

Study of unquenching effects on meson masses and decay constants

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- the sap algorithm
- simulation parameters
- plateau analysis
- partially quenched QCD reference point
- full QCD reference point
- conclusions

the SAP procedure is based on a “geometric” separation of UV and IR modes

[Lüscher 04]

- Dirichlet b.c. within the blocks provide $q \geq \pi/L$
- block interactions are small $S(x, y) \sim 1/|x - y|^3$

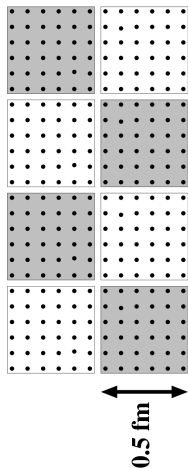
- the Wilson Dirac operator can be decomposed as

$$\begin{aligned} D &= \frac{1}{2} \left\{ \gamma_\mu \left(\nabla_\mu^* + \nabla_\mu \right) - \nabla_\mu^* \nabla_\mu \right\} + m \\ &= D_{\Omega^*} + D_\Omega + D_{\partial\Omega^*} + D_{\partial\Omega} \end{aligned}$$

- where

$$D_{\Omega^*} = \sum_{\text{white } \Lambda} D_\Lambda \quad D_\Omega = \sum_{\text{black } \Lambda} D_\Lambda$$

- Ω^* contains the white blocks while Ω contains the black ones
- $\partial\Omega^*$ and $\partial\Omega$ are the “exterior boundaries”



- the determinant of the full operator can be decomposed as

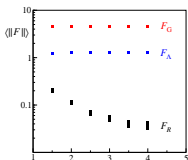
$$\det D = \det \hat{D}_\Lambda \det R$$

where

$$R = 1 - P_{\partial\Omega^*} D_\Omega^{-1} D_{\Omega^*}^{-1} D_{\partial\Omega}$$

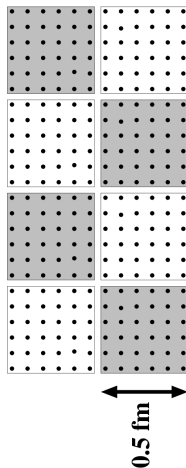
- two flavor QCD can be simulated by introducing two scalar fields

$$S_{N_f=2} = \sum_{\text{all } \Lambda} \|\hat{D}_\Lambda^{-1} \phi_\Lambda\|^2 + \|R^{-1} \chi_{\partial\Omega^*}\|^2$$



- as a consequence, there is a “natural” hierarchy of the forces

$$\epsilon_G \|F_G\| \sim \epsilon_\Lambda \|F_\Lambda\| \sim \epsilon_R \|F_R\|$$



nick	β	lattice	c_{SW}	k	N_{conf}
A_1	5.6	32×24^3	0.0	0.15750	64
A_2				0.15800	109
A_3				0.15825	100
A_4				0.15835	100
B_1	5.8	64×32^3	0.0	0.15410	100
B_2				0.15440	101
B_3				0.15455	104
B_4				0.15462	102
D_1	5.3	48×24^3	1.90952	0.13550	104
D_2				0.13590	171
D_3				0.13610	168
D_4				0.13620	168
D_5				0.13625	151

All the results presented in this talk have been obtained in **collaboration** with:

Luigi Del Debbio
Leonardo Giusti
Martin Lüscher
Roberto Petronzio

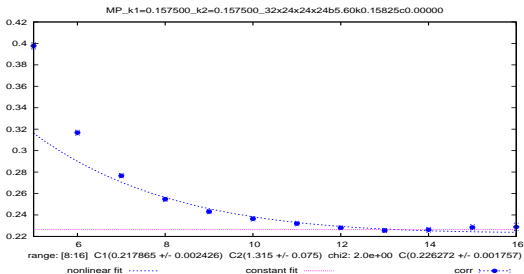
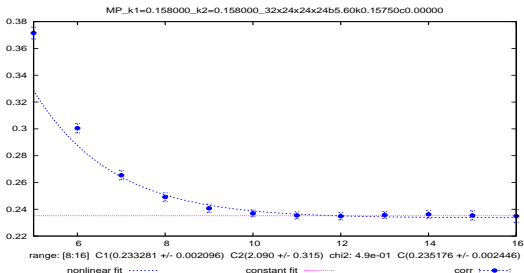
$N_f = 2$ QCD is more subtle than $N_f = 0$ QCD ... excited states contain multi-pion states

$$M_{12}^{\text{eff}}(t) \simeq M_{12} + c e^{-(M_{12}+2M_\pi)t}$$

● A_1 lattice: $k_1 = k_2 \neq k_{\text{sea}}$

● A_3 lattice: $k_1 = k_2 \neq k_{\text{sea}}$

● the two ground states practically coincide while in the up plot the pion is heavier

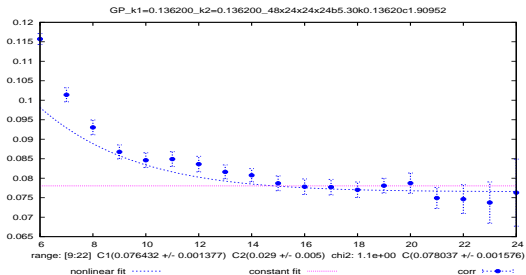


plateau analysis: pseudoscalar residue

the $\langle P(t)P(0) \rangle$ residue is related to the pseudoscalar decay constant through the PCAC relation

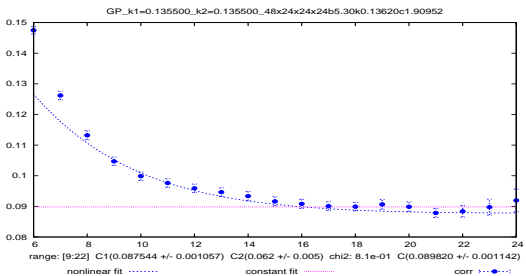
- D_4 lattice: $k_1 = k_2 = k_{sea}$

$$G_{\pi}^{\text{eff}}(t) \simeq G_{\pi} + c e^{-3M_{\pi}t}$$



- D_4 lattice: $k_1 = k_2 \neq k_{sea}$

$$G_{12}^{\text{eff}}(t) \simeq G_{12} + c e^{-(M_{12}+2M_{\pi})t}$$

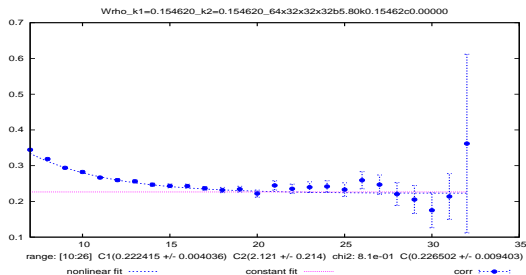


we have used a variational technique (correlation matrix with different Jacobi smearing steps)

- B_4 lattice: $k_1 = k_2 = k_{sea}$

$$W_\rho^{\text{eff}}(t) \simeq W_\rho + c e^{-E_1 t}$$

$$E_1 = \sqrt{M_\pi^2 + \left(\frac{2\pi}{L}\right)^2}$$



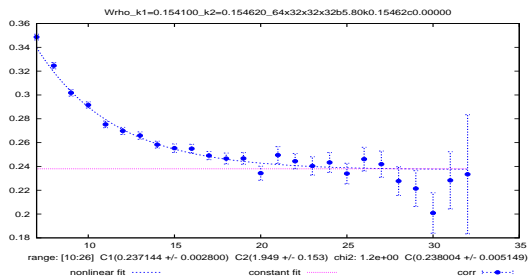
- B_4 lattice: $k_1 \neq k_2$

$$W_{V12}^{\text{eff}}(t) \simeq W_{V12} + c e^{-E_1 t}$$

$$E_1 = E_{1sea} + E_{sea2}$$

$$E_{1sea} = \sqrt{M_{1sea}^2 + \left(\frac{2\pi}{L}\right)^2}$$

$$E_{sea2} = \sqrt{M_{sea2}^2 + \left(\frac{2\pi}{L}\right)^2}$$



- on each lattice we study the ratio:

$$\left[\frac{M_P(k_{\text{sea}}, k_1)}{M_V(k_{\text{sea}}, k_1)} \right]^2 \quad \text{v.s.} \quad \frac{1}{k_1}$$

and we find the value $k_1 = k_{\text{ref}}$ such that

$$R(k_{\text{sea}}) = \left[\frac{M_P(k_{\text{sea}}, k_{\text{ref}})}{M_V(k_{\text{sea}}, k_{\text{ref}})} \right]^2 = \left[\frac{M_K}{M_{K^*}} \right]^2$$

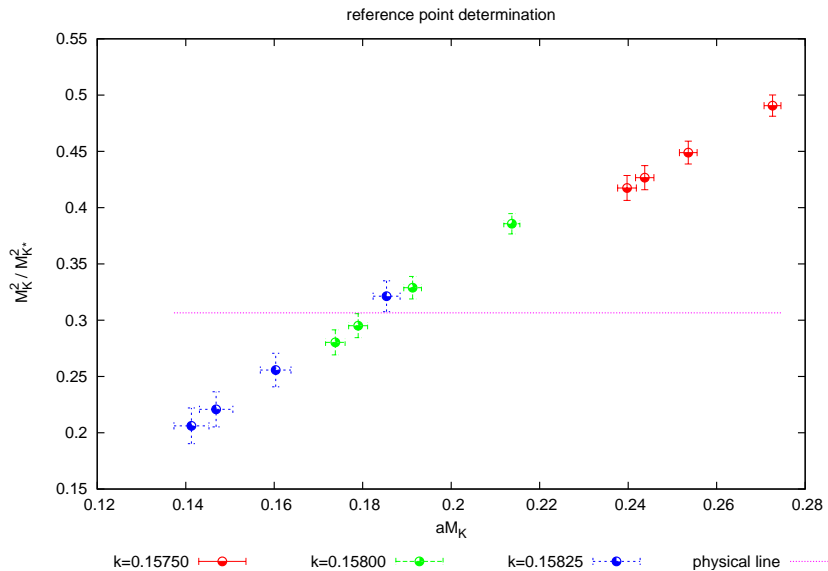
- then, at fixed β , we study

$$\mathcal{O}(k_{\text{sea}}, k_{\text{ref}}) \quad \text{v.s.} \quad \frac{1}{k_{\text{sea}}}$$

obtaining that for sufficiently small sea quark masses the observable is flat

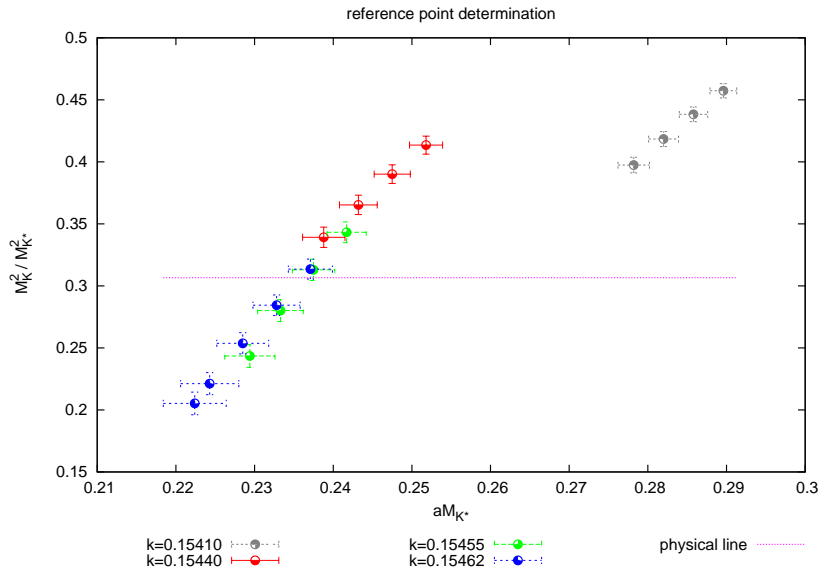
- in this way we have a possible definition of m_{strange} , M_K , M_{K^*} and F_K

partially quenched reference point: A^* lattices



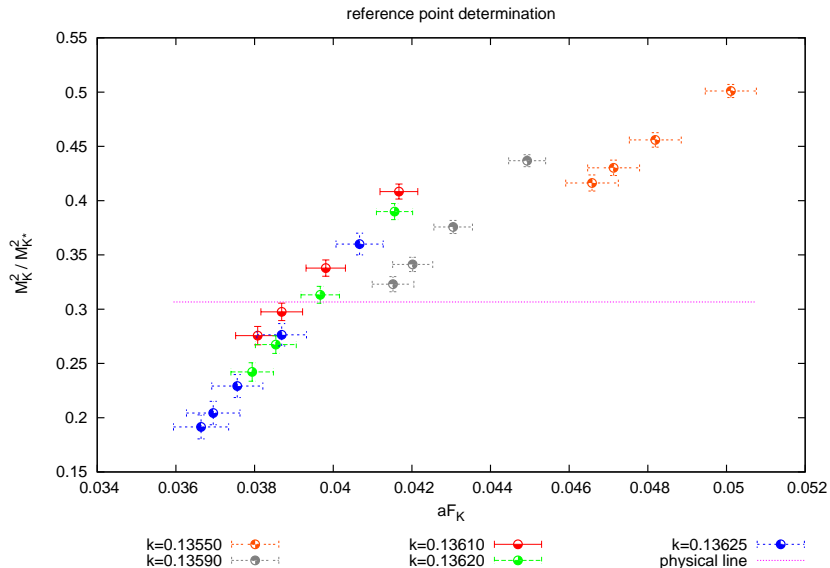
the physical volume is $L \simeq 1.7$ fm

partially quenched reference point: B^* lattices



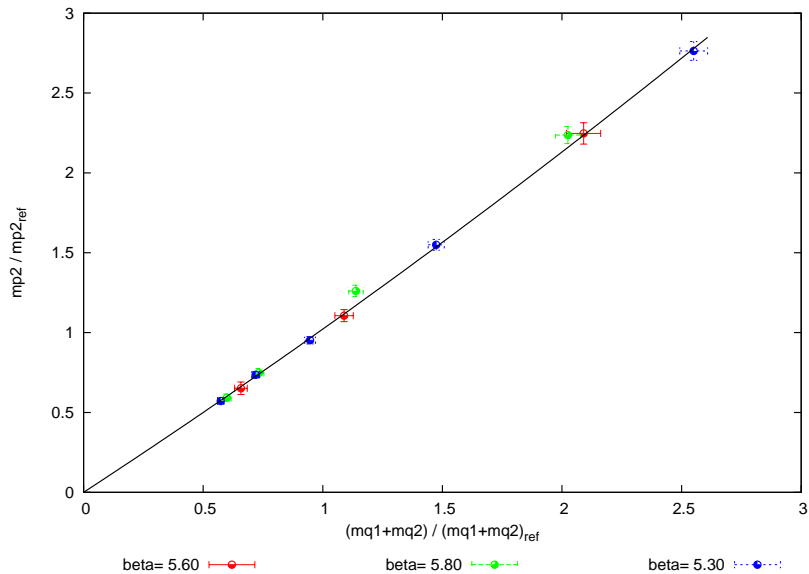
the physical volume is $L \simeq 1.7$ fm

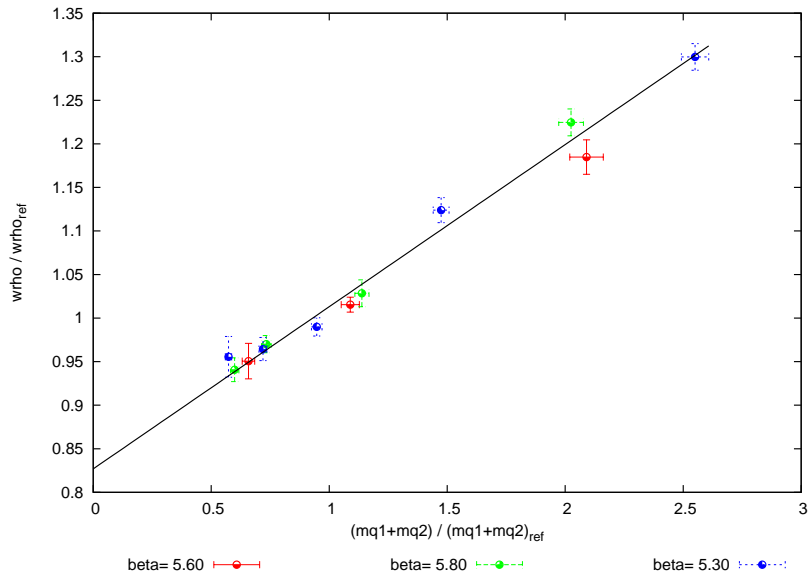
partially quenched reference point: D^* lattices



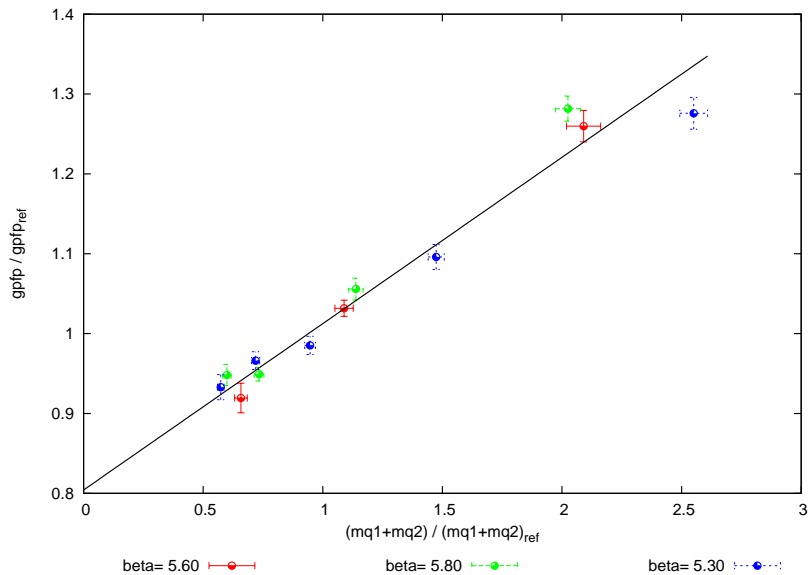
the physical volume is $L \simeq 1.9$ fm

universality: pseudoscalar meson masses





universality: pseudoscalar meson decay constants

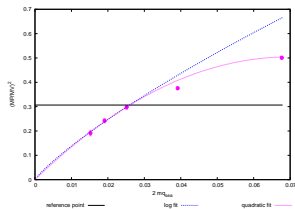
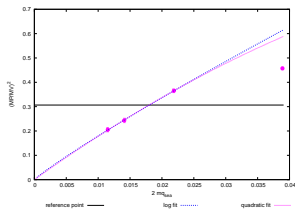
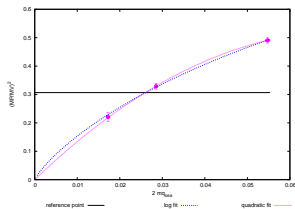


another possibility to fix the reference point is:

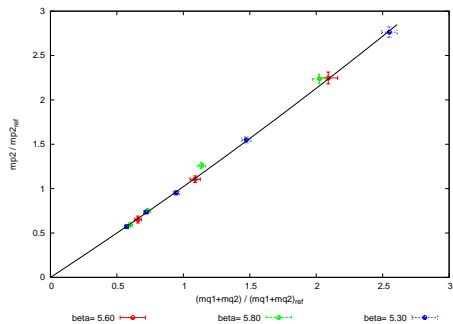
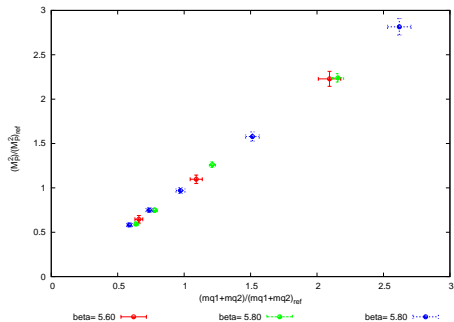
- at fixed bare coupling we consider only the full QCD observables $\mathcal{O}(m_{\text{sea}}, m_{\text{sea}})$ and we find the value m_{ref} such that

$$R(m_{\text{ref}}) = \left[\frac{M_P(m_{\text{ref}}, m_{\text{ref}})}{M_V(m_{\text{ref}}, m_{\text{ref}})} \right]^2 = \left[\frac{M_K}{M_{K^*}} \right]^2$$

- then we interpolate $\mathcal{O}(m_{\text{sea}}, m_{\text{sea}})$ in order to get $\mathcal{O}(m_{\text{ref}}, m_{\text{ref}})$



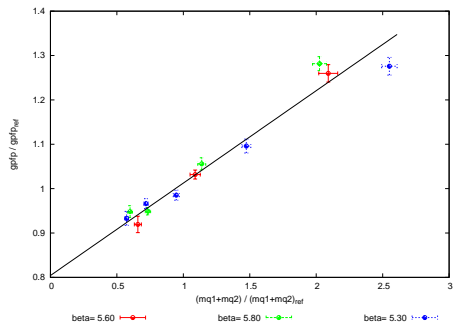
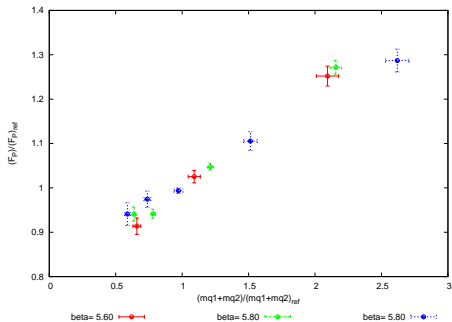
partially quenched reference point



full QCD reference point

reference points comparison: pseudoscalar meson decay constants

partially quenched reference point



full QCD reference point

- we are able to simulate with reasonable volumes and reasonable lattice spacings pions of masses from 500 MeV down to 300 MeV.
- the inclusion of the sea quarks complicates the extraction of the hadronic observables
 - multi hadron states contaminate 2pt correlation functions
 - previous results for the vector meson masses could have been over estimated
- masses and decay constants of the mesons containing a light and a “non-light” quark can be computed with good precision (in the “chiral limit”)
 - unquenching effects saturate
- as a consequence, the p.q. reference point can be extracted with good precision
 - note: we use p.q. **only** to set the scale
- to check possible systematics due to finite volume effects we are currently simulating a volume of about 2.5 fm.