Renormalization constants for Lattice QCD: new results from NSPT

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The overall picture

Despite the fact that there is no theoretical obstacle to computing finite and log-div RC in PT, on the lattice one tries to compute them NP. Popular (intermediate) schemes are RI'-MOM (Rome group) and SF (alpha Coll).

\>> LPT is very hard so that usually computations are 1 LOOP (analytic 2 LOOP on their way).

\>> LPT converges badly!
Often people have to rely on Boosted PT (Parisi, Lepage & Mackenzie).

\>> BUT ... we can compute to 3 (or even 4, ...) LOOPS!

Last year (LAT05) we presented results for (Wilson fermions, $n_f=2$) Z_p/Z_s

\>> We now have some more new results: Z_p/Z_s, Z_v/Z_a, Z_v, Z_a, ...
(at various numbers of flavours)

F. Di Renzo et al, to be issued soon

Outline

- The scheme we work in: RI'-MOM.
- Some technical details of our computations (with just a flavour of what NSPT is).
- A caveat on anomalous dimensions (work on Z_s in progress).
- Ratios (Z_p/Z_s, Z_v/Z_a) work really pretty well: 3 loops results for n_f = 0,2,3,4 (so that one can fit results for generic n_f).
- Z_v and Z_a are well under control to 3 loops for $n_f = 0,2$.
- All the results are extended to 4 loops (at the moment) for $n_f=2$.
- Some comments on improved (Clover) fermions.

Renormalization scheme (definitions)

Martinelli & al NP 445 (1995) 81

We work in the RI'-MOM scheme: compute quark bilinears operators between (off-shell p) quark states and then amputate to get Γ functions

$$\langle p | \overline{\psi} \Gamma \psi | p \rangle = G(p) \rightarrow \Gamma(p)$$

project on the tree level structure

$$O(p) = \mathrm{Tr}\left(\hat{P}_O \Gamma(p)\right)$$

Renormalization conditions read

$$Z_O \, Z_q^{-1} \, O(p)\Big|_{p^2=\mu^2} = 1$$

where the field renormalization constant is defined via

$$Z_q = -i \frac{1}{12} \frac{\mathrm{Tr}(\not{p} S^{-1})}{p^2}\Big|_{p^2=\mu^2}$$

One wants to work at zero quark mass in order to get a mass-independent scheme.

Computational setup (NSPT)

F. Di Renzo, G. Marchesini, E. Onofri, Nucl.Phys. B457 (1995), 202

F. Di Renzo, L. Scorzato, JHEP 0410 (2004), 73

From Langevin equation ...
(remember: η is gaussian noise)

$$\frac{\partial \phi(x,t)}{\partial t} = -\frac{\partial S[\phi]}{\partial \phi(x,t)} + \eta(x,t)$$

... (see Parisi & Wu) Stochastic Quantization gets an evolution which converges to the Path Integral.

$$\langle \mathcal{O}[\phi_\eta(t)] \rangle_\eta \xrightarrow[t\to\infty]{} \frac{1}{Z}\int \mathcal{D}[\phi]\mathcal{O}[\phi]e^{-S[\phi]}$$

If you look for the solution to the Langevin equation as a series in the coupling

$$\phi = \phi_0 + g\phi_1 + g^2\phi_2 + \ldots$$

... your Langevin simulation becomes a collection of equations to be integrated on a computer, returning results for observables which are given order by order.

In recent years we implemented NSPT for unquenched Lattice QCD! So ...

- We computed quark bilinears to 3 (4) loops for the Wilson (gauge) - Wilson (fermions) action in the $n_f=0,2,3,4$ cases.
- The gauge was fixed to Landau (FFT-accelerated algorithm).
- Lattice sizes 32^4 and 16^4 (check for finite size effects in the quenched case).
- The relevant mass counterterms (critical mass) to stay at zero quark mass were taken from the literature up to 2 loops and computed to 3 (and 4) loops and inserted properly.

How do we deal with results?

At loop L a renormalization constant is made of a finite part, a divergent one (log's) and irrelevant pieces. γ's are taken from J.Gracey (2003): 3 loops!

$$z_L = c_L + \sum_i^L d_i(\gamma)\log(\hat{p})^i + F(\hat{p}) \qquad (\hat{p} = pa)$$

We take small values for (lattice) momentum and look for "hypercubic symmetric" Taylor expansions to fit the finite parts we want to get.

RI'-MOM is an infinite-volume scheme, while we have to perform finite V computations! Care will be taken of this (crucial) aspect.

Our standard example: Z_q at 1Loop

$$Z_q = -i\frac{1}{12}\frac{\mathrm{Tr}(\not{p}S^{-1})}{p^2}\Big|_{p^2=\mu^2}$$

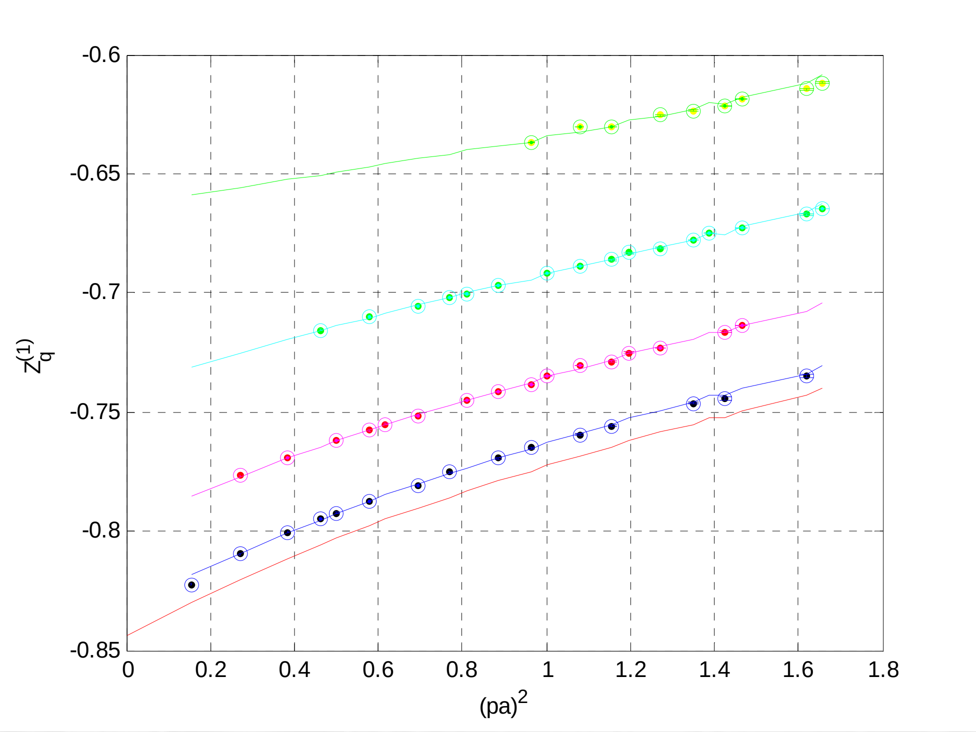
It is an easy example (no log in Landau gauge). We can see our "hypercubic symmetric" Taylor expansions at work!

$$\gamma_\mu p_\mu \Sigma \hookrightarrow \gamma_\mu p_\mu \left(\Sigma + \hat{p_\mu}^2 \Sigma_1 + \hat{p_\mu}^4 \Sigma_2 + \ldots\right)$$

In the end it is not so difficult to understand if you think about tree level ...

$$2\,\gamma_\mu \sin(\frac{\hat{p_\mu}}{2})$$

It works pretty well!

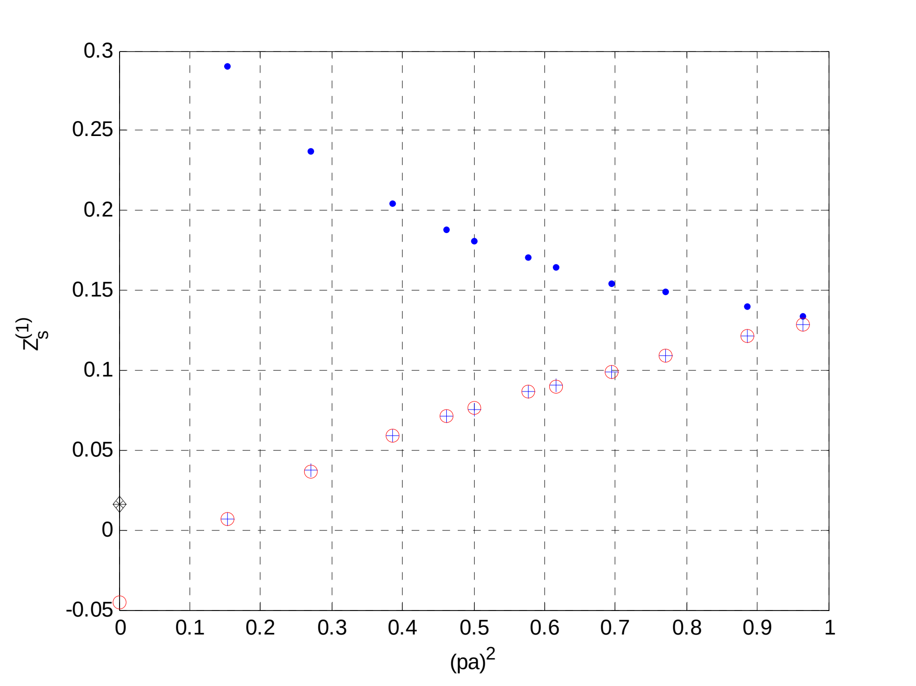


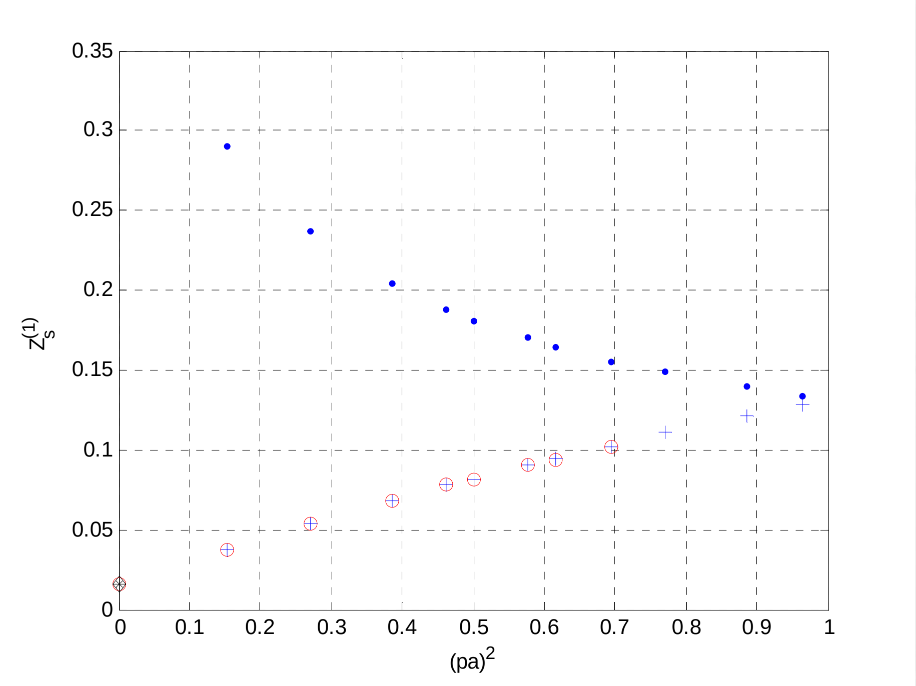
For a complete list of reference to analytic results we compare to, please refer to Capitani Phys Rep 382(03)113

Let's talk about the care needed when dealing with anomalous dimensions. We recall the situation for the scalar current (1 loop): from the master formula

$$\left(1-\frac{z_q^{(1)}}{\beta}+\ldots\right)\left(1+\frac{z_s^{(1)}-\gamma_s^{(1)}\log(p^2)}{\beta}+\ldots\right)\left(1-\frac{o_s^{(1)}}{\beta}+\ldots\right)\Big|_{p^2=\mu^2}=1$$

i.e. you have to subtract a log $-z_q^{(1)}+z_s^{(1)}-\gamma_s^{(1)}\log(\hat{p}^2)+o_s^{(1)}=0$



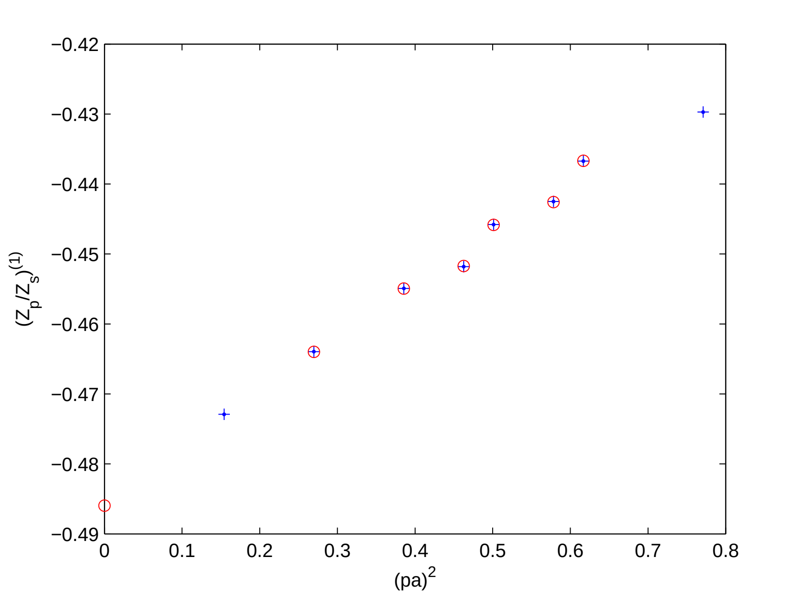


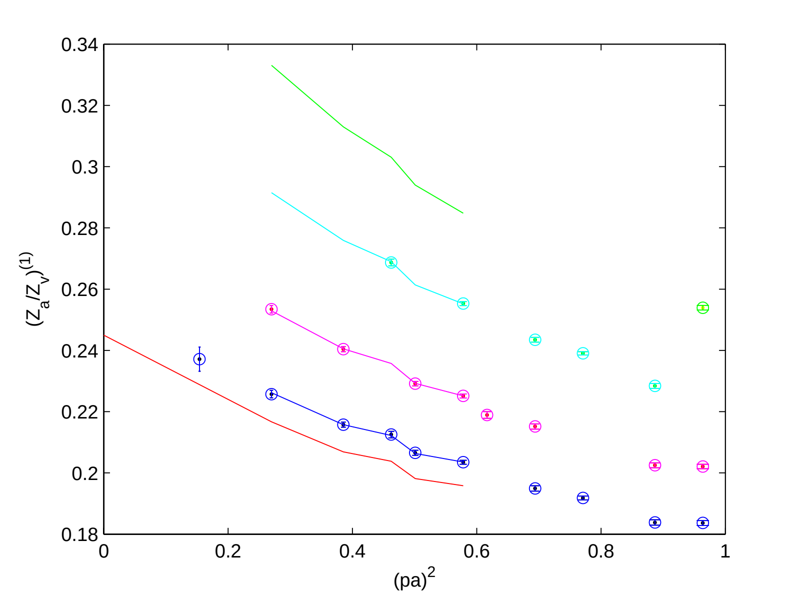
On the right the right thing to do: subtract a "tamed log" (finite volume!)
We are working out corrections at 2 and 3 loops: hard, but precious!

Ratios are always safe to compute!

- The quark field renormalization constant cancels out in the Z_p/Z_s and Z_v/Z_a ratios.
- Both ratios are finite, so there is no log to subtract.
- Finite size effects (as checked on 32^4 and 16^4) are well under control.
- We now have results for different numer of flavours (and within errors they scale with n_f as they should).
- For all the computations we will refer to, the relevant mass counterterms (critical mass) have been computed to 3 (and 4) loops and inserted properly.

Below a couple of examples of our fits





Z_p/Z_s

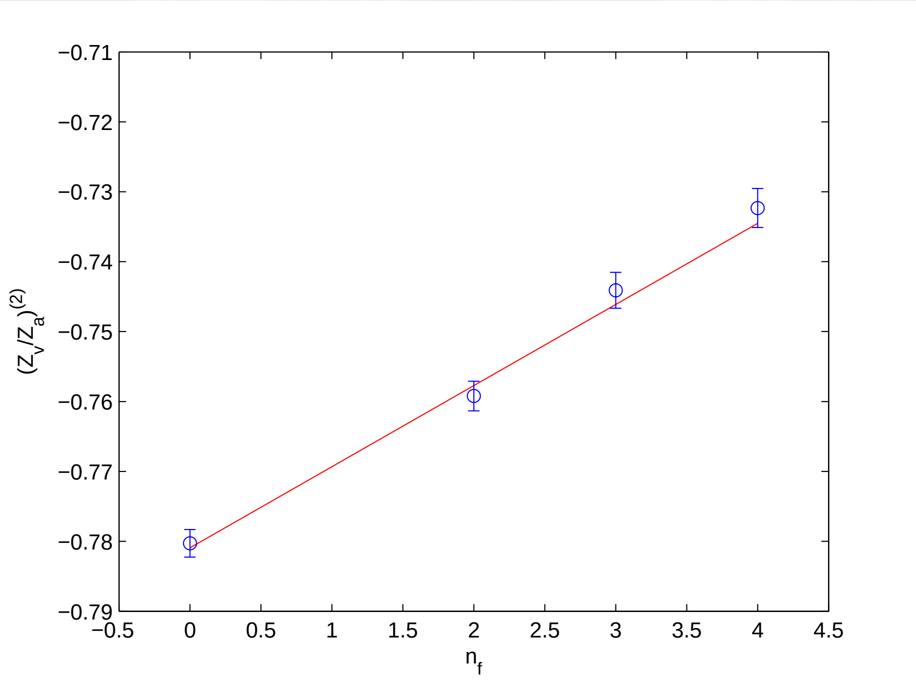
n_f	$O(\beta^{-1})$	$O(\beta^{-2})$	$O(\beta^{-3})$
0	-0.487(1)	-1.50(1)	-5.72(3)
2	-0.487(1)	-1.46(1)	-5.35(3)
3	-0.487(1)	-1.43(1)	-5.13(3)
4	-0.487(1)	-1.40(1)	-4.86(3)

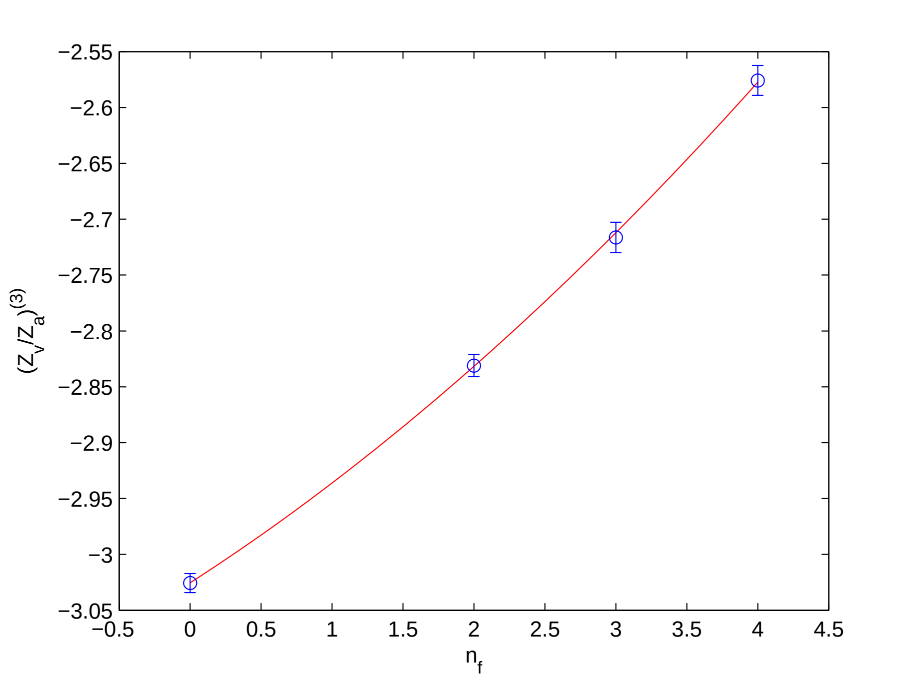
We also computed Z_s/Z_p and the two series are actually inverse of each other.

Z_v/Z_a

n_f	$O(\beta^{-1})$	$O(\beta^{-2})$	$O(\beta^{-3})$
0	-0.244(1)	-0.780(5)	-3.02(2)
2	-0.244(1)	-0.759(5)	-2.83(2)
3	-0.244(1)	-0.744(6)	-2.72(2)
4	-0.244(1)	-0.732(6)	-2.57(2)

About the n_f dependence



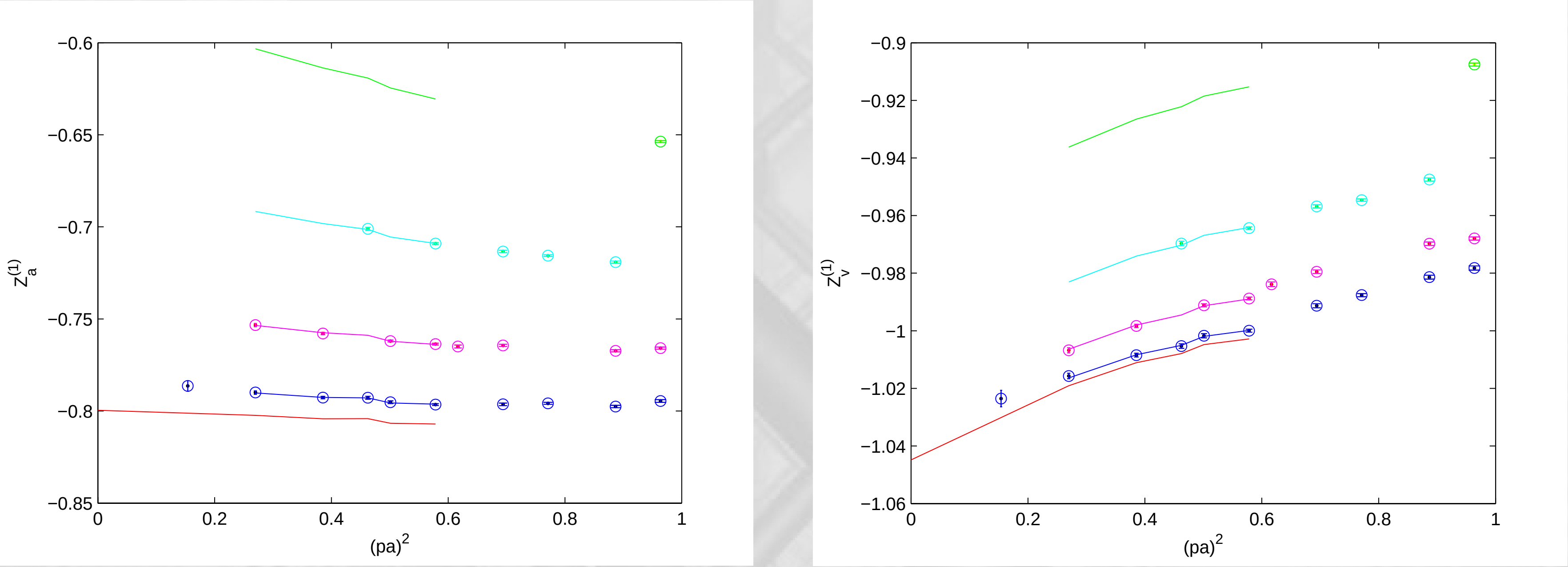


Z_a and Z_v

- They are finite, so again no problems from log. In our master formula they are interlaced with (log's coming from) Z_q ...

$$Z_O \, Z_q^{-1} \, O(p)\Big|_{p^2=\mu^2} = 1$$

- ... which can anyway be eliminated either directly from measurements of the propagator ...
- ... or by taking a ratio with the conserved vector current.



Z_a

n_f	$O(\beta^{-1})$	$O(\beta^{-2})$	$O(\beta^{-3})$
0	-0.800(2)	-1.39(3)	-4.04(4)
2	-0.800(2)	-1.31(3)	-3.50(8)

Z_v

n_f	$O(\beta^{-1})$	$O(\beta^{-2})$	$O(\beta^{-3})$
0	-1.044(2)	-1.98(3)	-6.10(8)
2	-1.044(2)	-1.88(3)	-5.42(8)

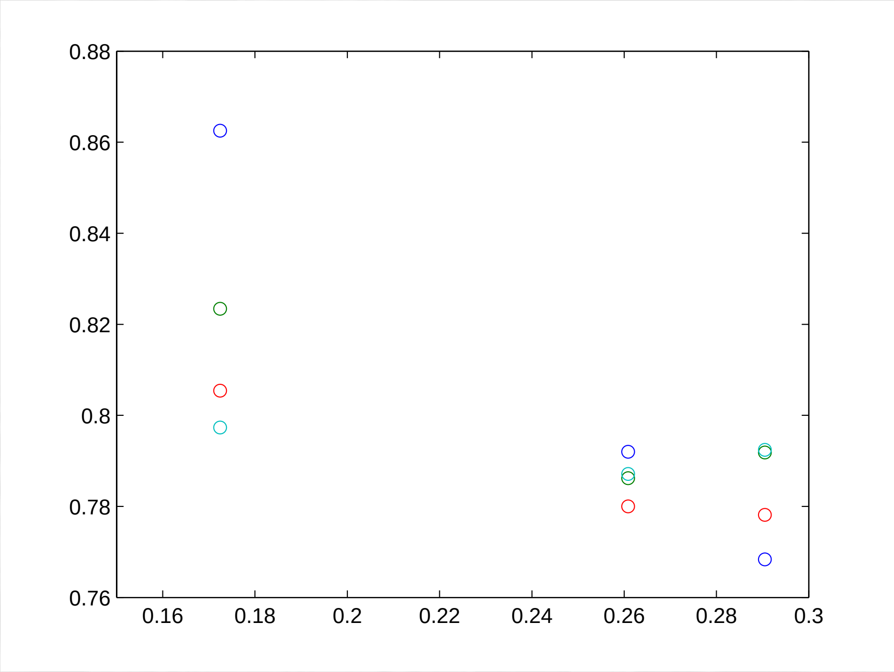
Resumming Z_a and Z_v (to 4 loops!)

One can compare to NP results from $SPQ_{CD}R$

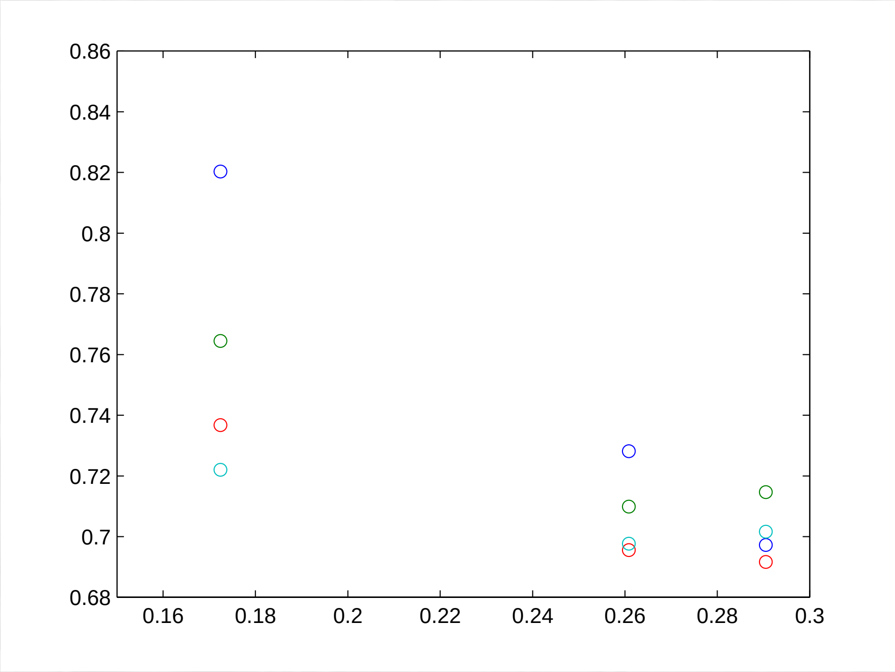
We can now have numbers for Z_a and Z_v. We resum (@ $\beta=5.8$) using different coupling definitions:

$$x_0 \equiv \beta^{-1} \qquad x_1 \equiv -\frac{1}{2}\log(P) \qquad x_2 \equiv \frac{\beta^{-1}}{P}$$

$Z_a = 0.79(1)$



$Z_v = 0.70(1)$



Just a few comments on Clover fermions

- Next step (while also fixing the scalar current) is to compute renormalization constants for improved (Clover) fermions (work in progress).

- First step is to make use of the already stored quenched configurations (as a precious byproduct, quenched non-perturbative determinations of Clover renormalization constants are after all the best testing ground to make a comparison).
- On top of that, the building block of measurements are the same needed for Langevin dynamics.

- The missing ingredient is the second loop of c_{SW}. Of course we will try to compute it. Still, there is something we are already doing to give estimates at three loops, i.e. making use of the Alpha Collaboration parametrization for their non-perturbative determination (below the quenched formula)

$$c_{SW} = \frac{1 - 0.454\, g_0^2 - 0.175\, g_0^4 + 0.012\, g_0^6 + 0.045\, g_0^8}{1 - 0.720\, g_0^2}$$