Testing UV-filtered clover fermions

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Overview

$$D_{\rm W} = \frac{1}{2}\sum_\mu \Big\{(\gamma_\mu - I)U_\mu(x)\delta_{x+\hat\mu,y} - (\gamma_\mu + I)U_\mu^\dagger(x-\hat\mu)\delta_{x-\hat\mu,y}\Big\} + \frac{1}{2\kappa}\delta_{x,y}$$

Wilson fermions are fast, since $D_{\rm W}$ is sparse, and they preserve flavour.

The main disadvantage is that they break chiral symmetry. Two reduction strategies:

- "$O(a)$-improvement": $D_{\rm W} \to D_{\rm SW} = D_{\rm W} - \frac{c_{\rm SW}}{2}\sum_{\mu<\nu}\sigma_{\mu\nu}F_{\mu\nu}\,\delta_{x,y}$
- "UV-filtering": $U_\mu(x) \to U_\mu^{\rm APE,HYP,\ldots}$ and $F_{\mu\nu}(x) \to F_{\mu\nu}^{\rm APE,HYP,\ldots}$

This talk: combining both approaches, one reaches $am_{\rm res} = O(10^{-2})$.

Smearing/filtering recipes

We use the same type of $U_\mu^{\mathrm{APE,HYP,\ldots}}$ in D_W and $F_{\mu\nu}$:

- the gauge-field is smeared and D_SW evaluated on this smooth background
- yields new UV-filtered fermion action (modified by ultralocal $\wedge$ irrelevant terms)

We use four recipes [APE Collab. 1987, Hasenfratz Knechtli 2001, Morningstar Peardon 2004, new: HEX is HYP with EXP inside]

APE with $\alpha^\mathrm{APE} = 0.6$ $\quad\stackrel{\mathrm{PT}}{\longleftrightarrow}\quad$ EXP with $\alpha^\mathrm{EXP} = 0.1$ ["stout"]

HYP with $\alpha^\mathrm{HYP} = (0.75, 0.6, 0.3)$ $\quad\stackrel{\mathrm{PT}}{\longleftrightarrow}\quad$ HEX with $\alpha^\mathrm{HEX} = (0.125, 0.15, 0.15)$

With fixed $(\alpha, n^\mathrm{iter})$ the UV-filtered ("fat-link") fermion action is in the same universality class as the naive ("thin-link") D_SW

Filtered ("fat-link") PT is not much harder than naive ("thin-link") PT [Bernard DeGrand 1999]

$$A_\mu^{(n)}(q) = \sum_\nu \Big(\underbrace{[1 - \frac{\alpha}{2(d-1)}\hat{q}^2]^n}_{f^{(n)}_\mathrm{APE}(\hat{q}^2)} (\delta_{\mu,\nu} - \frac{\hat{q}_\mu \hat{q}_\nu}{\hat{q}^2}) + \frac{\hat{q}_\mu \hat{q}_\nu}{\hat{q}^2} \Big) A_\nu(q)$$

Critical mass in PT

$$am_{\rm crit} = \Sigma_0 = -\frac{g_0^2}{16\pi^2} C_F S + O(g_0^4) \qquad [<0]$$

S	thin link	1 APE	2 APE	3 APE	1 HYP
$c_{\rm SW}=0$	**51.43471**	13.55850	7.18428	4.81189	**6.97653**
$c_{\rm SW}=1$	**31.98644**	4.90876	1.66435	0.77096	**1.98381**
$c_{\rm SW}=2$	1.10790	-7.11767	-5.48627	-4.23049	-4.41059

naive ⟶ $\div\mathbf{7.4}$

↓ ↓

$\div\,\mathbf{1.6}$ ⟶ $\div\mathbf{26}$

In PT the two ingredients clover-improvement and link-fattening pile up to suppress $-am_{\rm crit}$ quite drastically !

Renormalization factors in PT

$$\langle .|O_j^{\rm cont}(\mu)|.\rangle = \sum_k Z_{jk}(a\mu)\langle .|O_k^{\rm latt}(a)|.\rangle$$

$$Z_{jk}(a\mu) = \delta_{jk} - \frac{g_0^2}{16\pi^2}(\Delta_{jk}^{\rm latt} - \Delta_{jk}^{\rm cont}) = \delta_{jk} - \frac{g_0^2}{16\pi^2}C_F z_{jk}$$

For point-like 2-fermion operators, the mixing is incorporated in "improved currents".

$$Z_S(a\mu) = 1 - \frac{g_0^2}{4\pi^2}\Big[\frac{z_S}{3} - \log(a^2\mu^2)\Big] + \ldots \qquad Z_V = 1 - \frac{g_0^2}{12\pi^2}z_V + \ldots$$

$$Z_P(a\mu) = 1 - \frac{g_0^2}{4\pi^2}\Big[\frac{z_P}{3} - \log(a^2\mu^2)\Big] + \ldots \qquad Z_A = 1 - \frac{g_0^2}{12\pi^2}z_A + \ldots$$

NB: $\frac{1}{2}(z_P - z_S) = z_V - z_A$ measures chiral symmetry breaking [overlap: $Z_S = Z_P, Z_V = Z_A$]

$c_{\mathrm{SW}}=0$	thin link	1 APE	2 APE	3 APE	1 HYP
z_S	12.95241	1.12593	-1.53149	-2.87223	-1.78317
z_P	22.59544	5.28288	1.07019	-0.98025	0.51727
z_V	20.61780	6.39810	3.62281	2.51381	3.38076
z_A	15.79628	4.31963	2.32197	1.56782	2.23054
$(z_P-z_S)/2$	**4.82152**	2.07848	1.30084	0.94599	**1.15022**
z_V-z_A	**4.82152**	2.07847	1.30084	0.94599	**1.15022**

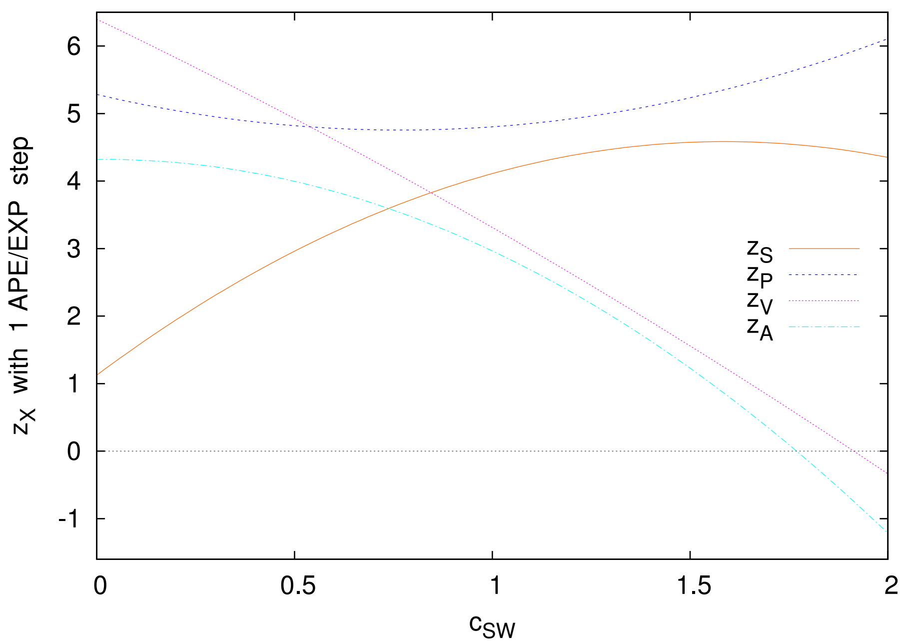
$c_{\mathrm{SW}}=1$	thin link	1 APE	2 APE	3 APE	1 HYP
z_S	19.30995	4.11106	0.40606	-1.43930	-0.03678
z_P	22.38259	4.80364	0.65185	-1.33218	0.12845
z_V	15.32907	3.31243	1.43934	0.82550	1.38517
z_A	13.79274	2.96614	1.31645	0.77195	1.30255
$(z_P-z_S)/2$	**1.53632**	0.34629	0.12290	0.05356	**0.08262**
z_V-z_A	**1.53633**	0.34629	0.12289	0.05355	**0.08262**

naive → $\div\mathbf{4.19}$

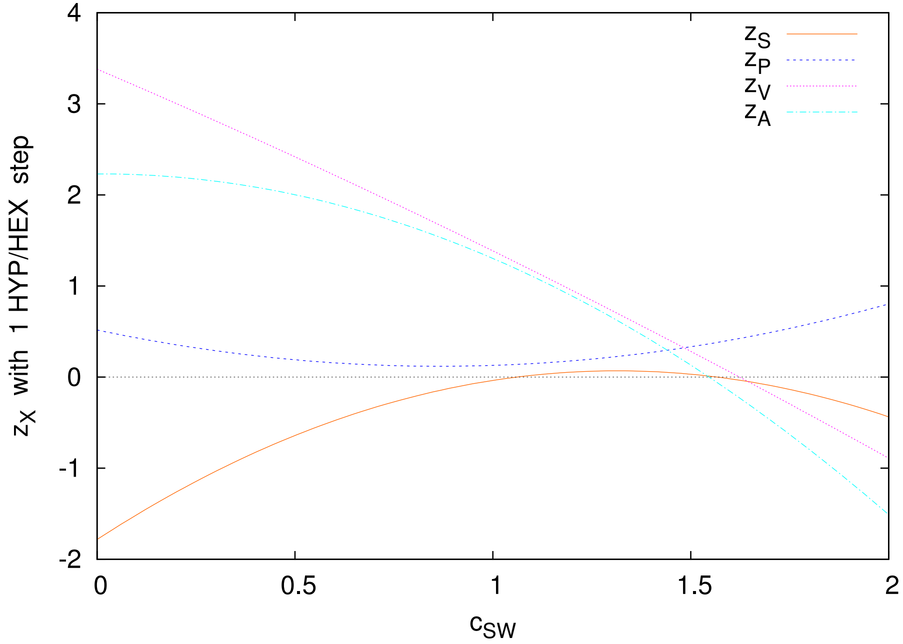
↓ ↓

$\div\mathbf{3.14}$ → $\div\mathbf{58.4}$

In PT the two ingredients clover-improvement and link-fattening pile up to to suppress $\frac{1}{2}(z_P - z_S) = z_V - z_A$ quite drastically !



1 APE: $c_{SW} = 1.2648$ realizes minimal XSB



1 HYP: $c_{SW} = 1.1653$ realizes minimal XSB and near $c_{SW} \simeq 1.5$ all $z_X(X = S, P, V, A)$ are simultaneously small

Irrelevance of tadpole resummation

$$S/(16\pi^2) = -\Sigma_0/(g_0^2 C_F) = -\underbrace{[\text{sunset}]_0/(g_0^2 C_F)}_{\text{(fairly) small }(c_{\text{SW}})} - \underbrace{[\text{tadpole}]_0/(g_0^2 C_F)}_{9.174788\text{ [Landau gauge]}}$$

Tadpole contribution ("thin-link": 9.174788) for various $(n^{\text{iter}}, \alpha^{\text{APE}})$:

	0.12	0.24	0.36	0.48	0.6	0.72	0.84	0.96
1	6.99558	5.19536	3.77414	2.73191	2.06867	1.78443	1.87918	2.35292
2	5.44459	3.26311	2.05185	1.39240	1.02644	0.85574	0.94215	1.50761
3	4.32095	2.22832	1.31922	0.89113	0.66450	0.55028	0.63677	1.39614
4	3.49281	1.62650	0.94513	0.64620	0.48918	0.40469	0.49903	1.64011
5	2.87228	1.25138	0.72799	0.50519	0.38709	0.32019	0.42898	2.29060

$\longrightarrow$ tadpole (and hence S) small for "generic" filtering (in Landau gauge)

$\longrightarrow$ good reason to hope that PT in g_0^2 converges nicely for UV-filtered actions

$\Longrightarrow$ with UV-filtering non need to resum tadpole contributions [Lepage Mackenzie 2002]

Non-perturbative tests

$$m^{\rm W} = m_0 - m_{\rm crit} \quad \text{where} \quad am_0 = \frac{1}{2\kappa} - 4\,, \; am_{\rm crit} = \frac{1}{2\kappa_{\rm crit}} - 4$$

$$\longrightarrow \qquad m^{\rm VWI}(\mu) = Z_m(a\mu)(1 + b_m am^{\rm W})m^{\rm W}$$

$$m^{\rm PCAC} = \frac{\langle \bar\partial_\mu [A^a_\mu(x) + ac_A \bar\partial_\mu P^a] O^a(0)\rangle}{2\langle P^a(x) O^a(0)\rangle}$$

$$\longrightarrow \qquad m^{\rm AWI}(\mu) = \frac{Z_A}{Z_P(a\mu)} \frac{1 + b_A am^{\rm W}}{1 + b_P am^{\rm W}} m^{\rm PCAC}$$

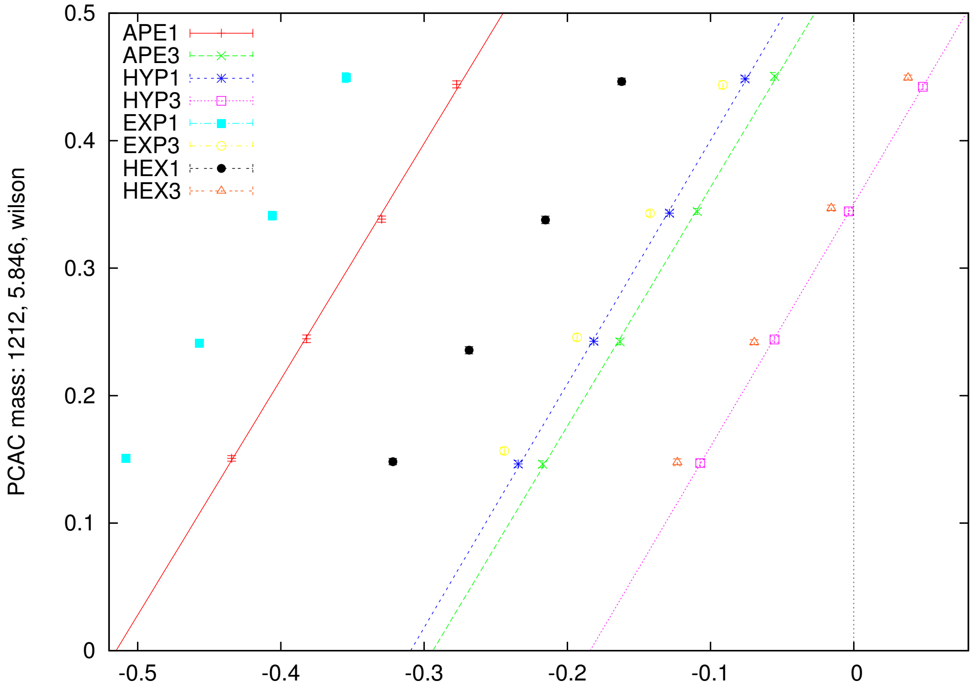
NP tests: quenched QCD in fixed physical volume $V \simeq (1.5\,\mathrm{fm})^4$ via fitted $r_0 = r_0(\beta)$ [Sommer Necco 2002]

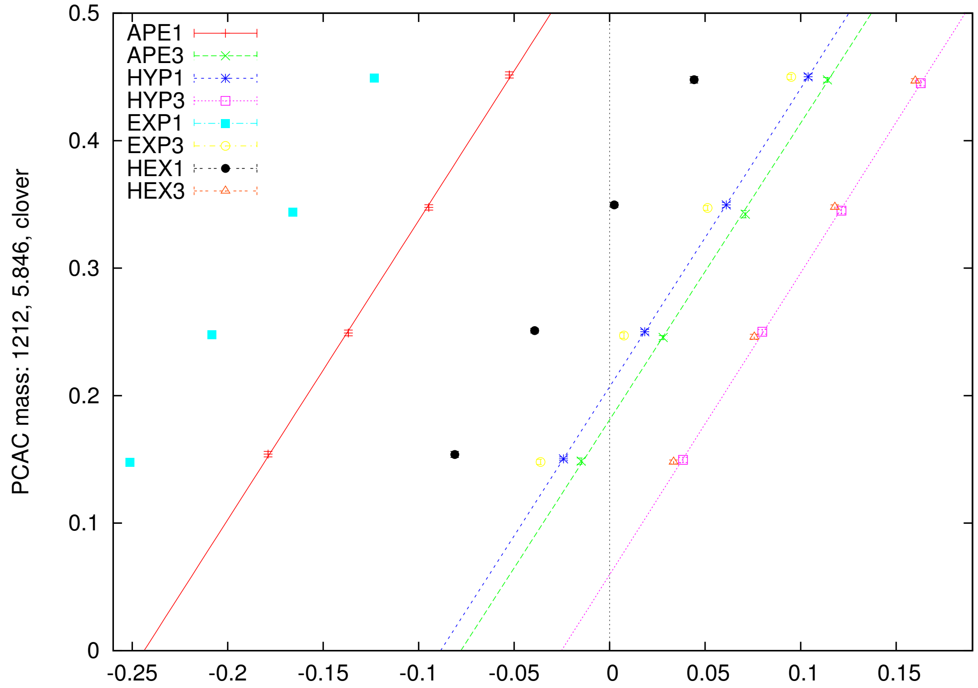
β	5.846	6.000	6.136	6.260	6.373
L/a	12	16	20	24	28
L/r_0	2.979	2.981	2.983	2.981	2.979
$a^{-1}\,[\mathrm{GeV}]$	1.590	2.118	2.646	3.177	3.709
$n_{\rm conf}$	64	32	16	8	4

Tests with
- tree-level improvement coefficients $(c_{\rm SW}=1, b_X=1, b_m \neq -\frac{1}{2}, c_{V,A}=0)$
- one-loop renormalization factors $(Z_X = 1 - \frac{g_0^2}{12\pi^2} z_X$ for $X = S, P, V, A)$

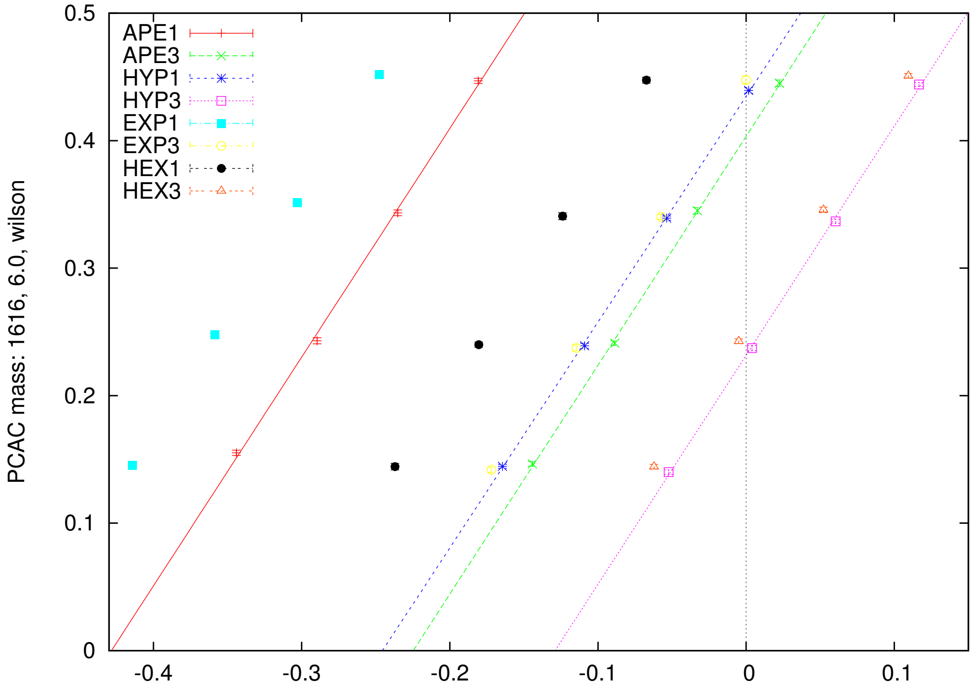
Plots: $2m^{\rm PCAC}$ versus m_0 has slope $\frac{2}{\tilde Z_A} = \frac{2}{Z_A}\frac{Z_P}{Z_S}$ and horizontal offset $-am_{\rm crit}$ ...

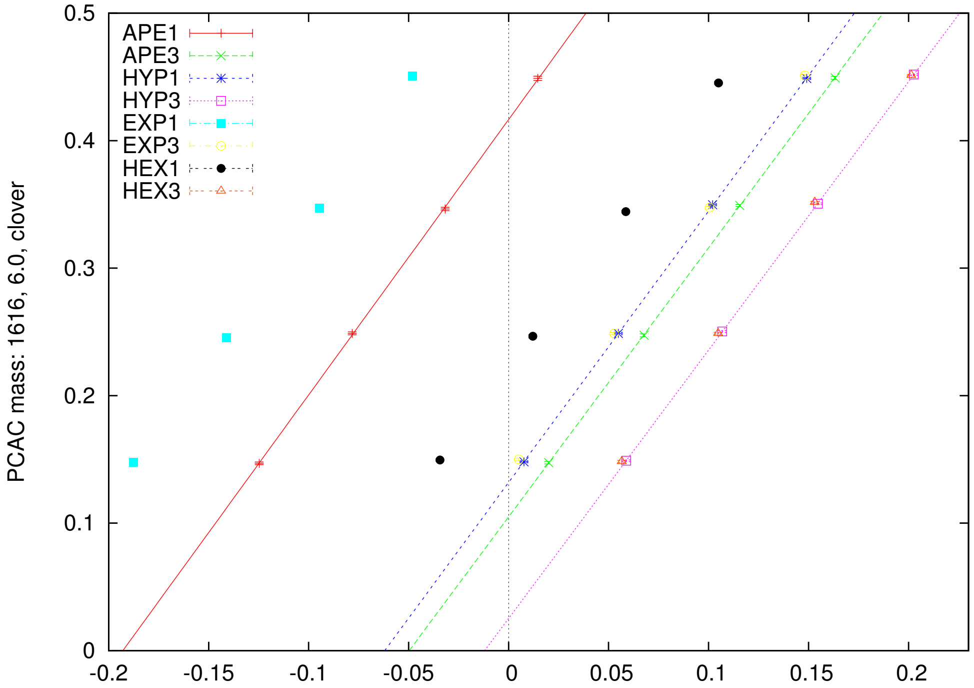
$2m^{\rm PCAC}$ versus m_0 with $c_{\rm SW}=0$ (left) and $c_{\rm SW}=1$ (right) at $\beta=5.846$:



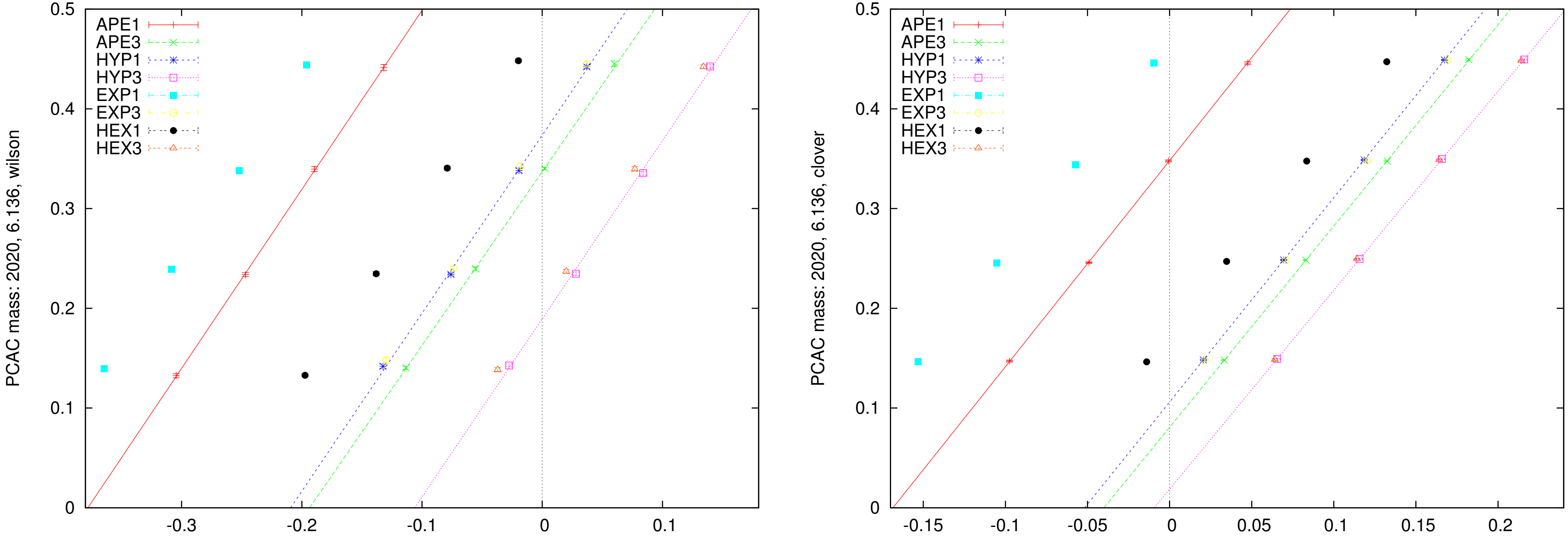


$2m^{\rm PCAC}$ versus m_0 with $c_{\rm SW}=0$ (left) and $c_{\rm SW}=1$ (right) at $\beta=6.000$:

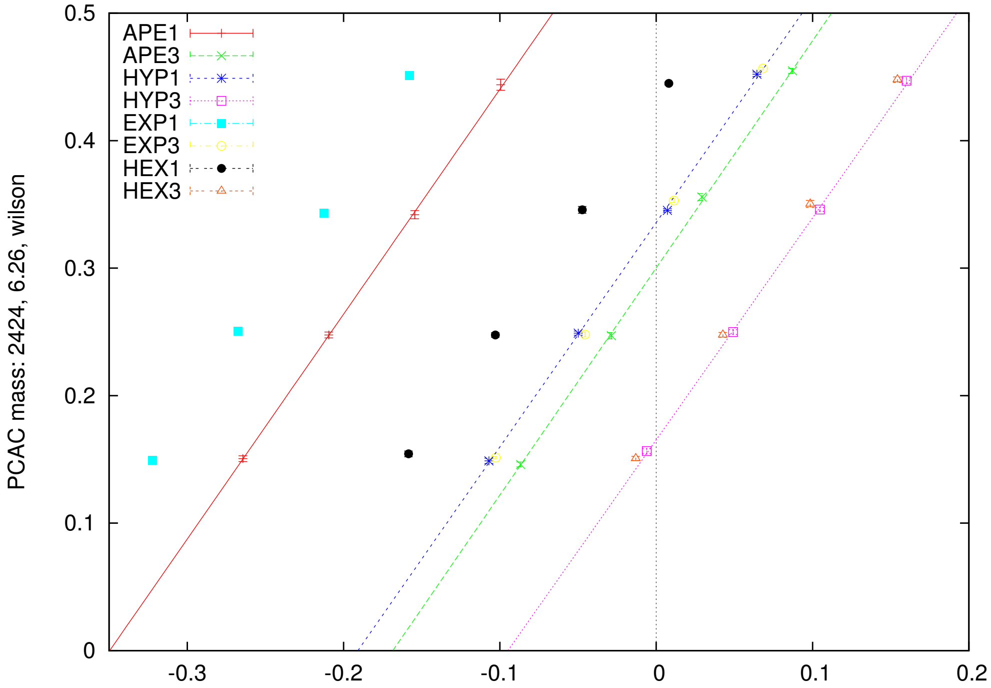


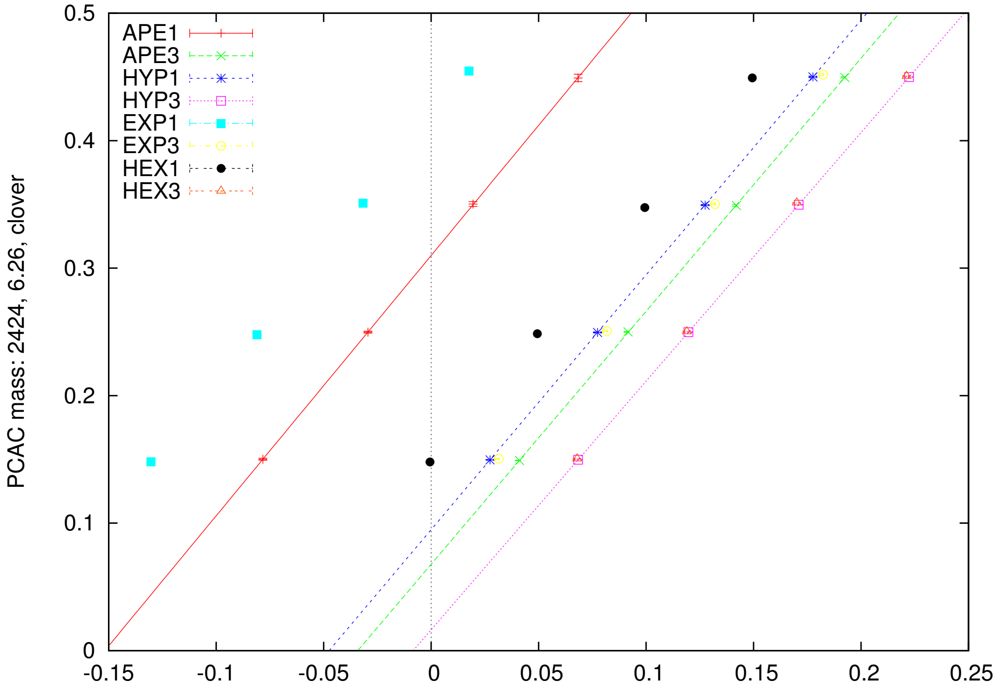


$2m^{\mathrm{PCAC}}$ versus m_0 with $c_{\mathrm{SW}}=0$ (left) and $c_{\mathrm{SW}}=1$ (right) at $\beta=6.136$:

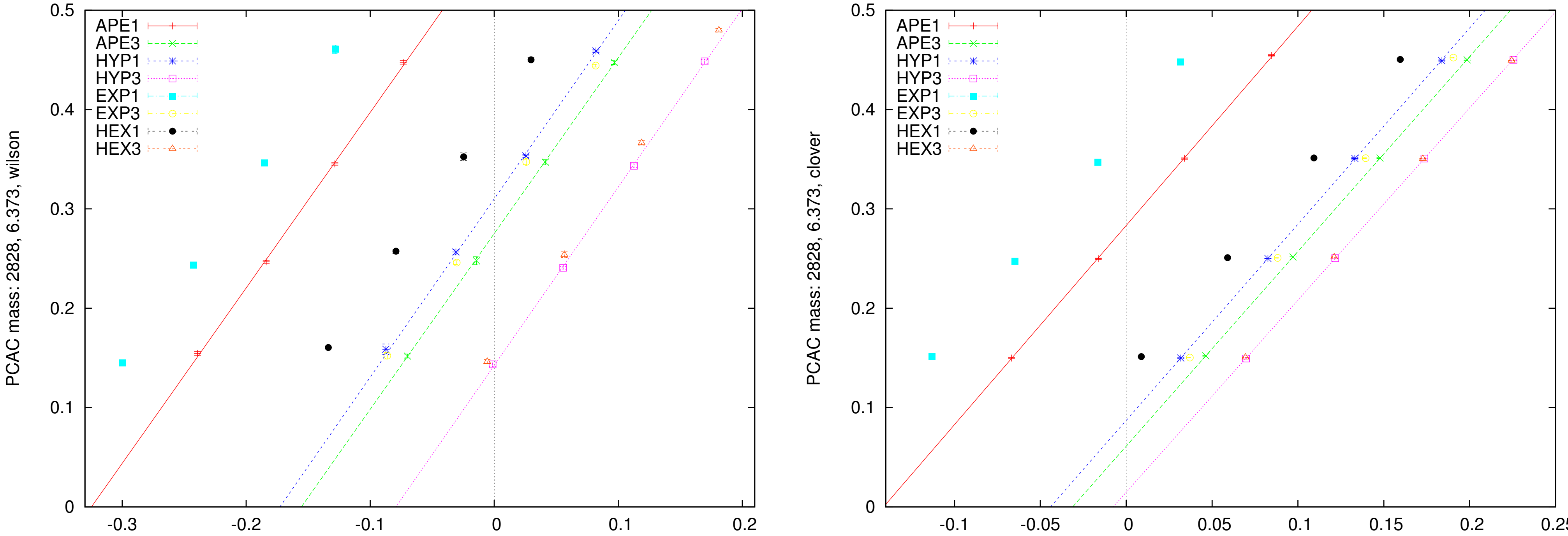


$2m^{\rm PCAC}$ versus m_0 with $c_{\rm SW}=0$ (left) and $c_{\rm SW}=1$ (right) at $\beta=6.260$:

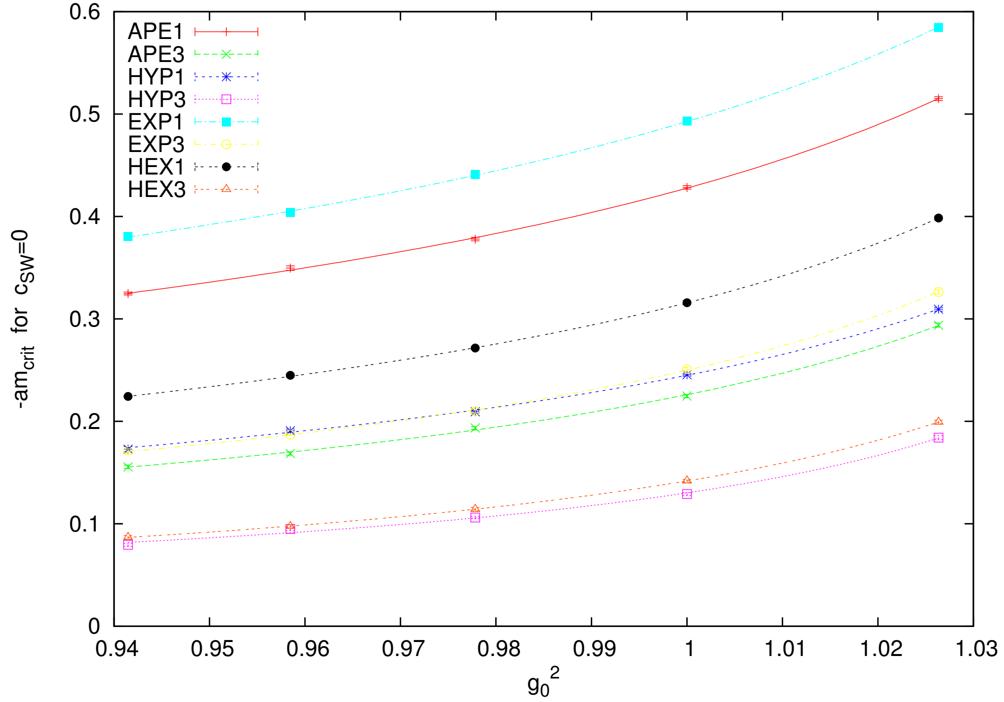


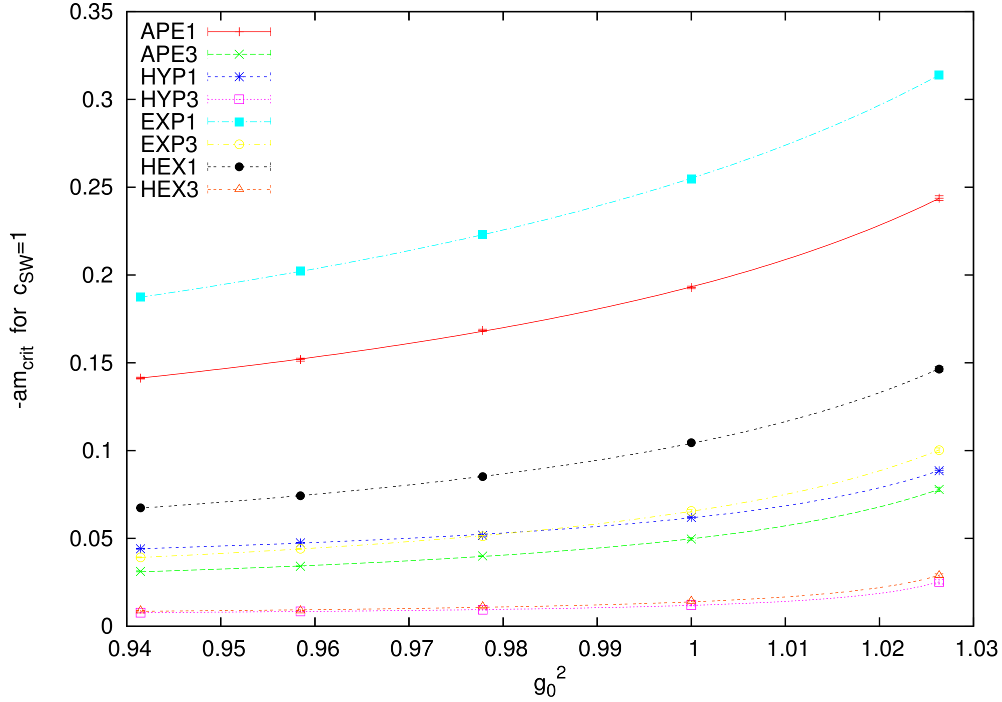


$2m^{\rm PCAC}$ versus m_0 with $c_{\rm SW}=0$ (left) and $c_{\rm SW}=1$ (right) at $\beta=6.373$:



Rational fits for $-am_{\rm crit}$





Asymptotically $-am_{\rm crit} \to \frac{S}{12\pi^2} g_0^2$ with S known for APE,HYP,EXP,HEX filtering

Note that $\tilde{Z}_A \simeq 1$ implies $|m_{\rm crit}| \simeq m_{\rm res}$

fit ansatz: $$-\,am_{\rm crit} = \frac{c_1 g_0^2 + c_2 g_0^4}{1 + c_3 g_0^2}$$

Compare fitted c_1 to the asymptotic predicition $\frac{S}{12\pi^2}$ with S given before [ALPHA] ...

	$c_{SW}=0$		$c_{SW}=1$	
pert.	0.114480		0.0414467	
1 APE / 1 EXP	0.213(12)	0.252(12)	0.0909(28)	0.1094(20)
pert.	0.040629		0.0065096	
3 APE / 3 EXP	0.077(14)	0.083(07)	0.0172(15)	0.0171(09)
pert.	0.058906		0.0167502	
1 HYP / 1 HEX	0.095(14)	0.121(04)	0.0338(12)	0.0332(16)
pert.	—		—	
3 HYP / 3 HEX	0.034(15)	0.026(01)	0.0060(02)	0.0060(15)

⟶ no quantitative agreement of c_1 with 1-loop PT in our range of couplings

⟶ disagreement smaller for more vigorous filtering (with $c_{SW}=1$)

⟶ consistency $c_1^{APE} \simeq c_1^{EXP}$ and $c_1^{HYP} \simeq c_1^{HEX}$ like in PT

	(5.846)	6.000	6.136	6.260	6.373
$m_{res}^{3\,APE}[\,MeV]$	(144)	111	107	108	113
$m_{res}^{3\,HYP}[\,MeV]$	(47)	27	25	26	27

- $O(30\,MeV)$ with UV-filtering and 0-loop c_{SW} !
- $O(10\,MeV)$ with UV-filtering and 1-loop c_{SW} ?
- compare: $O(\geq 3\,MeV)$ with thin-link DW fermions

Summary

(1) UV-filtering of D_{SW} yields a legal action for fixed $(\alpha^{\mathrm{rec}}, n^{\mathrm{iter}})$

(2) PT at 1-loop suggests that the series in g_0^2 at $O(a)$ converges well

(3) My personal guess: the non-perturbative series in a converges well

(4) Chiral symmetry breaking is reduced: $m^{\mathrm{res}} \simeq 30\,\mathrm{MeV}$ and $\frac{1}{2}(z_P - z_S) = z_V - z_A \ll 1$

(5) With EXP/stout (and maybe 1-loop c_{SW}) ready for dynamical simulations