Charmed Spectroscopy from a Nonperturbatively Determined Relativistic Heavy Quark Action in Full QCD

Huey-Wen Lin
for the RBC collaboration

Columbia University

Motivations

- Flavor physics and CP violation play an important role in particle physics.
- Problem:
  - $(ma) \ll 1$ no longer true; $(ma)^n$ terms are significant
  - Too expensive to directly simulate heavy quarks by brute force
- Multiple fermion actions in lattice calculations:
  - NRQCD has no continuum limit
  - HQET cannot be applied to quarkonia
  - Anisotropic lattice might contain $O(\alpha_s m a_s)$
- Relativistic heavy quark (RHQ) action
  - Systematically absorb mass factor into the coefficients
  - Small cutoff effects: $\alpha_s \Lambda_{\rm QCD} a$ and $(\Lambda_{\rm QCD} a)^2$
- High precision calculations require better determination of the action

Outline

- Introduction
  - Relativistic Heavy Quark (RHQ) action
- Nonpertubatively determined RHQ action
  - Proposal
  - Simulations details
  - Analysis and results
  - Charmonium and Charm quark mass
- Full QCD
  - Test run: applying quenched coefficients directly on dynamical configurations
  - NP tune RHQ action in full QCD
  - Charmonium and D/D_s
- Conclusion and Outlook

Relativistic Heavy Quark (RHQ) Action

- Operators up to $O(a)$ constrained by parity and charge conjugation

Order	Operators
$O(a^{-1})$	$m\overline{\psi}\psi$
$O(a^0)$	$\overline{\psi}\gamma_0 D_0\psi,\ \overline{\psi}\vec{\gamma}\cdot\vec{D}\psi$
$O(a^1)$	$a\overline{\psi}D_0^2\psi,\ a\overline{\psi}(\vec{D})^2\psi,$ $a\overline{\psi}\left[\gamma_0,\gamma_i\right]\left[D_0,D_i\right]\psi, a\overline{\psi}\left[\gamma^i,\gamma^j\right]\left[D^i,D^j\right]\psi,$ $a\overline{\psi}\left[\gamma^i,\gamma^0\right]\left[D^i,D^0\right]\psi$

- Field transformations are useful to determine the redundant operators

A. El-Khadra et. al., Phys. Rev. D55 (1997) 3933

- "RHQ" power counting & $O(a^2)$ field transformation

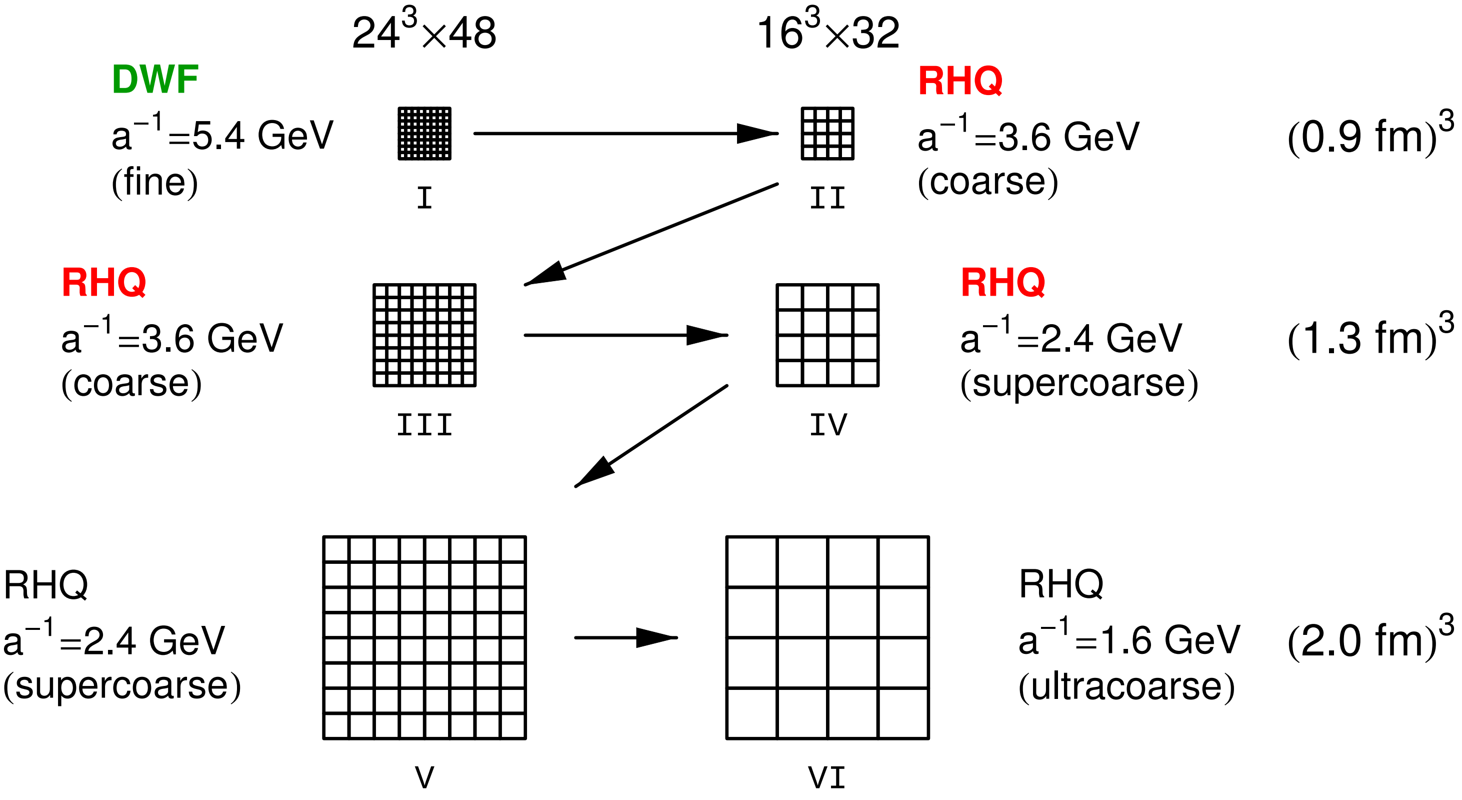
As discussed in Norman Christ's talk

$$S = \sum_n \overline{\psi_n}\left\{m_0+\gamma_0 D_0-\frac{1}{2}aD_0^2+\zeta\left[\vec{\gamma}\cdot\vec{D}-\frac{1}{2}a(\vec{D})^2\right]-\sum_i\frac{i}{4}c_P a\sigma_{\mu\nu}F_{\mu\nu}\right\}\psi_n$$

- 3 parameters: m_0, ζ, c_P

How do we determine m_0, ζ, c_P?

- Lattice perturbation calculation: hard to control errors
- Fit to experimental values: lose predictive power
- Step-scaling technique



$24^3\times48$ $16^3\times32$

DWF
a^{-1}=5.4 GeV
(fine)

I → II

RHQ
a^{-1}=3.6 GeV
(coarse)

$(0.9\text{ fm})^3$

RHQ
a^{-1}=3.6 GeV
(coarse)

III → IV

RHQ
a^{-1}=2.4 GeV
(supercoarse)

$(1.3\text{ fm})^3$

RHQ
a^{-1}=2.4 GeV
(supercoarse)

V → VI

RHQ
a^{-1}=1.6 GeV
(ultracoarse)

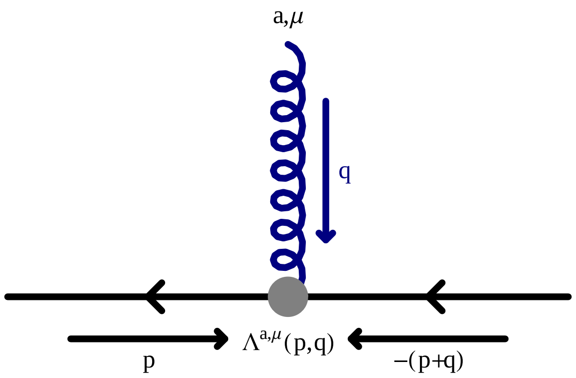
$(2.0\text{ fm})^3$

with scaling factor $\sigma = 1.5$

Matched Quantities

- Off-shell quantities: quark propagators and quark-gluon vertex

Quark–GluonVertex



H.-W. Lin, Phys. Rev. D73 (2006) 094511

Disadvantage:
- More parameters to determine at one time
- Need more computational resources

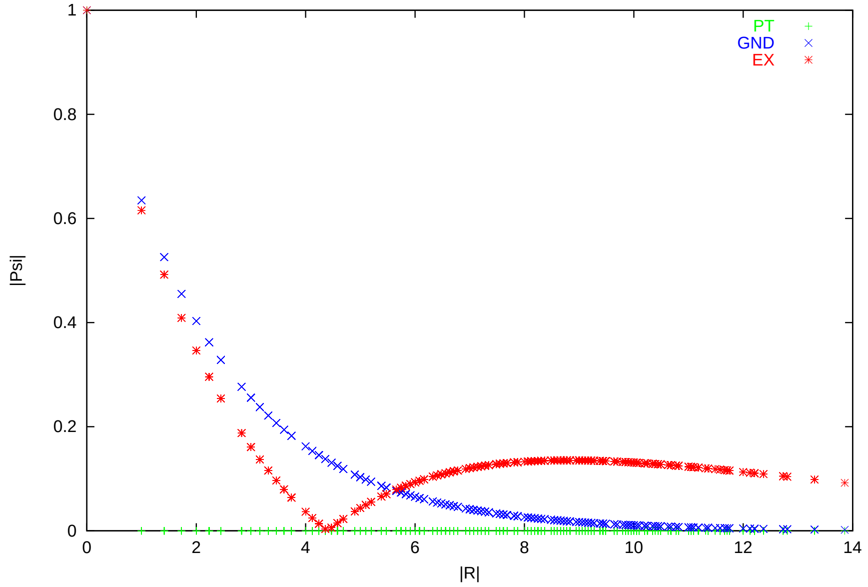
- **On-shell quantities**:

  1. Spin-averaged: $m_{\text{sa}}^{hh} = \frac{1}{4}\left(m_{\text{PS}}^{hh} + 3m_{\text{V}}^{hh}\right)$, $m_{\text{sa}}^{hl} = \frac{1}{4}\left(m_{\text{PS}}^{hl} + 3m_{\text{V}}^{hl}\right)$
  2. Hyperfine splitting: $m_{\text{hs}}^{hh} = m_{\text{V}}^{hh} - m_{\text{PS}}^{hh}$, $m_{\text{hs}}^{hl} = m_{\text{V}}^{hl} - m_{\text{PS}}^{hl}$
  3. Spin-orbit averaged and splitting: $m_{\text{soa}}^{hh} = \frac{1}{4}\left(m_{\text{S}}^{hh} + 3m_{\text{AV}}^{hh}\right)$, $m_{\text{sos}}^{hh} = m_{\text{AV}}^{hh} - m_{\text{S}}^{hh}$
  4. Dispersion relation: $E^2 = m^2 + c^2p^2$.

H.-W. Lin and N. Christ, PoS LAT2005, 225

Simulations

- Hardware: QCDOC 512-node machine at frequency 420 MHz, reproducibility checked.
- Quenched Wilson gauge action with heatbath algorithm.
- 20,000 sweeps for thermalization and 10,000 sweeps separation.
- Coulomb gauge-fixed hydrogenic source smearing



- 100 configurations.

How to...

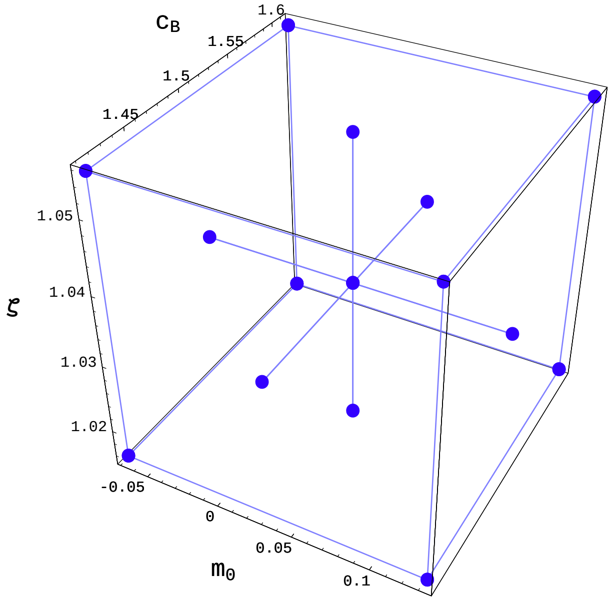
1. Choose the coarse lattice parameters?

- Guess
- Develop a linear interpretation: $Y^{i,d}_{\text{coarse}} = A^d + J^d \cdot X^i_{\text{RHQ}}$.

$\Uparrow$ predicted masses

$\Uparrow$ coarse lattice parameters

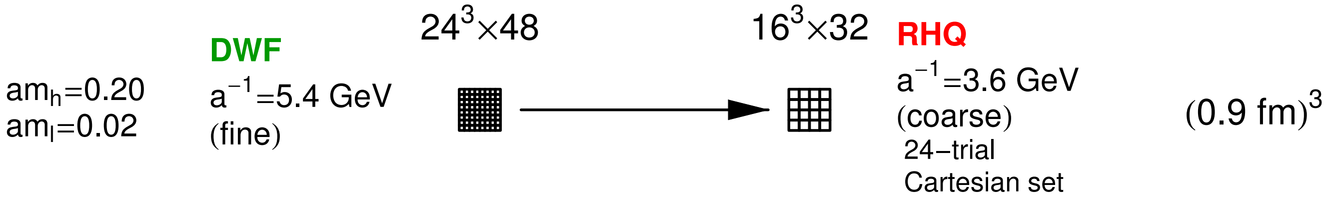
2. Get J and A?



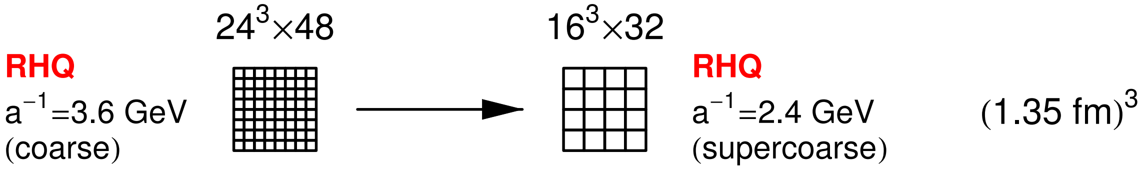
- Fitting parameters
- Finite differences directly from Cartesian set
- These two approaches agree within errors with output parameters

Results

- A variety of $a_f m_h$, ranging from 0.16 to 0.30
- First matching:



- Second matching:



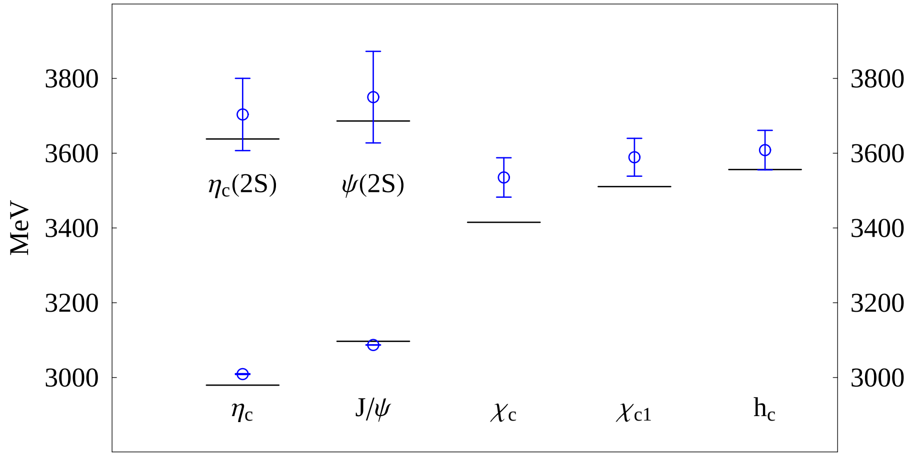
- **Result:** coefficients for $a^{-1} = 2.4$ GeV lattice, as functions of mass

$$\begin{aligned}
c_P(m_0) &= 1.65(3) + 0.12(6)m_0 + 1.06(4)m_0^2 \\
\zeta(m_0) &= 1.090(10) + 0.318(16)m_0 - 0.092(10)m_0^2, \\
m_0(am_h) &= -0.536(24) + 3.80(13)am_h + 1.32(22)(am_h)^2
\end{aligned}$$

Charmonium

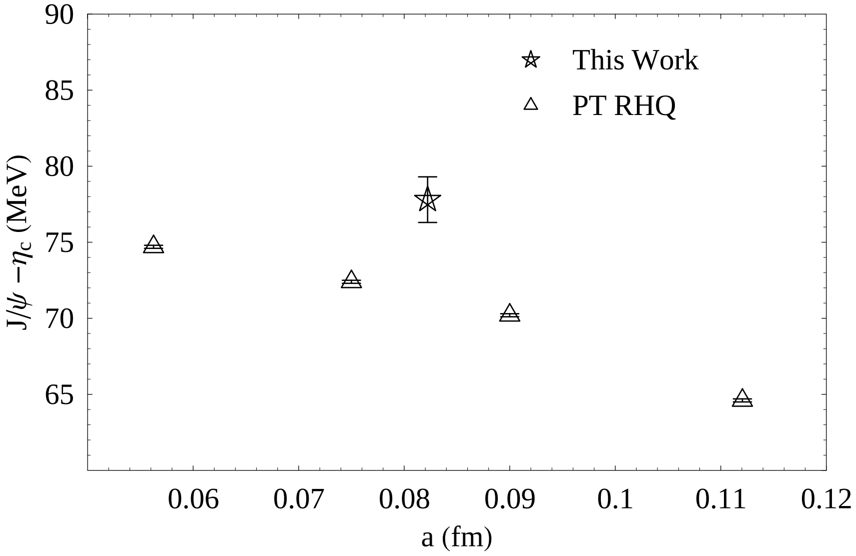
- Simulate three charm quark points
- $m_{\overline{1S}} = [m_{\eta_c} + 3m_{J/\Psi}]/4$ to determine the bare charm quark mass
- Step-scaling procedure allows us to perform the Z_m^{lat} in RI/MOM scheme using DWF $\Rightarrow m_c^{\overline{\text{MS}}}(\overline{m}_c) = 1.314(18)$ GeV.

- Spectrum



- Hyperfine splitting

Kuramashi et. al. PoS, LAT2005, 226



- The hyperfine splitting is 77.8(15) MeV
  - about 40% smaller than the experimental splitting (116 MeV)
  - but 10% higher than the calculation using one-loop coefficients.
- Resolvable when we apply our nonperturbative approach in a full-QCD calculation?

Application to Full QCD?

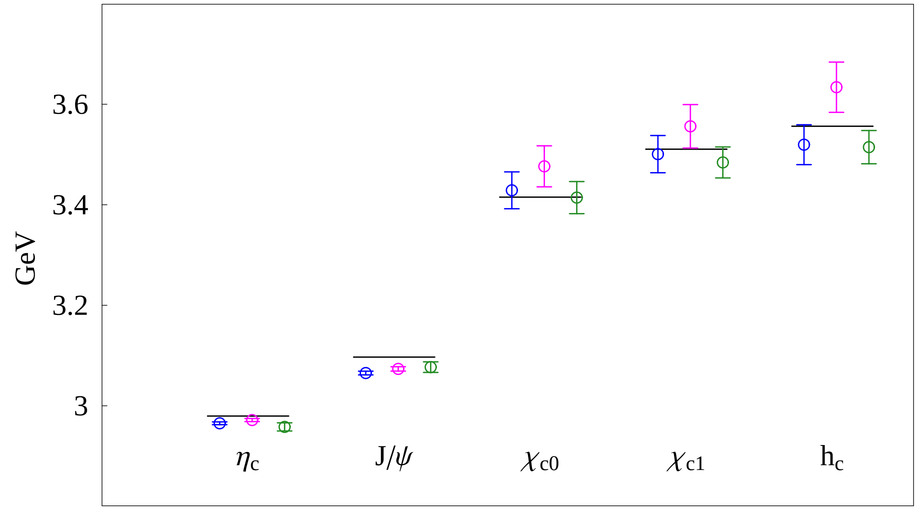
2+1 Flavor Dynamical DWF Ensembles by RBC/UKQCD

Gauge action	Iwasaki
β	2.13
Algorithm	RHMC
a^{-1}	$\approx$ 1.6 GeV ($a \approx$ 0.12 fm)
Lattice vol	$16^3 \times 32 \times 16$, $24^3 \times 64 \times 16$
am_{res}	$\approx$ 0.003
$am_{\text{up, down}}$	{0.01,0.02,0.03}
am_{strange}	0.04
M_5	1.8
Sommer scale	0.5 fm

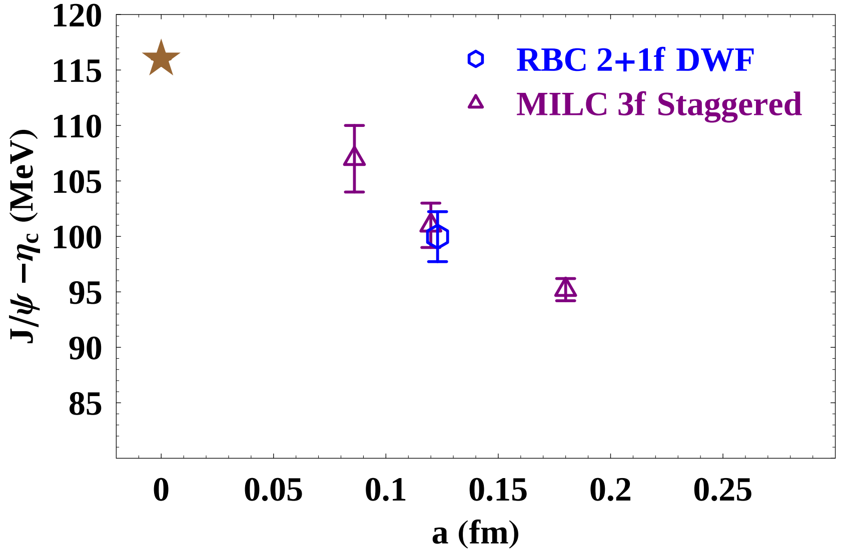
Test Run

- Use quenched coefficients on 2+1-flavor dynamical configurations

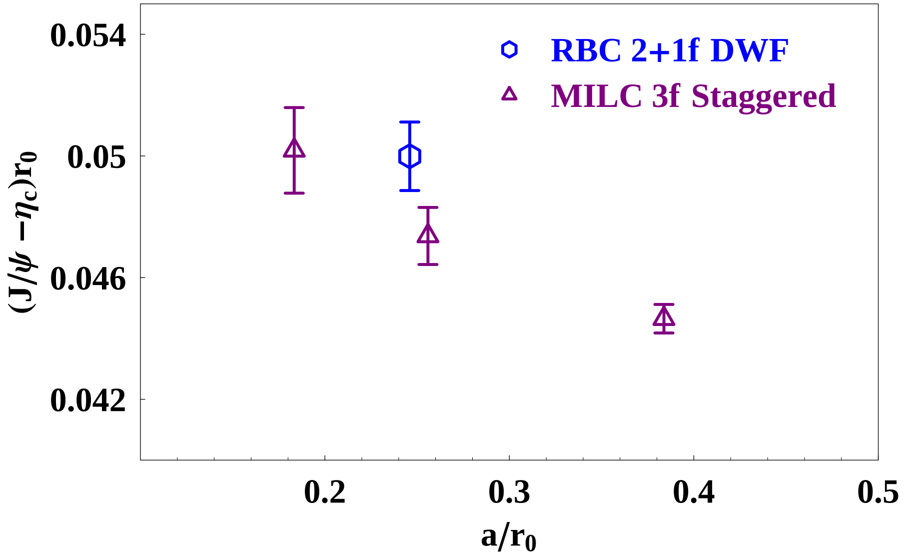
Charmonium



Hyperfine splitting

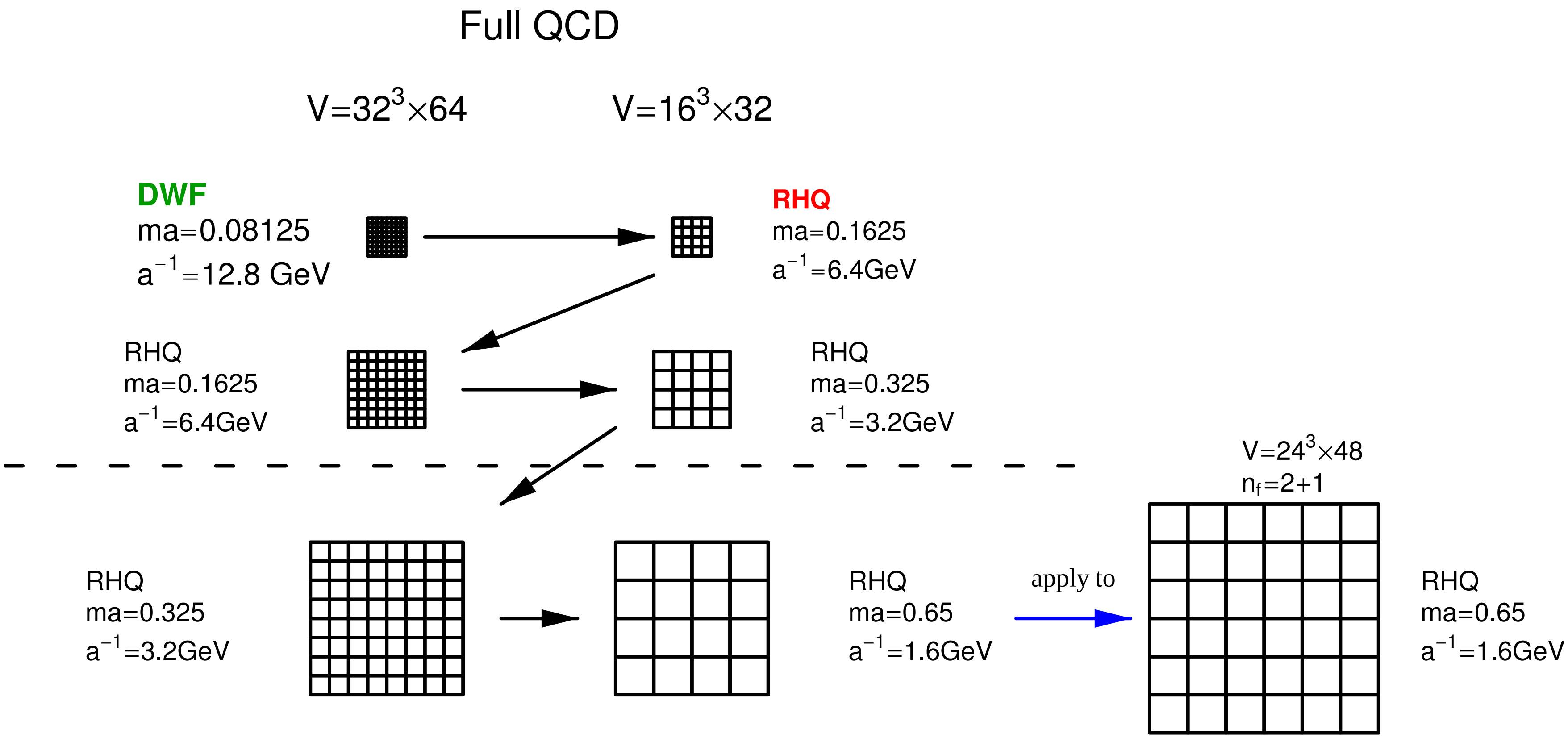


- Instant boost to hyperfine splitting value!
- Not so fast!
  Mismatched gauge action: Iwasaki
- $O(a^2)$ effects in the gauge action
- Can be further improved...



Next Running Plan

- Matching requires $N_f = 3$ but not a physical value of m_{sea}.
- Can make m_{sea} large and the calculation easy provided $m_{\text{sea}} a \ll 1$.
- Must have $m_{\text{sea}}/\Lambda_{\text{QCD}}$ equal for each pair of systems being matched.



Full QCD

$V=32^3\times64$

$V=16^3\times32$

DWF
ma=0.08125
a^{-1}=12.8 GeV

RHQ
ma=0.1625
a^{-1}=6.4GeV

RHQ
ma=0.1625
a^{-1}=6.4GeV

RHQ
ma=0.325
a^{-1}=3.2GeV

$V=24^3\times48$
n_f=2+1

RHQ
ma=0.325
a^{-1}=3.2GeV

RHQ
ma=0.65
a^{-1}=1.6GeV

apply to

RHQ
ma=0.65
a^{-1}=1.6GeV

What to do immediately?
(Before starting dynamical step-scaling)

- Use RBC-UKQCD full QCD lattices
- Tune RHQ parameters nonpertubatively

$$\begin{aligned} m_0 &\rightarrow \text{charmonium spin-averaged mass} \\ c_P &\rightarrow \text{charmonium spin-splitting mass} \\ \zeta &\rightarrow \text{dispersion relation} \end{aligned}$$

- Accurate through $|\vec{p}a|$ and to all order of $(ma)^n$
- What's left to do?
  - Charmed baryons
  - Heavy-heavy P-wave and radial excited states
  - Charm-light systems
  - More meson states (including exotic ones)

Operator	O_h	J^{PC}	remark
1	A_1	0^{++}	$^3P_0(\chi_{c0})$
γ_5	A_1	0^{-+}	$^1S_0(\eta_c)$
γ_i	T_1	1^{--}	$^3S_1(J/\psi)$
$\gamma_5\gamma_i$	T_1	1^{++}	$^3P_1(\chi_{c1})$
$\gamma_i\gamma_j$	T_1	1^{+-}	$^1P_1(h_c)$
$\gamma_5\nabla_i$	T_1	1^{+-}	
∇_i	T_1	1^{--}	
$\gamma_4\nabla_i$	T_1	1^{-+}	
$\gamma_i\nabla_i$	A_1	0^{++}	$^3P_0(\chi_{c0})$
$\epsilon_{ijk}\gamma_j\nabla_k$	E	1^{++}	$^3P_1(\chi_{c1})$
$s_{ijk}\gamma_j\nabla_k$	T_2	2^{++}	$^3P_2(\chi_{c2})$
$\gamma_5\gamma_i\nabla_i$	A_1	0^{--}	exotic
$\gamma_5 s_{ijk}\gamma_j\nabla_k$	T_2	2^{--}	
$\gamma_5 S_{\alpha jk}\gamma_j\nabla_k$	T_2	2^{--}	
$\gamma_4\gamma_5\epsilon_{ijk}\gamma_j\nabla_k$	T_1	1^{-+}	exotic
$\gamma_4 s_{ijk}\nabla_j\nabla_k$	T_2	2^{+-}	exotic
$\gamma_5\gamma_i D_i$	A_2	3^{++}	

Operator	O_h	J^{PC}	remark
$\gamma_5 S_{\alpha jk}\gamma_j D_k$	E	2^{++}	
$\gamma_5 s_{ijk}\gamma_j D_k$	T_1	1^{++}	
$\gamma_5\epsilon_{ijk}\gamma_j D_k$	T_2	2^{++}	
$\gamma_4\gamma_5 s_{ijk}\gamma_i\nabla_j\nabla_k$	A_2	3^{+-}	
$\gamma_4\gamma_5 S_{\alpha jk}\gamma_j D_k$	E	2^{+-}	
$\gamma_4\gamma_5 s_{ijk}\gamma_j D_k$	T_1	1^{+-}	
$\gamma_4\gamma_5\epsilon_{ijk}\gamma_j D_k$	T_2	3^{+-}	
$\gamma_i D_i$	A_2	3^{--}	
$s_{ijk}\gamma_j D_k$	T_1	1^{--}	
$\epsilon_{ijk}\gamma_j D_k$	T_2	2^{--}	
$\gamma_4\gamma_5 s_{ijk}\nabla_j\nabla_k$	T_2	2^{-+}	
$\gamma_5 B_i$	T_1	1^{--}	
$\epsilon_{ijk}\gamma_j B_k$	T_1	1^{-+}	exotic
$s_{ijk}\gamma_j B_k$	T_2	2^{-+}	
$\gamma_5\gamma_i B_i$	A_1	0^{+-}	exotic
$\gamma_5\epsilon_{ijk}\gamma_j B_k$	T_1	1^{+-}	
$\gamma_5 s_{ijk}\gamma_j B_k$	T_2	2^{+-}	exotic

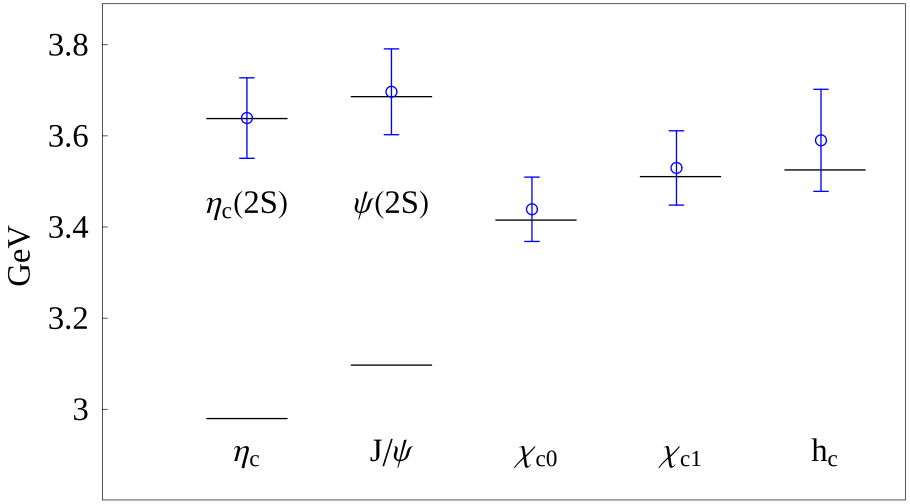
Nonperturtatively Determined Coefficients

- Determine coefficients: $Y^{i,d} = A^d + J^d \cdot X^i_{\text{RHQ}}$.

$$\underset{m_{\eta_c}, m_{J/\Psi}, c^2}{\Uparrow} \qquad \underset{\text{parameters}}{\Uparrow}$$

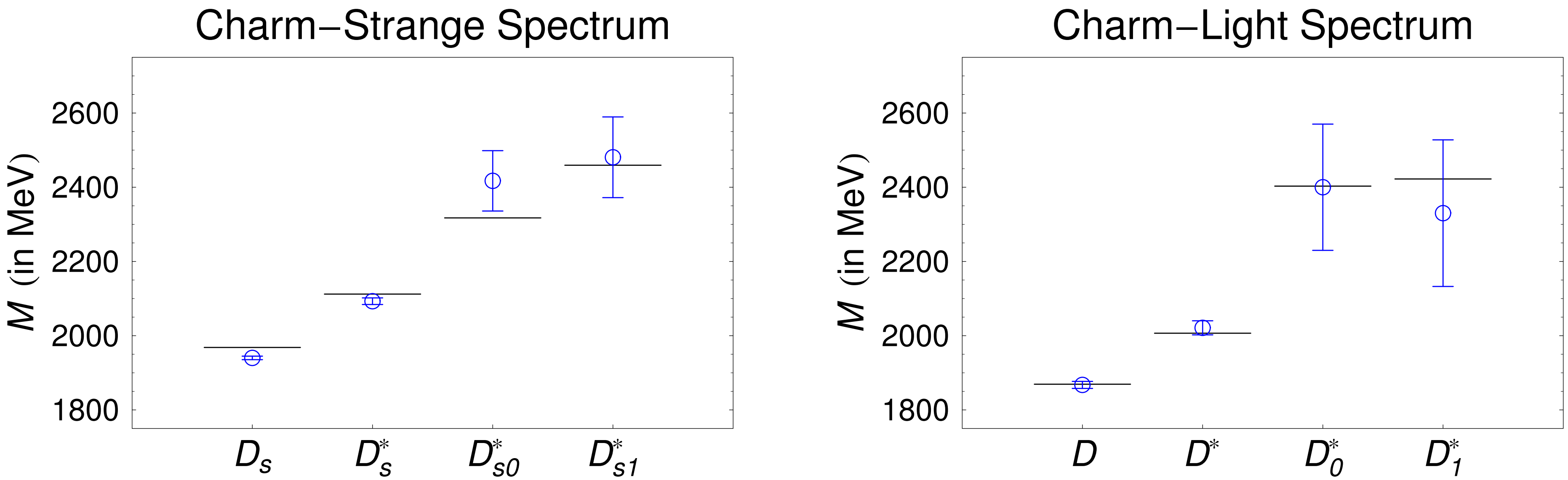
$$X_{\text{RHQ}} = \{m_0, c_P, \zeta\} = \{0.46(4), 2.50(9), 1.285(19)\}$$

- Only run on the lightest sea quark mass $am_{u,d} = 0.01$, 126 configurations
- Charmonium spectrum



D and D_s

- Only run on the lightest sea quark mass $am_{u,d} = 0.01$
- $m_{\text{valence}} = \{0.01, 0.04\}$, 126 configurations each



- $m_{D^*} - m_D = 154(16)$ MeV (exp: 142 MeV)
  $m_{D_s^*} - m_{D_s} = 153(7)$ MeV (exp: 144 MeV)

Conclusion and Outlook

- m_0, c_P and ζ can be determined nonperturbatively with step scaling.
- Interesting quenched physics:
  - Improvement on hyperfine splitting in charmonium
  - Charm quark mass: $m_c^{\overline{\mathrm{MS}}}(\overline{m}_c) = 1.314(18)$ GeV.

- Application to the full QCD:
  - Directly applying NP quenched coefficients to dynamical configurations gives promising charmonium features
  - Better hyperfine splitting than existing dynamical result
  - Full QCD NP determined RHQ action in the future

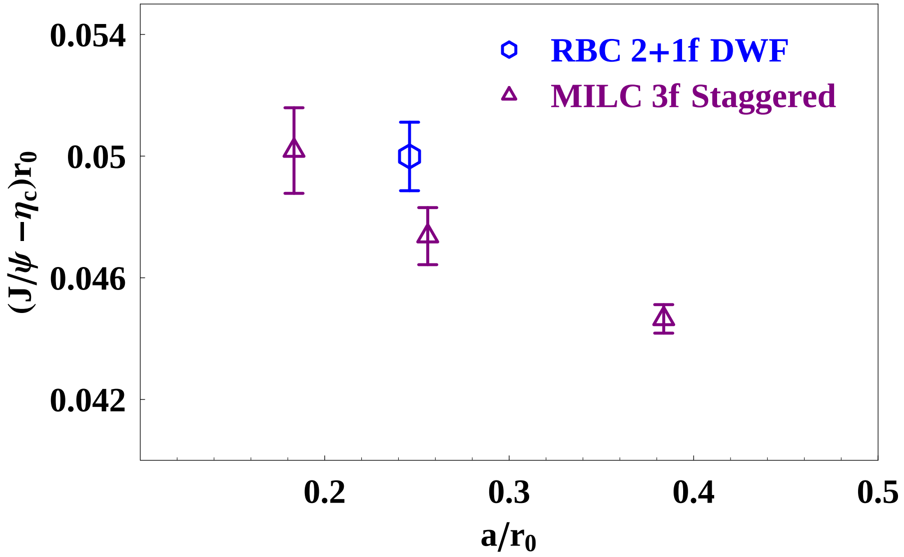
- Tune RHQ action with experimental numbers
  - Reproduce experimental numbers in charmonium and charm-light systems
  - Calculate more meson states involved derivative operators (including exotic ones)
  - Tune one more coefficient to do charmed baryons

- Applications to the B-system?
- Search for $O(a)$-improved coefficients in the currents
  - Decay constants, form factors involving one or two heavy quarks

Backup slides

Test Run — Hyperfine Splitting

- Sensitive to the choice of the reference scale
- QCD-TARO (hep-lat/0307004): 65(1) MeV–85(4) MeV
- Kentucky group (hep-lat/0507027): 88(4) MeV–121(4) MeV
- Rescale the comparison:



- Can be further improved with better tuned coefficients

Relativistic Heavy Quark (RHQ) Action

- Operators up to $O(a)$ constrained by parity and charge conjugation

Order	Operators
$O(a^{-1})$	$m\overline{\psi}\psi$
$O(a^0)$	$\overline{\psi}\gamma_0 D_0\psi,\ \overline{\psi}\vec{\gamma}\cdot\vec{D}\psi$
$O(a^1)$	$a\overline{\psi}D_0^2\psi,\ a\overline{\psi}(\vec{D})^2\psi,$ $a\overline{\psi}\left[\gamma_0,\gamma_i\right]\left[D_0,D_i\right]\psi, a\overline{\psi}\left[\gamma^i,\gamma^j\right]\left[D^i,D^j\right]\psi,$ $a\overline{\psi}\left[\gamma^i,\gamma^0\right]\left[D^i,D^0\right]\psi$

- Field transformations are useful to determine the redundant operators
- Relativistic heavy quark (RHQ) action

A. El-Khadra et. al., Phys. Rev. D55 (1997) 3933

$$S = \sum_n \overline{\psi_n}\left\{m_0 + \gamma_0 D_0 + \zeta\vec{\gamma}\cdot\vec{D} - \frac{1}{2}aD_0^2 - \frac{\zeta}{2}a(\vec{D})^2 - \sum_i \frac{i}{2}c_E a\sigma_{0i}F_{0i} - \sum_{i\le j}\frac{i}{2}c_B a\sigma_{ij}F_{ij}\right\}\psi_n \qquad (1)$$

- 4 parameters: m_0, ζ, c_E, c_B

RHQ Power Counting

- RHQ power counting: D^0 is treated the same as m_0, $O(1/a)$
$\Rightarrow aD^0$ and am_0 are $O(a^0)$.
- Continuum effective Symmanzik lagrangian

$$\begin{aligned}
\mathcal{L} &= \overline{\psi}\,\{C_1 + \gamma^0 D^0 C_2 \\
&+ \{\vec{\gamma}\cdot\vec{D}, C_3\} + a\{[\vec{\gamma}\cdot\vec{D}, \gamma^0 D^0], C_4\} \\
&+ \vec{D}^2 C_5 + a\{\vec{D}^2, \gamma^0 D^0\} C_6 \\
&+ [\gamma^i, \gamma^j][D^i, D^j] C_7 + a\{[\gamma^i, \gamma^j][D^i, D^j], \gamma^0 D^0\} C_8 \\
&+ [\gamma^i, \gamma^0][D^i, D^0] C_9 + a[[\gamma^i, \gamma^0][D^i, D^0], \gamma^0 D^0] C_{10}\}\,\psi. \qquad (2)
\end{aligned}$$

- $C_i = f(ma, g^2, (D_0 a)^2)$
- Reduce Eq. 2 to $\overline{\psi}(m + \gamma^\mu D^\mu)\psi$ with field transformations

Field Transformation Revisited

Starting from Fermilab action (Eq. 1):

- Shift $c_E \to c_E + 2ma\xi_E$ after field transformations of

$$\begin{aligned} \psi' &= (1 - a^2[\gamma^i, \gamma^0][D^i, D^0]\xi_E)\psi \\ \overline{\psi}' &= \overline{\psi}(1 - a^2[\gamma^i, \gamma^0][\overleftarrow{D}^i, \overleftarrow{D}^0]\xi_E), \end{aligned} \quad (3)$$

  plus $O(a^2)$ term of $[\gamma^i, \gamma^0]\gamma^0[[D^i, D^0], D^0] \to$ ignorable.

- Shift $c_B \to c_B + 2ma\xi_B$ after field transformations of

$$\begin{aligned} \psi' &= (1 - a^2[\gamma^i, \gamma^j][D^i, D^j]\xi_B)\psi \\ \overline{\psi}' &= \overline{\psi}(1 - a^2[\gamma^i, \gamma^j][\overleftarrow{D}^i, \overleftarrow{D}^j]\xi_B). \end{aligned} \quad (4)$$

  plus $O(a^1)$ term of $[\gamma^i, \gamma^j]\gamma^0\{[D^i, D^j], D^0\} \to$ important.

- $c_E = c_B = c_\mathrm{P}$ in Fermilab action
- Three parameters: m_0, ζ, c_P to be tuned.

Step-scaling parameters

	I	II
Volume	$24^3 \times 48$	$16^3 \times 32$
$1/a$	5.4 GeV (fine)	3.6 GeV (coarse)
L	~ 0.9 fm	~ 0.9 fm
β	6.638	6.351
ma	0.2	0.3
Action	DWF	RHQ
	III	IV
Volume	$24^3 \times 48$	$16^3 \times 32$
$1/a$	3.6 GeV (coarse)	2.4 GeV (supercoarse)
L	~ 1.35 fm	~ 1.35 fm
β	6.351	6.074
ma	0.3	0.45
Action	RHQ	RHQ
	V	VI
Volume	$24^3 \times 48$	$16^3 \times 32$
$1/a$	2.4 GeV (supercoarse)	1.6 GeV (ultracoarse)
L	~ 2.0 fm	~ 2.0 fm
β	6.074	5.849
ma	0.3	0.45
Action	RHQ	RHQ

Eigenspace

- The eigenvalues of the $J^T \cdot J$ matrix are:

$$\{9.58(15), 1.39(10), 0.000144(22), 0.000037(12)\}, \tag{5}$$

with corresponding eigenvectors

$$\begin{array}{cccccc}
 & m_0. & c_B. & c_E. & \zeta. & \\
\{ & 0.832(4), & -0.1101(6), & -0.1081(7), & 0.533(6) & \}, \\
\{ & -0.523(7), & 0.062(6), & 0.086(3), & 0.846(4) & \}, \\
\{ & 0.182(7), & 0.81(7), & 0.56(10), & -0.004(6) & \}, \\
\{ & 0.040(23), & -0.57(10), & 0.82(7), & -0.0157(29) & \}.
\end{array} \tag{6}$$

- The smallest eigenvalue is dominated by the eigenvector that has large component in c_E and that leads to large errorbar in the c_E coefficient.

Quadratic Effects

- The simple Taylor expansions up to second-derivative terms are

$$Y_{\text{coarse},qua} = Y_{\text{coarse},ave} + J^{(3)} \cdot (X_i - X_{14}) + \frac{1}{2}(X_i - X_{14}) \cdot Q \cdot (X_i - X_{14}) \quad (7)$$

- Consider quadratic terms

$$\begin{aligned}\chi^2_{\text{fine,qua}} &= \left(Y_{\text{fine}} - A^{(3)} - J^{(3)} \cdot X^{(3)}_{\text{out}} - 1/2(X^{(3)}_{\text{out}})^T \cdot Q^{(3)} \cdot X^{(3)}_{\text{out}}\right)^T \\ &\quad \cdot W_{\text{f}} \cdot \left(Y_{\text{fine}} - A^{(3)} - J^{(3)} \cdot X^{(3)}_{\text{out}} - 1/2(X^{(3)}_{\text{out}})^T \cdot Q^{(3)} \cdot X^{(3)}_{\text{out}}\right) \quad (8)\end{aligned}$$

The result is $\{m_0, c_{\text{P}}, \zeta\} = \{0.034(8), 1.50(3), 1.035(5)\}$ when considering quadratic effects.