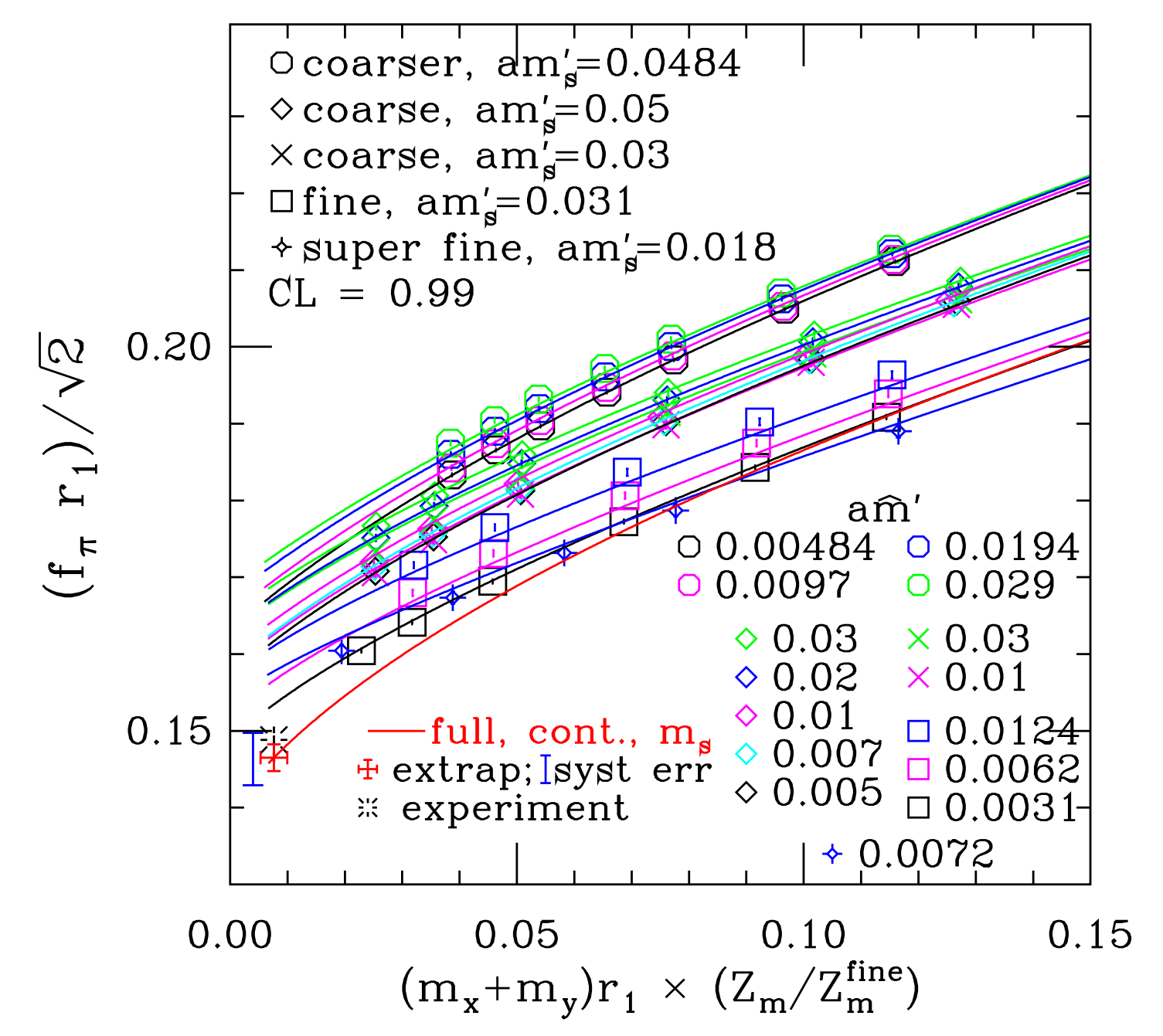
Squared pion masses as a function of the light quark mass. The lines are fits from LO SχPT. The degeneracies predicted by Ref. [5] are clearly visible.

7. Chiral Logs in the Pion Mass



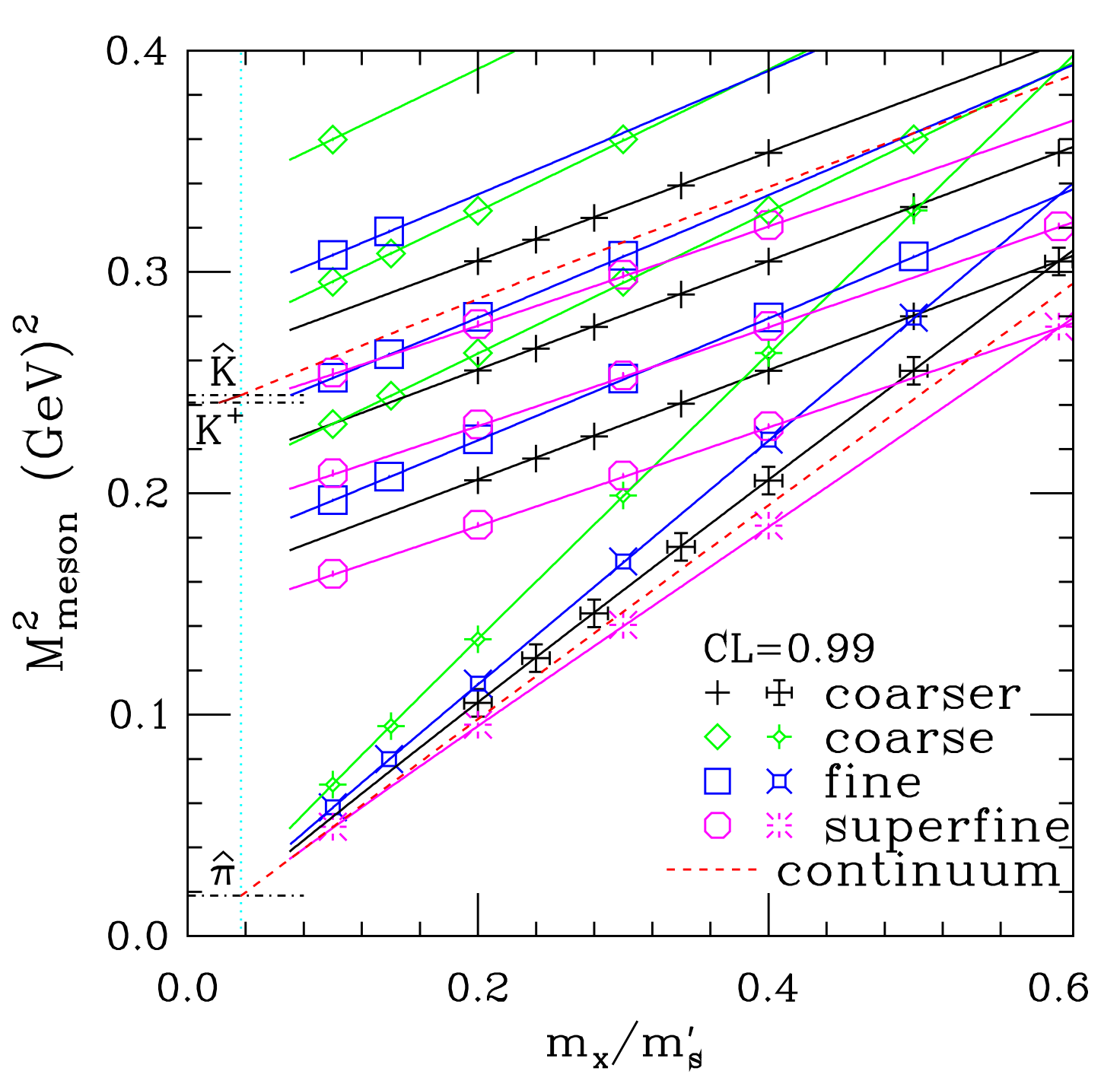
The square of the pion mass as a function of the sum of the valence quark masses ($m_x + m_y$) in units of r_1. The mass renormalization constant, relative to that of the fine lattices, has been included so that data from all lattice spacings can be presented on the same plot. Lines through the data points come from a SχPT fit to the entire data set for decay constants and masses. The upward slope in these lines at small valence quark masses is caused by partially quenched chiral logs. The effect is especially pronounced in the super-fine case because, not only are the taste-violating mass splittings rather small, but the light sea quark mass is rather large, $\hat{m}' = 0.4 m'_s$. The red line is the fit function in "full QCD" (valence and sea quark masses set equal) after extrapolation of parameters to the continuum limit. It is much smoother because it does not have partially quenched chiral logs.

8. Leptonic Decay Constant of the Pion



The pion decay constant as a function of the valence quark masses, in units of the scale r_1. Lines through the data points come from the same SχPT fit as in the preceding figure. The red line represents the fit function in "full QCD" (valence and sea masses set equal) after extrapolation of parameters to the continuum limit, and with the strange sea quark fixed at its physical mass. The red plus shows our extrapolated value of f_π, in agreement with experiment (black burst) within systematic errors (blue bar).

9. Determining the Quark Masses



Squared meson masses as a function of the light valence mass m_x divided by the simulation value of the strange quark mass, m'_s. Three sets of kaon points, with various heavier valence quark masses m_y near the strange mass are plotted for each lattice spacing. Pion points have $m_x = m_y$. The statistical errors in the points are not visible on this scale. The dashed red lines give the continuum-extrapolated fit, and the cyan vertical dotted line shows the physical value of $\hat{m}/m_s$ obtained. Extending the red kaon line (short dark red continuation) until it intersects the K^+ value then gives m_u/m'_s, from which we find $m_u/\hat{m}$ or m_u/m_d.

10. Preliminary Results for Decay Constants

$$
\begin{aligned}
f_\pi &= 128.6 \pm 0.4 \pm 3.0\ \mathrm{MeV} & [129.5 \pm 0.9 \pm 3.5\ \mathrm{MeV}]\\
f_K &= 155.3 \pm 0.4 \pm 3.1\ \mathrm{MeV} & [156.6 \pm 1.0 \pm 3.6\ \mathrm{MeV}]\\
f_K/f_\pi &= 1.208(2)(^{+\ 7}_{-14}) & [1.210(4)(13)]
\end{aligned}
$$

Old results in red are from Refs. [1, 2, 3]. The first error is statistical and the second systematic.

- The experimental rate for $\pi \to \mu\nu$ + V_{ud} from (superallowed) nuclear beta decays ⇒ $f_\pi = 130.7 \pm 0.1 \pm 0.4$ MeV.
- The result for f_K/f_π, plus V_{ud} and $K \to \mu\nu$ (expt), gives $|V_{us}|$. Following Marciano [14], we find

$$|V_{us}| = 0.2223(^{+26}_{-14}) \quad [0.2219(26)]$$

(lattice errors dominate)

- The Particle Data Group (2004) gives $V_{us} = 0.2200(26)$ [15] from $K \to \pi\mu\nu$ (expt) and (non-lattice) theory; newer experiments (KTEV, KLOE, NA48) give $0.2262(23)$ (E. Blucher [16]).

11. Preliminary Results for Quark Masses

$$
\begin{aligned}
m_s^{\overline{\mathrm{MS}}} &= 90(0)(5)(4)(0)\ \mathrm{MeV} & [76(0)(3)(7)(0)\ \mathrm{MeV}]\\
\hat{m}^{\overline{\mathrm{MS}}} &= 3.3(0)(2)(2)(0)\ \mathrm{MeV} & [2.8(0)(1)(3)(0)\ \mathrm{MeV}]\\
m_s/\hat{m} &= 27.2(0)(4)(0)(0) & [27.4(1)(4)(0)(1)]\\
m_u^{\overline{\mathrm{MS}}} &= 2.0(0)(1)(1)(1)\ \mathrm{MeV} & [1.7(0)(1)(2)(2)\ \mathrm{MeV}]\\
m_d^{\overline{\mathrm{MS}}} &= 4.6(0)(2)(2)(1)\ \mathrm{MeV} & [3.9(0)(1)(4)(2)\ \mathrm{MeV}]\\
m_u/m_d &= 0.42(0)(1)(0)(4) & [0.43(0)(1)(0)(8)]
\end{aligned}
$$

Old results in red are from Refs. [1, 13, 18]

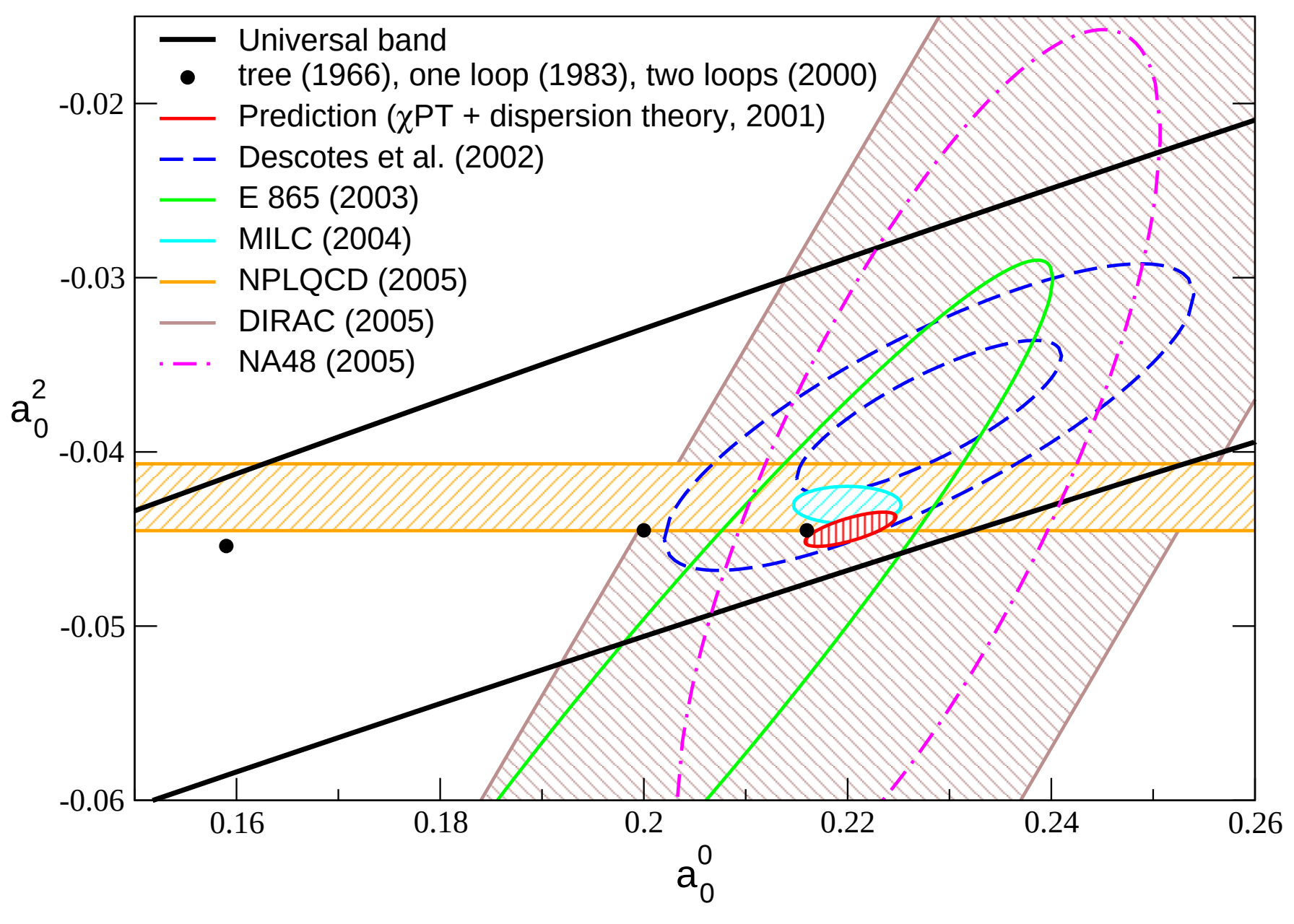
- Errors are from statistics, simulation, perturbation theory, and EM effects. (Scale $\mu = 2\,\mathrm{GeV}$.)
- The main difference between the new and old results comes from perturbation theory (matching lattice masses to $\overline{\mathrm{MS}}$ masses). Here we use the new two-loop results of Mason, Trottier and Horgan [19]. A non-perturbative mass renormalization calculation is in progress.
- Note that $m_u/m_d = 0.42(0)(1)(0)(4)$ [or even our old result $0.43(0)(1)(0)(8)$] means that $m_u = 0$ is ruled out (at least at the low energy scale of $2\,\mathrm{GeV}$).

12. Preliminary Results for Low Energy Constants

- We also find at the chiral scale m_η (in units of 10^{-3}):

$$
\begin{aligned}
2L_6 - L_4 &= 0.5(1)(2) & [0.5(2)(4)]\\
2L_8 - L_5 &= -0.1(1)(1) & [-0.2(1)(2)]\\
L_4 &= 0.1(2)(2) & [0.2(3)(3)]\\
L_5 &= 2.0(3)(2) & [1.9(3)(3)]
\end{aligned}
$$

- These results are consistent with "conventional" ones summarized in Ref. [20]: $L_5 = 2.2(5)$, $L_6 = 0.0(3)$, $L_4 = 0.0(5)$.
- Our result for $2L_8 - L_5$ is far from range $-3.4 \leq 2L_8 - L_5 \leq -1.8$ that would allow $m_u = 0$.
- This result is consistent with, but not independent of, our direct determination of m_u.
- One consequence of our old results for the Low Energy Constants is shown in the figure below, where various constraints on the pion scattering lengths a_0^2 and a_0^0 [21] are shown. The previously published MILC results give the light blue horizontal ellipse. We thank G. Colangelo for providing us with this plot.



13. Conclusions and Outlook

- Our result for $|V_{us}|$ is consistant with, and as accurate as, the world average from alternative methods. Runs now in progress, as well as those planned for the near future, should reduce our error significantly.
- m_u is clearly not zero, so this cannot be a solution [22] of the Strong CP puzzle [23].
- The largest error in our determination of m_u/m_d comes from continuum estimates of electromagnetic effects. We plan to make lattice evaluations of these effects, perhaps along the lines of Ref. [24].
- Lattice evaluations of the Low Energy Constants have reached the level of accuracy needed to have a significant impact on phenomonology.
- SχPT works very well.

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