Chiral Extrapolations in 2+1 Flavor Domain Wall Fermion Simulations

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Outline

- Introduction

  domain wall fermions, residual mass, $\mathcal{O}(a)$ correction, ...
- Simulation Details

  the ensembles, measurement parameters
- Numerical Results

  $m_{\rm res}$, light pseudoscalar masses and decay constants
- Chiral Extrapolations

  NLO chiral fits for M_π^2 and f_π, $O(a)$ effect
- Conclusions

Introduction

- Domain wall fermions have good chiral symmetry at finite lattice spacing.
- 2+1 flavor simulations with domain wall fermions are in production on QCDOC machines.
- However, quark masses in the simulations are not physical (too heavy!).
- Chiral extrapolations are essential to extrapolate the measured quantities to the physical points.
- We want to know if the results from DWF simulations can be described by continuum chiral perturbation theory.
- In this talk I will focus on the chiral extrapolations of the pseudoscalar masses and decay constants.

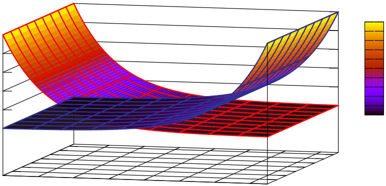
Domain Wall Fermions

The Dirac operator for domain wall fermions is defined as

T.Blum *et al*, PRD 69:074502,2004

$$D_{x,s;x',s'} = \delta_{s,s'} D^{\parallel}_{x,x'} + \delta_{x,x'} D^{\perp}_{s,s'}$$

$D^{\parallel}_{x,x'}$: 4D Wilson Dirac operator
$D^{\perp}_{s,s'}$: derivative in the 5th dimension



- Bound states on the left and right walls with left-handed and right-handed chirality
- Finite L_s would allow mixing between two bound states , which leads to residual chiral symmetry breaking

Domain Wall Fermions

Axial Ward identity for DWF

Furman & Shamir, Nucl.Phys.B 439:54-78,1995

$$\Delta_\mu \langle \mathcal{A}^a_\mu(x) O(y) \rangle = 2m_f \langle J^a_5(x) O(y) \rangle + 2\langle J^a_{5q}(x) O(y) \rangle + i\langle \delta^a O(y) \rangle.$$

$J^a_{5q}(x)$: "mid-point" contribution, comes from mixing between quark states on left and right walls, another explicit chiral symmetry breaking term besides m_f.

Close to continuum :

$$J^a_{5q} = m_{\mathrm{res}} J^a_5 + \mathcal{O}(a)$$

m_{res} : residual mass, additive mass renormalization to the input quark mass

$$m_{\mathrm{res}} \propto \frac{1}{a} e^{-\alpha L_s}$$

Domain Wall Fermions

To $\mathcal{O}(a)$, the effective Lagrangian for DWF can be written as

$$L_{eff} = \bar{\psi}(\vec{x},t)[i\not{D} + m_q]\psi(\vec{x},t) + ae^{-\alpha L_s} c_{dwf}\bar{\psi}(\vec{x},t)\sigma^{\mu\nu}F_{\mu\nu}\psi(\vec{x},t)$$

where $m_q = m_f + m_{\text{res}}$.

- Similar to Wilson fermions (Rupak & Shoresh, PRD 66:054503,2002), but the $\mathcal{O}(a)$ lattice artifact is exponentially small for domain wall fermions
- Ignoring the $\mathcal{O}(a)$ correction,
  - Effective field theory for simple quantities like meson masses with DWF would be the same as the continuum theory, but with m_f replaced by $m_f + m_{\text{res}}$.
  - Pion masses vanish at $m_q = 0$ or $m_f = -m_{\text{res}}$

The Ensembles

Gauge Configurations

- Domain wall fermion @ $L_s = 16$, $M_5 = 1.8$
  + Iwasaki gauge @ $\beta = 2.13$
- 2+1f sea quarks : $am_{u/d} = 0.01, 0.02, 0.03$, $am_s = 0.04$
- Two lattice volumes : $16^3 \times 32$, $24^3 \times 64$
- $a^{-1} \sim 1.6$ GeV
- Generated jointly by the RBC and UKQCD Collaborations on QCDOC machines

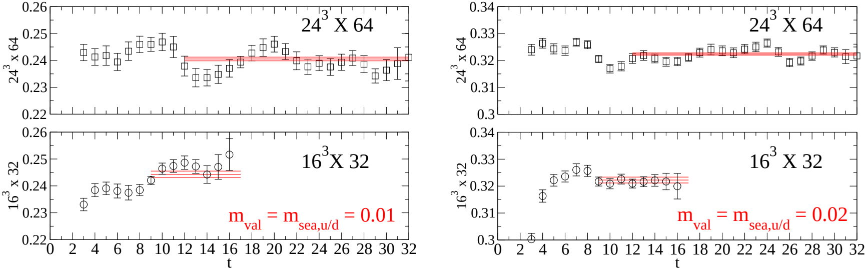
Measurements

- Coulomb gauge fixed wall source, spatial size 16^3
- Degenerate valence quarks

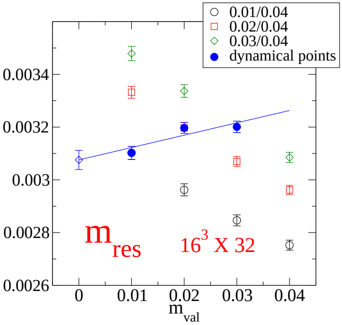
Volume	valence quarks	ncfgs	separation	bin size
$16^3 \times 32$	0.01 to 0.04	$\sim$ 700	5	10
$24^3 \times 64$	0.005 to 0.04	$\sim$ 40 - 60	80	1

Pseudoscalar Effective Masses

- The typical pseudoscalar effective masses are shown below
- Fluctuations outside errorbars are seen on the left.
- Good plateaux on the right.



The Residual Mass



- The ratio $R(t)$ is used to compute m_{res} on the lattice

$$R(t) = \frac{\langle \sum_{\vec{x}} J^a_{5q}(\vec{x}, t)\pi^a(0)\rangle}{\langle \sum_{\vec{x}} J^a_5(\vec{x}, t)\pi^a(0)\rangle}$$

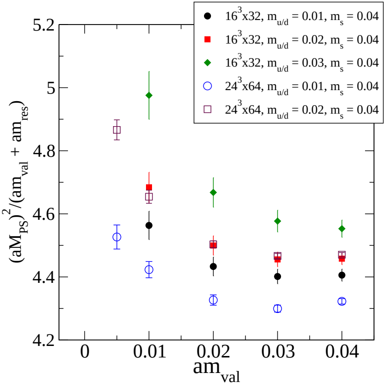
- Fit $R(t)$ where plateaux are reached to a constant to get m_{res}

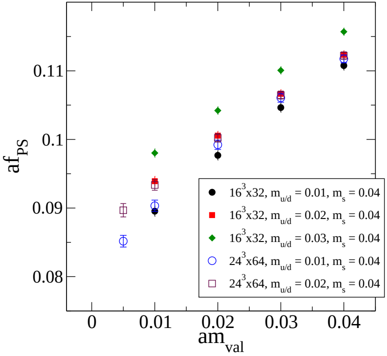
- Quark mass dependence of the measured m_{res} is visible
- This is due to the finite lattice spacing correction and of the order $\mathcal{O}(m_f a)$
- The residual mass is defined in the chiral limit $(m_f \to 0)$

$$am_{\text{res}} = 0.00308(3)$$

First Glance at the Results of M_π^2 and f_π

- The results of M_π^2/m_q and f_π from both volumes are shown below
- Nonlinearities are present in both the masses and decay constants
- Are they consistent with continuum χPT?





Continuum PQχPT NLO Formulae

- 2+1 flavor in sea, degenerate in valence

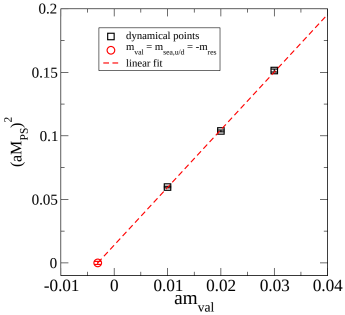
Sharpe & Shoresh, hep-lat/0006017

$$
\begin{aligned}
M_\pi^2 &= \chi_V\Big\{1+\frac{48}{f^2}(2L_6-L_4)\bar{\chi}+\frac{16}{f^2}(2L_8-L_5)\chi_V \\
&+\frac{1}{24f^2\pi^2}\Big[\frac{2\chi_V-\chi_l-\chi_s}{\chi_V-\chi_\eta}\chi_V\ln\chi_V-\frac{(\chi_V-\chi_l)(\chi_V-\chi_s)}{(\chi_V-\chi_\eta)^2}\chi_V\ln\chi_V \\
&+\frac{(\chi_V-\chi_l)(\chi_V-\chi_s)}{\chi_V-\chi_\eta}(1+\ln\chi_V)+\frac{(\chi_\eta-\chi_l)(\chi_\eta-\chi_s)}{(\chi_V-\chi_\eta)^2}\chi_\eta\ln\chi_\eta\Big]\Big\} \\
f_\pi &= f\Big\{1+\frac{8}{f^2}(3L_4\bar{\chi}+L_5\chi_V) \\
&-\frac{1}{16\pi^2 f^2}\Big[(\chi_V+\chi_l)\ln\frac{\chi_V+\chi_l}{2}+\frac{\chi_V+\chi_s}{2}\ln\frac{\chi_V+\chi_s}{2}\Big]\Big\}
\end{aligned}
$$

- $\chi_i = 2B_0 m_i$
- 6 free parameters : f, $2L_6 - L_4$, $2L_8 - L_5$, B_0, L_4, L_5.
- B_0 and f are shared by both M_π^2 and f_π.

the Chiral Limit for DWF

- As already mentioned, if we neglect the chiral symmetry breaking from the $\mathcal{O}(a)$ contribution, the pion masses should vanish at $m_f = -m_{\rm res}$
- Fit pion mass results to $M_\pi^2 = 2B_0(m_f + m_{\rm res}) + C$. If the above argument holds true, then $C \to 0$.

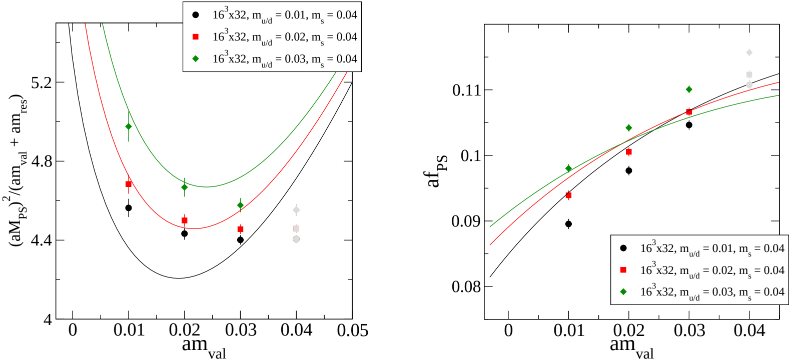


- Linear fit to the dynamical points on $16^3 \times 32$ lattices
- $C = 0.000(1)$, consistent with 0
- We will use the continuum PQχPT formulae to do the extrapolations, but with

$$m_q = m_f + m_{\rm res}$$

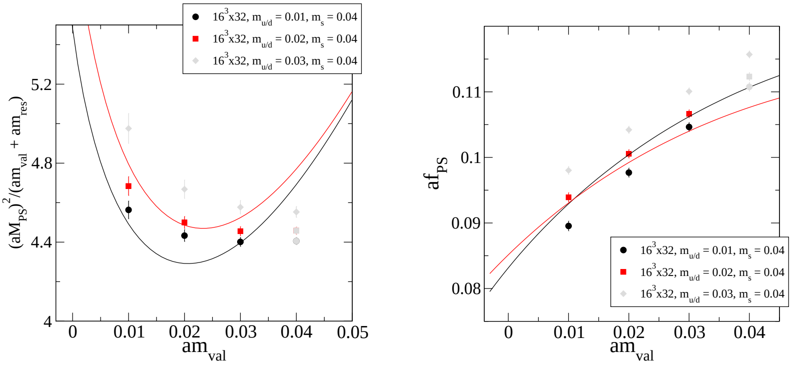
Combined fits : $16^3 \times 32$

- The simultaneous fits to both M_π^2 and f_π are shown below
- Mass range : $m_{val} = 0.01 - 0.03$, $m_{sea,u/d} = 0.01, 0.02$ or $M_\pi = 390 - 630$ MeV
- The fit does NOT represent our data : $\chi^2/dof = 24(4)$ (uncorr.)



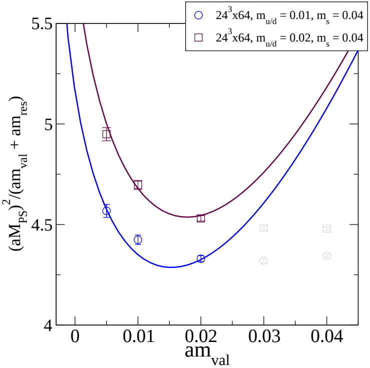
Combined fits : $16^3 \times 32$

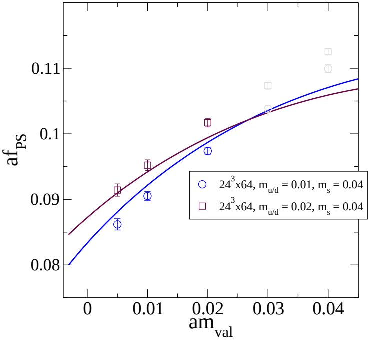
- The simultaneous fits to both M_π^2 and f_π are shown below
- Mass range : $m_{val} = 0.01 - 0.03$, $m_{sea,u/d} = 0.01, 0.02$ or $M_\pi = 390 - 615$ MeV
- The fit does NOT represent our data : $\chi^2/dof = 17(5)$ (uncorr.)



Combined fits : $24^3 \times 64$

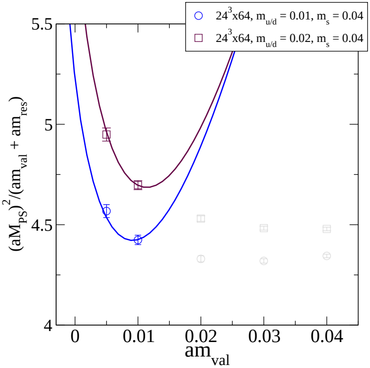
- The simultaneous fits to both M_π^2 and f_π are shown below
- Mass range : $m_{val} = 0.005 - 0.02$, $m_{sea,u/d} = 0.01, 0.02$ or $M_\pi = 300 - 520$ MeV
- Fit quality improves with lighter masses : $\chi^2/dof = 8(3)$ (uncorr.)

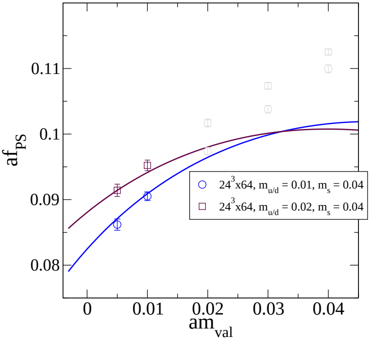




Combined fits : $24^3 \times 64$

- The simultaneous fits to both M_π^2 and f_π are shown below
- Mass range : $m_{val} = 0.005 - 0.01$, $m_{sea,u/d} = 0.01, 0.02$ or $M_\pi = 300 - 420$ MeV
- Fit quality improves with lighter masses : $\chi^2/dof = 2(2)$ (uncorr.)





Summary of NLO fits

- When the pion masses are in the range of 390 to 630 MeV, our data can not be described by continuum NLO χPT
- Adding lighter masses and excluding the heavier ones improves fit quality significantly
- Fits are poorly constrained due to limited data points, but the trend is clear and promising

Chiral Expansions at Finite Lattice Spacing

The size of the finite lattice spacing corrections for DWF is estimated to be

$$ae^{-\alpha L_s} c_{dwf} \sim m_{\rm res}(a\Lambda_{QCD})^2 \sim 0.1 m_{\rm res}$$

which is comparable to m_q^2, thus should only appear in the NLO chiral Lagrangian

$$\begin{aligned} \mathcal{L}^a_{LO} &= \mathcal{L}^{cont}_{LO} \\ \mathcal{L}^a_{NLO} &= \frac{f^2}{8}\mathrm{Tr}[\Sigma\rho + (\Sigma\rho)^\dagger] + \mathcal{L}^{cont}_{NLO} \end{aligned}$$

where $\rho = 2B_0 M_a$.

- NLO chiral formulae with $\mathcal{O}(a)$ correction for pion masses and decay constants :
  - Only one new low energy constant M_a is needed
  - Only changes the expression for pion masses by an additive constant $\rho = 2B_0 M_a$

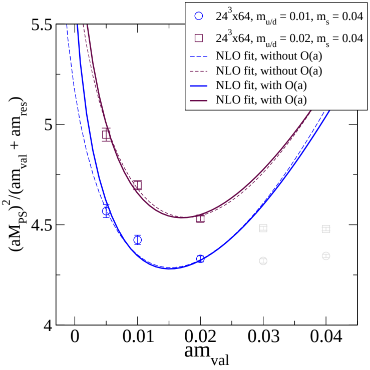
NLO Formulae with $\mathcal{O}(a)$ correction

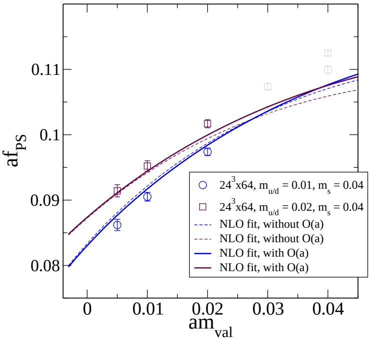
$$
\begin{aligned}
M_\pi^2 &= \rho + \chi_V\Big\{1 + \frac{48}{f^2}(2L_6 - L_4)\bar{\chi} + \frac{16}{f^2}(2L_8 - L_5)\chi_V \\
&+ \frac{1}{24f^2\pi^2}\Big[\frac{2\chi_V - \chi_l - \chi_s}{\chi_V - \chi_\eta}\chi_V \ln\chi_V - \frac{(\chi_V - \chi_l)(\chi_V - \chi_s)}{(\chi_V - \chi_\eta)^2}\chi_V \ln\chi_V \\
&+ \frac{(\chi_V - \chi_l)(\chi_V - \chi_s)}{\chi_V - \chi_\eta}(1 + \ln\chi_V) + \frac{(\chi_\eta - \chi_l)(\chi_\eta - \chi_s)}{(\chi_V - \chi_\eta)^2}\chi_\eta \ln\chi_\eta\Big]\Big\} \\
f_\pi &= f\Big\{1 + \frac{8}{f^2}(3L_4\bar{\chi} + L_5\chi_V) \\
&- \frac{1}{16\pi^2 f^2}\Big[(\chi_V + \chi_l)\ln\frac{\chi_V + \chi_l}{2} + \frac{\chi_V + \chi_s}{2}\ln\frac{\chi_V + \chi_s}{2}\Big]\Big\}
\end{aligned}
$$

- 7 free parameters : f, $2L_6 - L_4$, $2L_8 - L_5$, B_0, L_4, L_5 and ρ.

Combined fits with $\mathcal{O}(a)$ correction : $24^3 \times 64$

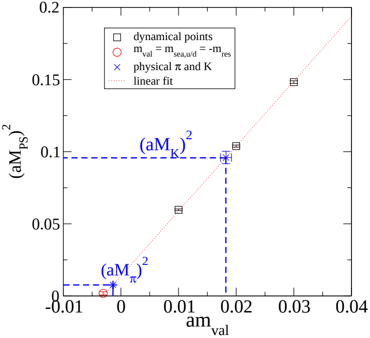
- NLO chiral fits with and without the $\mathcal{O}(a)$ correction: 24^3, $am_{val} = 0.005$ to 0.02.
- The constant M_a is found to be $9(4) \times 10^{-4}$ or 30% of $m_{\rm res}$
- Consistent with the naive estimation (10% of $m_{\rm res}$)

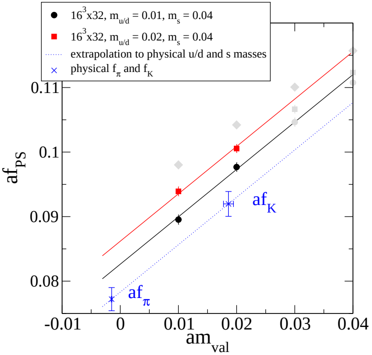




Extracting physical quark masses and decay constants?

- NLO chiral fits for DWF are still under investigation
- Resort to naive linear fits to extract the physical quark masses (see talk by R.Tweedie) and decay constants





Results for Decay Constants

- From these linear fits we get

Volume	f_π (MeV)	f_K (MeV)	f_K/f_π
$16^3 \times 32$	125(4)	148(4)	1.18(1)

  Analysis for the large volume data is still in progress
  Experimental values (PDG 2004):

$$f_\pi \approx 131 \text{ MeV}, \; f_K \approx 160 \text{ MeV}, \; f_K/f_\pi \approx 1.22$$

- Large systematic errors are to be expected from the chiral extrapolations

Conclusions

- The NLO continuum PQχPT fails to describe our data when the pion masses are in the range of 390 to 630 MeV.
- The chiral fits improve significantly when the masses are lighter.
- We have also included the $\mathcal{O}(a)$ correction in the fits and found its effect is small.
- Our data points are very limited and the analyses are not conclusive yet. More mass points are needed to constrain the fits.

Estimate Finite Volume Effect

- Finite volume effects can be estimated by (C.Bernard, hep-lat/0111051)

$$M_\pi^2 \ln\frac{M_\pi^2}{\Lambda_\chi^2} \;\rightarrow\; M_\pi^2\left(\ln\frac{M_\pi^2}{\Lambda_\chi^2} + \delta_1(M_\pi L)\right)$$

$$\delta_1(M_\pi L) \;\sim\; \frac{12\sqrt{2\pi}}{(M_\pi L)^{3/2}} e^{-M_\pi L}$$

- In our current simulations,

$$L = 16,\ \frac{\delta_1(M_\pi L)}{\ln\frac{M_\pi^2}{\Lambda_\chi^2}} \text{ as large as } 4\%$$

$$L = 24,\ \frac{\delta_1(M_\pi L)}{\ln\frac{M_\pi^2}{\Lambda_\chi^2}} \text{ as large as } 1\%$$