Domain decomposition improvement of quark propagator estimation and the static-light meson spectrum

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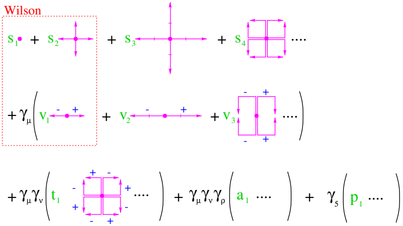
Lattice 2006

Outline

Chirally improved (CI) fermions

- Truncated pathwise expansion of the Dirac operator
- Use the Ginsparg-Wilson relation to determine coefficients

$$\gamma_5 D + D\gamma_5 = 2aD\gamma_5 RD$$



C. Gattringer, PRD63, 114501 (2001)

C. Gattringer, I. Hip, and C.B. Lang, NPB597, 451 (2001)

Dynamical CI configurations: C. B. Lang, P. Majumdar, and W. Ortner, PRD73, 034507 (2006)

Domain decomposition

Domain decomposition:
Decompose the lattice in two regions $1, 2$

$$\begin{aligned} S(\psi, \bar\psi, U) &= \bar\psi\, M\, \psi \\ &= \bar\psi_1\, M_{11}\, \psi_1 + \bar\psi_2\, M_{22}\, \psi_2 + \bar\psi_1\, M_{12}\, \psi_2 + \bar\psi_2\, M_{21}\, \psi_1 \end{aligned}$$

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \;\; ; \quad M^{-1} = P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} .$$

[see, e.g., C. Michael and J. Peisa, PRD58, 034506 (1998); M. Lüscher, CompPhysComm165, 199 (2005)]

“Open” contributions

Estimate $P = M^{-1}$ with N random sources $\vec{\chi}^n$ ($n = 1, ..., N$):

$$P\vec{\chi}^n = \vec{\eta}^n \quad \rightarrow \quad P_{ij} \approx \frac{1}{N}\eta_i^n \chi_j^{n\dagger}$$

“Recreate” sources in one region from resultant vectors everywhere:

$$\begin{aligned} \chi_1^n &= M_{11}\eta_1^n + M_{12}\eta_2^n \\ M_{11}^{-1}\chi_1^n &= [\eta_1^n] + M_{11}^{-1}M_{12}\eta_2^n \end{aligned}$$

Substitute back into the original estimator:

$$\begin{aligned} P_{12} &\approx \frac{1}{N}\left[M_{11}^{-1}\left(\cancel{\chi_1^n} - M_{12}\eta_2^n\right)\right]\chi_2^{n\dagger} \quad \xrightarrow{N\to\infty} \\ &= -M_{11}^{-1}M_{12}P_{22} \quad \text{(exact!)} \end{aligned}$$

(Schur complement)

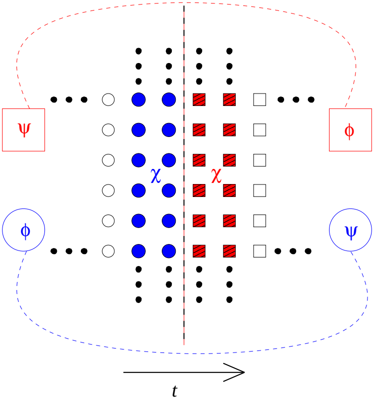
Source re-insertion and γ_5-Hermiticity

$$
\begin{aligned}
P_{12} &= -M_{11}^{-1} M_{12} P_{22} \\
&\approx -M_{11}^{-1} M_{12} \frac{1}{N} \chi_2^n \chi_2^{n\dagger} P \\
&\approx -\frac{1}{N} \left(M_{11}^{-1} M_{12} \chi_2^n \right) \left(\gamma_5 P \gamma_5 \chi_2^n \right)^\dagger \\
&\approx -\frac{1}{N} \psi_1^n \phi_2^{n\dagger} \, . \qquad \text{(“open” contributions)}
\end{aligned}
$$

Note: No sources needed in region 1 and those in region 2 should reach region 1 with one application of M.

Similar to “maximal variance reduction” (Michael & Peisa 1998), but we can use M, rather than $M^\dagger M$.

Implementation



$$\psi_1^n = M_{11}^{-1} M_{12} \chi_2^n$$

$$\phi_2^n = \gamma_5 P \gamma_5 \chi_2^n$$

- Sources on time slices connected to the other region by *M*
(8 for CI, 4 shown here)
- Spin dilution

“Closed” contributions

The propagator within one region:

$$
\begin{aligned}
P_{11} &= M_{11}^{-1} - M_{11}^{-1} M_{12} P_{21} \\
&= M_{11}^{-1} + M_{11}^{-1} M_{12} \gamma_5 \left(M_{11}^{-1} M_{12} P_{22} \right)^{\dagger} \gamma_5
\end{aligned}
$$

Keeping region 1 small (1 point or + Nearest Neighbors):

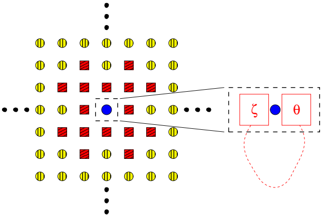
$$
\begin{aligned}
P_{11} &\approx M_{11}^{-1} + \frac{1}{N} \left(M_{11}^{-1} M_{12} \gamma_5 \chi_2^{n} \right) \left(\gamma_5 M_{11}^{-1} M_{12} P \chi^{n} \right)^{\dagger} \\
&\approx M_{11}^{-1} + \frac{1}{N} \zeta_1^{n} \theta_1^{n\dagger}
\end{aligned}
$$

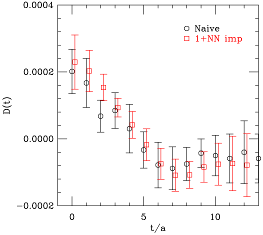
Some improvement: e.g.,

$D(t) = \sum_{t_0,\vec{x},\vec{y}} \mathrm{Tr}(\gamma_5 P_{\vec{x},t_0;\vec{x},t_0}) \mathrm{Tr}(\gamma_5 P_{\vec{y},t_0+t;\vec{y},t_0+t})$

@ $m_q \approx 0.23 m_s$ on 1 quenched config.

(jackknifed over the 12 random spin-color sources):





Correlator matrix

Using different source and sink operators, construct a matrix of correlators:

$$\begin{aligned} C_{ij}(t) &= \langle O_j O_i^\dagger \rangle \\ &= \sum_k a_j^{(k)} a_i^{(k)*} e^{-t\,E_k} \end{aligned}$$

Generalized eigenvalue problem:

$$C(t)\vec{\psi}^{(k)} = \lambda^{(k)}(t,t_0)\,C(t_0)\vec{\psi}^{(k)} \ ,$$

$$\lambda^{(k)}(t,t_0) \propto e^{-t\,E_k}[1+\mathcal{O}(e^{-t\,\Delta M_k})]$$

C. Michael, NPB259, 58 (1985)

M. Lüscher and U. Wolff, NPB339, 222 (1990)

Hadron interpolators

$$\bar{Q}\, O(\Gamma, D_i, \vec{D}^2, S)\, q$$

Local source for static quark ($\bar{Q}$)
Jacobi smearing $S(\kappa_{sm}, N_{sm})$ for light quark (q), then Laplacians:

$$q' = \quad S(0.2, 8)\, q \quad , \quad \vec{D}^2 S(0.2, 12)\, q \quad , \quad \vec{D}^2\vec{D}^2 S(0.2, 16)\, q$$

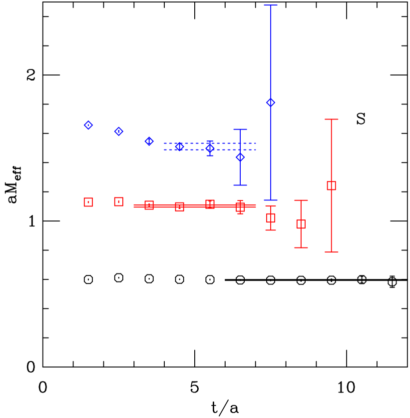
oper.	J^P	$\bar{Q}\, O(\Gamma, D_i)\, q'$
S	$0^-, 1^-$	$\bar{Q}\, \gamma_5\, q'$
P_-	$0^+, 1^+$	$\bar{Q}\, \sum_i \gamma_i D_i\, q'$
P_+	$1^+, 2^+$	$\bar{Q}\, (\gamma_1 D_1 - \gamma_2 D_2)\, q'$
$D_\pm$	$1^-, 2^-, 3^-$	$\bar{Q}\, \gamma_5 (D_1^2 - D_2^2)\, q'$

Static-light correlators

$$C_{ij}(t) = \left\langle 0 \left| (\bar{Q}\, O_j\, q)_t\ (\bar{q}\, \bar{O}_i\, Q)_0 \right| 0 \right\rangle$$
$$= \left\langle \sum_x \mathrm{Tr}\left[\frac{1+\gamma_4}{2} \prod_{i=0}^{t-1} U_4^\dagger(x+i\hat{4})\, O_j P_{x+t\hat{4},x} \bar{O}_i \right] \right\rangle_{\{U\}}$$

All points in region 1 ($N_s^3 \times N_t/2$) may be used for starting point x.

S-wave effective masses



$$aM_{eff}^{(k)}\left(t+\frac{1}{2}\right)=\ln\left(\frac{\lambda^{(k)}(t)}{\lambda^{(k)}(t+1)}\right)$$

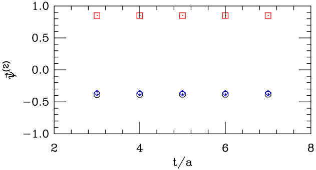
12 random spin-color sources (48 spin-diluted)

100 configs with LW gauge action, $\beta = 7.90$.

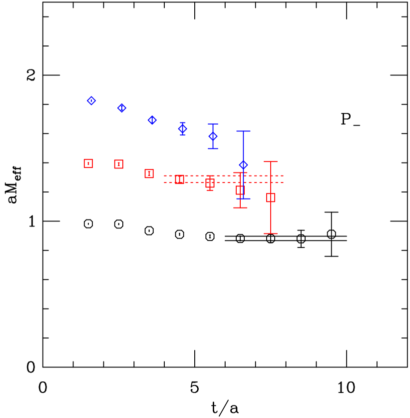
$a \approx 0.15$ fm ($a^{-1} \approx 1330$ MeV)

$L \approx 1.8$ fm

Require plateaus in $\vec{\psi}^{(k)}$ also; e.g., $2S$:



P-wave effective masses



$$aM_{\mathit{eff}}^{(k)}\left(t+\frac{1}{2}\right)=\ln\left(\frac{\lambda^{(k)}(t)}{\lambda^{(k)}(t+1)}\right)$$

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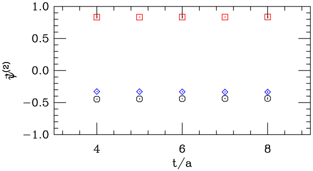
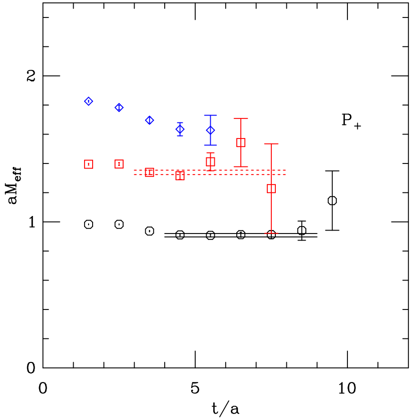


P-wave effective masses



$$aM_{eff}^{(k)}\left(t+\frac{1}{2}\right) = \ln\left(\frac{\lambda^{(k)}(t)}{\lambda^{(k)}(t+1)}\right)$$

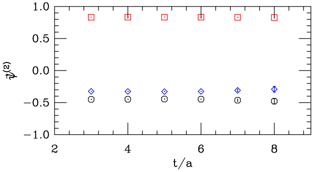
12 random spin-color sources (48 spin-diluted)

100 configs with LW gauge action, $\beta = 7.90$.

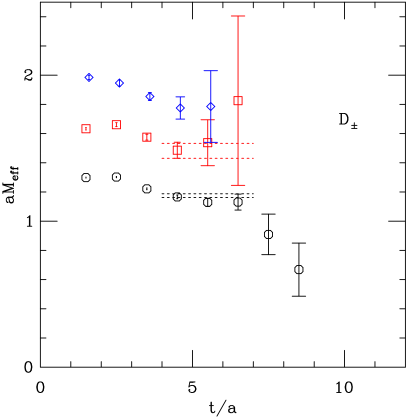
$a \approx 0.15$ fm ($a^{-1} \approx 1330$ MeV)

$L \approx 1.8$ fm

Require plateaus in $\vec{\psi}^{(k)}$ also; e.g., $2P_+$:



D-wave effective masses



$$aM^{(k)}_{\mathit{eff}}\left(t+\frac{1}{2}\right)=\ln\left(\frac{\lambda^{(k)}(t)}{\lambda^{(k)}(t+1)}\right)$$

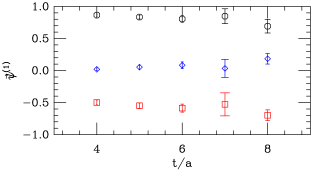
12 random spin-color sources (48 spin-diluted)

100 configs with LW gauge action, $\beta = 7.90$.

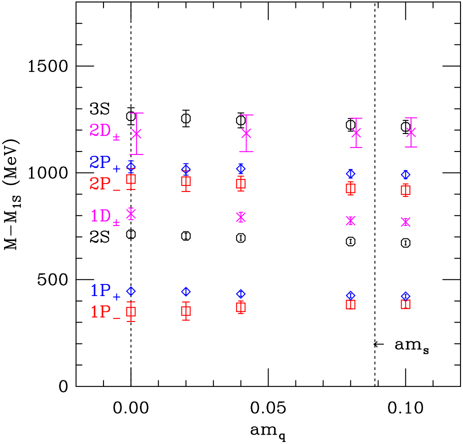
$a \approx 0.15$ fm ($a^{-1} \approx 1330$ MeV)

$L \approx 1.8$ fm

Require plateaus in $\vec{\psi}^{(k)}$ also; e.g., $1D_{\pm}$:



Chiral extrapolations of mass differences



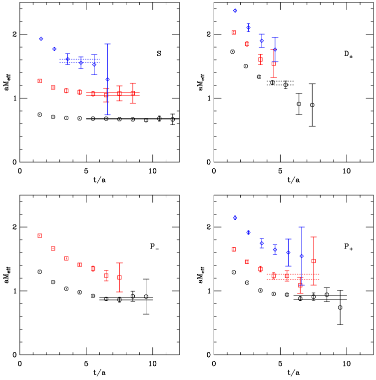
$M_{\pi,\text{sea}} = \infty$

$a^{-1} \approx 1330$ MeV

$L \approx 1.8$ fm

Errors are only statistical for the chosen fit ranges.

Effective masses (dynamical)



$$aM_{\mathit{eff}}^{(k)}\left(t+\frac{1}{2}\right)=\ln\left(\frac{\lambda^{(k)}(t)}{\lambda^{(k)}(t+1)}\right)$$

12 random spin-color sources (48 spin-diluted)

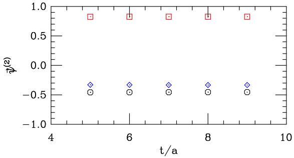
74 configs with LW gauge action and

$N_f = 2$ CI dynamical quarks, $M_{\pi,\mathrm{sea}} \approx 500$ MeV.

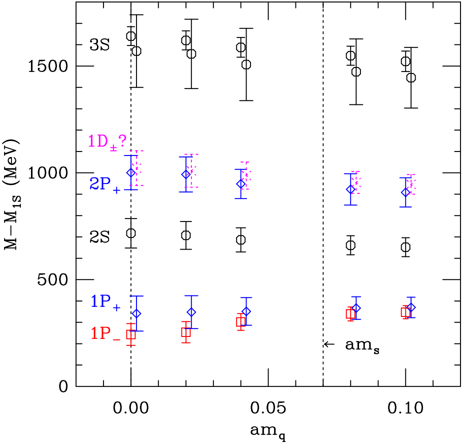
$a \approx 0.115$ fm ($a^{-1} \approx 1710$ MeV)

$L \approx 1.4$ fm

Require plateaus in $\vec{\psi}^{(k)}$ also; e.g., $2S$:



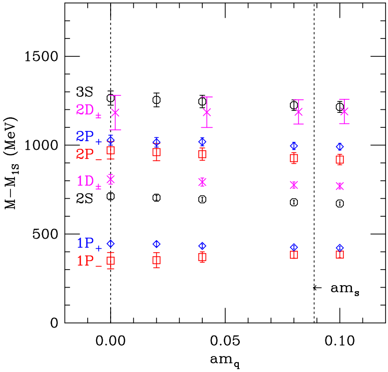
Chiral extrapolations of mass differences (dyn.)



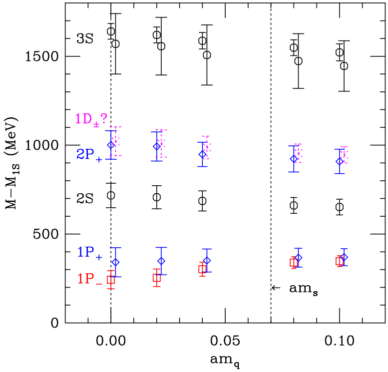
$M_{\pi,\text{sea}} \approx 500$ MeV
$a^{-1} \approx 1710$ MeV
$L \approx 1.4$ fm
Errors are only statistical for the chosen fit ranges.

Chiral extrapolations of mass differences

$N_f = 0, L \approx 1.8$ fm:



$N_f = 2, L \approx 1.4$ fm:



Mass differences for *B* mesons

B mesons:

state	J^P	$M - M_{1S}$ (MeV)		
		$M_{\pi,\text{sea}} = \infty$	$M_{\pi,\text{sea}} \approx 500$ MeV	PDG
		$L \approx 1.8$ fm	$L \approx 1.4$ fm	
$2S$	$0^-, 1^-$	712(14)	717(69)	-
$3S$	$0^-, 1^-$	$1265(40)(^{+0}_{-130})$	$1640(44)(^{+55}_{-200})$	-
$1P_-$	$0^+, 1^+$	350(46)	243(51)	384(8)
$2P_-$	$0^+, 1^+$	$971(49)(^{+50}_{-90})$	-	-
$1P_+$	$1^+, 2^+$	446(15)	341(82)	384(8)
$2P_+$	$1^+, 2^+$	$1028(28)(^{+160}_{-80})$	$1001(80)(^{+130}_{-20})$	-
$1D_\pm$	$1^-, 2^-, 3^-$	$808(27)(^{+0}_{-90})$	1022(81)(??)	-
$2D_\pm$	$1^-, 2^-, 3^-$	$1183(97)(^{+130}_{-150})$	-	-

Mass differences for B_s mesons

B_s mesons:

state	J^P	$M - M_{1S}$ (MeV)		
		$M_{\pi,\text{sea}} = \infty$	$M_{\pi,\text{sea}} \approx 500$ MeV	PDG
		$L \approx 1.8$ fm	$L \approx 1.4$ fm	
$2S$	$0^-, 1^-$	675(10)	665(45)	-
$3S$	$0^-, 1^-$	$1220(30)(^{+20}_{-50})$	$1560(45)(^{+35}_{-190})$	-
$1P_-$	$0^+, 1^+$	384(20)	330(34)	448(16)
$2P_-$	$0^+, 1^+$	$923(30)(^{+10}_{-60})$	-	-
$1P_+$	$1^+, 2^+$	424(10)	363(55)	448(16)
$2P_+$	$1^+, 2^+$	$993(20)(^{+130}_{-50})$	$930(75)(^{+0}_{-80})$	-
$1D_\pm$	$1^-, 2^-, 3^-$	$773(17)(^{+0}_{-80})$	960(55)(??)	-
$2D_\pm$	$1^-, 2^-, 3^-$	$1188(68)(^{+170}_{-80})$	-	-

Conclusions / Outlook

Conclusions:

- Quark propagator estimation between regions is greatly improved.
  Some improvement for closed loops.
- Successful isolation of excited B mesons via variational method (diagonalization of correlator matrix).
  (For more applications of this method, see talk by C. Hagen and poster by C. Ehmann.)
- A number of excited states found: $2S$, $3S$, $1P$, $2P$, $1D$, $2D$.
- $1P$-$1S$ splitting is consistent with experiment for B systems (except dynamical $1P_-$), but too low for B_s.
  These splittings are also consistently lower (by $\approx 1\sigma - 2\sigma$) on the smaller, dynamical lattices.
- Possible finite-volume effects ($M_{3S} - M_{1S}$ increases by $\approx 300 - 400$ MeV as $L \approx 1.8 \longrightarrow 1.4$ fm).
- Better statistics and larger volumes are needed to resolve matters.

For the future:

- More statistics: 3 other quark masses of dynamical CI lattices at the same volume; and larger ones are on the way...
- More interpolators: e.g., local operators, different smearings, D_-, D_+, $F_\pm$.
- Heavy-quark spin interactions to remove degeneracies: NRQCD-light.

Hope to see you next year!

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Nur echt mit dem Bock!