Taste breaking effects in scalar meson correlators

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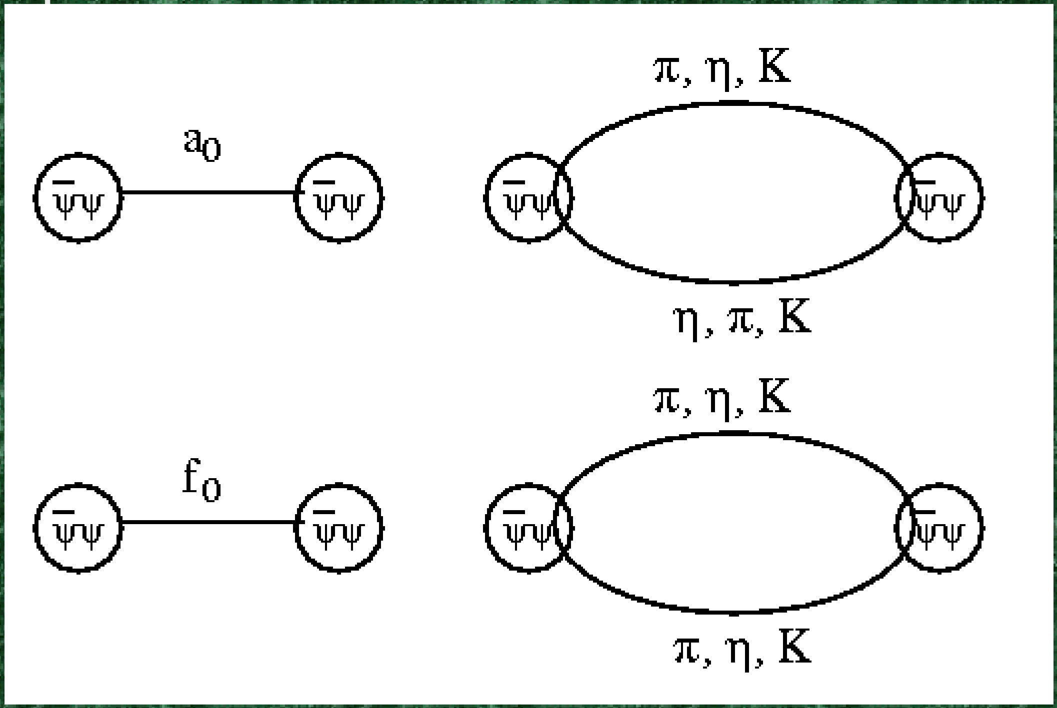
Scalar density correlator

pole term bubble term



Compute using rooted staggered fermions

Model using SXPT – test of fourth root

What is new?

- Lat05 Prelovsek: a0 bubble formalism
- This year
  - f0 formalism
  - Fits to Asqtad simulation data– several momenta – both a0 and f0

Staggered Chiral Perturbation Theory

- Assume a perturbative expansion in lattice spacing (a)
- Map rooted lattice theory to SXPT (Lee Sharpe & Aubin Bernard)
- Use replica trick to count ¼ factors

SXPT Tree Level Spectrum

$$\mathcal{L} = \frac{f^2}{8}\mathrm{Tr}\left(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma\right) - \frac{\mu f^2}{4}\mathrm{Tr}[\mathcal{M}\Sigma^\dagger + \Sigma\mathcal{M}^\dagger)] + \frac{m_0^2}{2}\phi_{0r}^2 + a^2\mathcal{V}(\Sigma)$$

- Mass matrix with tastes and replicas

$$\mathcal{M}_{frb,f'r'b'} = \delta_{ff'}\delta_{rr'}\delta_{bb'}m_f,$$

- Meson spectrum is obtained by diagonalizing the meson mass matrix or by summing diagrams

Tree level meson spectrum

- Meson taste labeling

$$a = \{P, V, A, T, I\}$$

- Masses

$$M^2_{ff'a} = \mu(m_f + m_{f'}) + a^2\Delta_a \quad \text{for } i = 1, 2, \ldots, N.$$

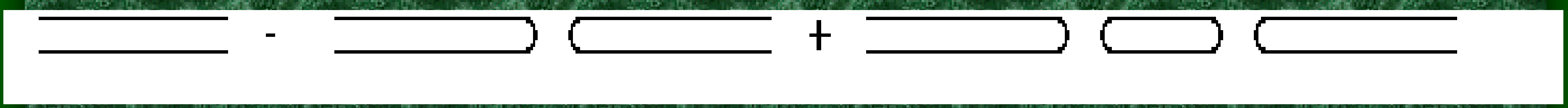
$$M^2_{\eta I} = \frac{2}{3}M^2_{SI} + \frac{1}{3}M^2_{UI}$$

- (Ignore A and V hairpins for now)

Series expansion for taste singlet eta

$$
\begin{aligned}
C_{\eta I}(p) &= u_f^\dagger \Bigg\{ \frac{\delta_{ff'}}{p^2+M^2_{ffI}} - n_r \frac{1}{p^2+M^2_{ffI}} \frac{m_0^2}{3} \frac{1}{p^2+M^2_{f'f'I}} \\
&+ n_r \frac{1}{p^2+M^2_{ffI}} \frac{m_0^2}{3} \frac{n_r}{p^2+M^2_{ggI}} \frac{m_0^2}{3} \frac{1}{p^2+M^2_{f'f'I}} - \dots \Bigg\} u_{f'}
\end{aligned}
$$

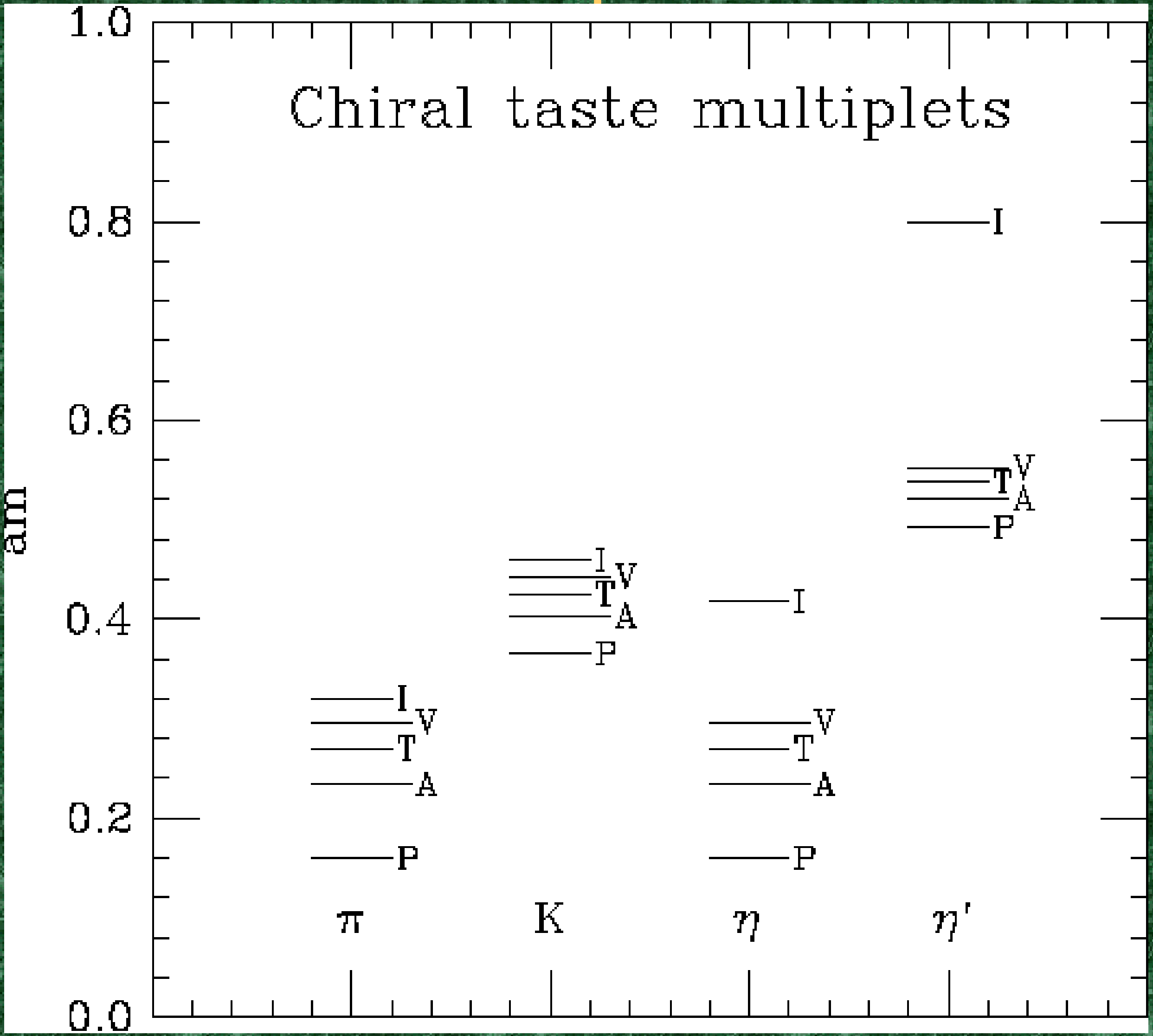
In terms of quark lines:



Sums cleanly to one pole:

$$
C_{\eta I} = \frac{1}{p^2+M^2_{\eta I}} \qquad M^2_{\eta I} = 2/3 M^2_{SI} + 1/3 M^2_{UI}
$$

Meson Spectrum



Matching Lattice QCD to SXPT

- Standard staggered correlator for a0

$$C_{\rm conn}(\vec{p},t) = \sum_{\vec{x}} (-)^{x} \cos(\vec{p}\cdot\vec{x}) \left\langle {\rm Tr}[M_u^{-1}(\vec{x},t;0,0) M_u^{-1\dagger}(\vec{x},t;0,0)] \right\rangle$$

- Taste-spin basis

$$q_f^{\alpha a}(2y) = \frac{1}{2}\sum_{\eta} \Gamma_{\eta}^{\alpha a} \chi_{f,2y+\eta}$$

- Scalar density for a0

$$\rho_{udI}(2y) = \frac{1}{4}\bar{q}_d(2y) I \otimes I q_u(2y) = \sum_{\eta} \bar{\chi}_{d,2y+\eta} \chi_{u,2y+\eta}$$

Matching Lattice QCD to SXPT

- Lattice QCD

$$Z(m_{ff'}) = \int dU \exp[-S_g(U)] \det[M(U, m_{ff'})]^{n_r}$$

$$\langle \bar{\rho}_{udI}(2y)\rho_{udI}(0)\rangle = \frac{\partial^2 \log Z}{\partial m_{I,ud}(2y)\partial m_{I,du}(0)}\Big|_{m_{ff'}(x)=\delta_{ff'}m_f}$$

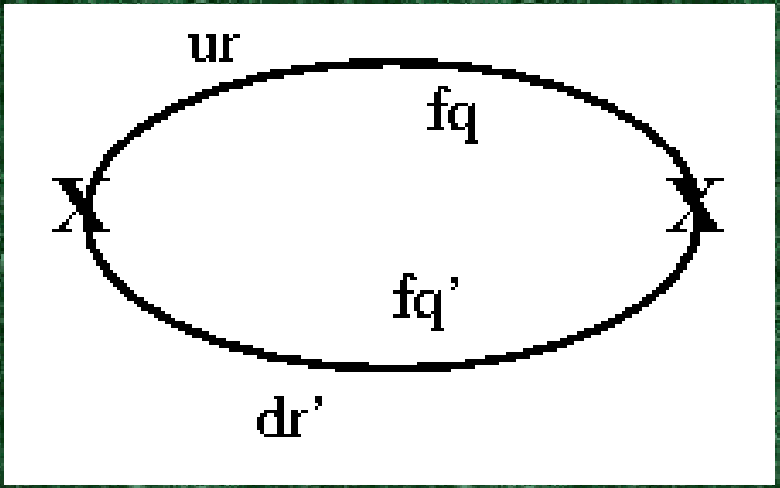
- SXPT

$$Z_{\mathrm{SXPT}}(m_{ff'}) = \int [d\Sigma] \exp[-S(\Sigma, m_{ff'})]$$

$$\langle \bar{\rho}_{udI}(x)\rho_{udI}(0)\rangle = \frac{\partial^2 \log Z_{\mathrm{SXPT}}}{\partial m_{ud}(x)\partial m_{du}(0)}\Big|_{m_{ff'}(x)=\delta_{ff'}m_f}$$

Two meson “bubble” term

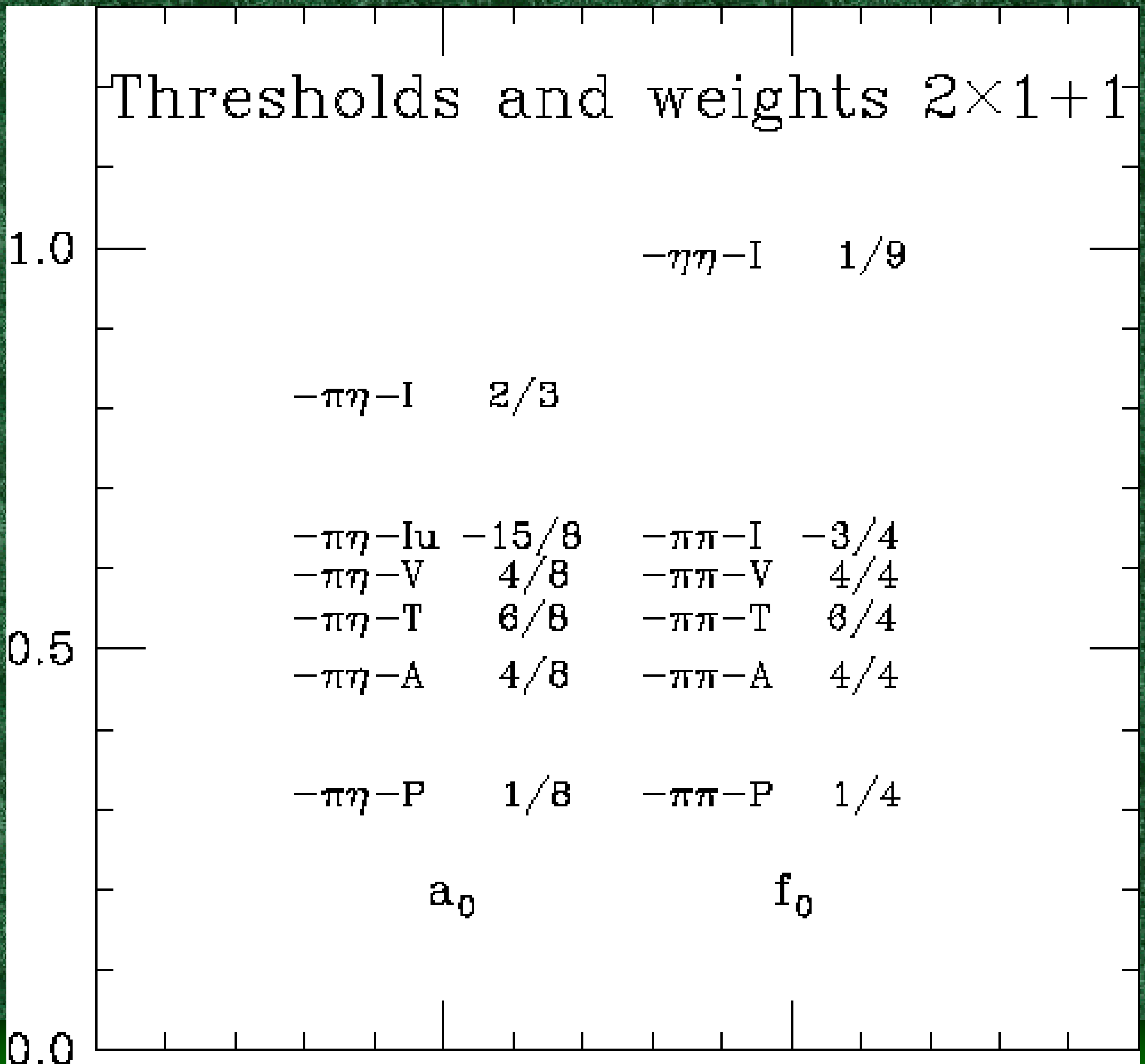
$$\langle \bar{\rho}_{udI}(x)\rho_{udI}(0)\rangle = (2a)^8\mu^2 \sum_{r,q,b,f} \sum_{r',q',b',f'} \langle \phi_{ur,fq}(x)\phi_{fq,dr}(x)\phi_{dr',fq'}(0)\phi_{fq',dr'}(0)\rangle$$



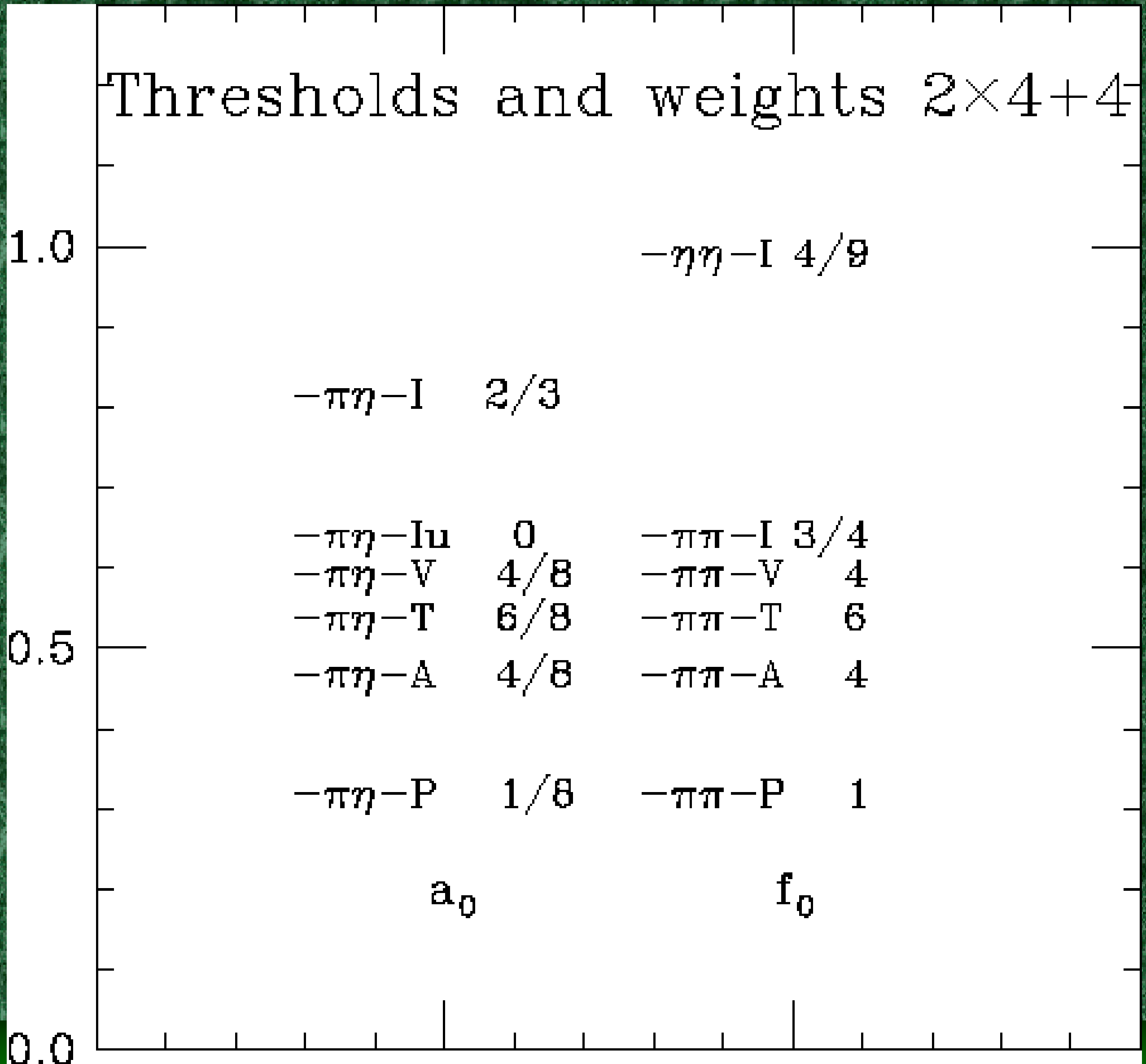
Prelovsek (Lat05) result for a0 “bubble”

$$
\begin{aligned}
C(p,t) &= B_0^2 \sum_k \Bigg\{ \frac{1}{16} \sum_a \Bigg[2\frac{1}{(k+p)^2+M_{Ua}^2}\frac{1}{k^2+M_{Ua}^2} + \frac{1}{(k+p)^2+M_{Ka}^2}\frac{1}{k^2+M_{Ka}^2} \Bigg] \\
&- 4\Bigg[\frac{1}{(k+p)^2+M_{UI}^2}\frac{1}{3}\frac{k^2+M_{SI}^2}{(k^2+M_{UI}^2)(k^2+M_{\eta}^2)} \\
&+ \frac{1}{(k+p)^2+M_{UV}^2} a^2\delta_V \frac{k^2+M_{SV}^2}{(k^2+M_{UV}^2)(k^2+M_{\eta V}^2)(k^2+M_{\eta' V}^2)} \\
&+ \frac{1}{(k+p)^2+M_{UA}^2} a^2\delta_A \frac{k^2+M_{SA}^2}{(k^2+M_{UA}^2)(k^2+M_{\eta A}^2)(k^2+M_{\eta' A}^2)} \Bigg] \Bigg\}
\end{aligned}
$$

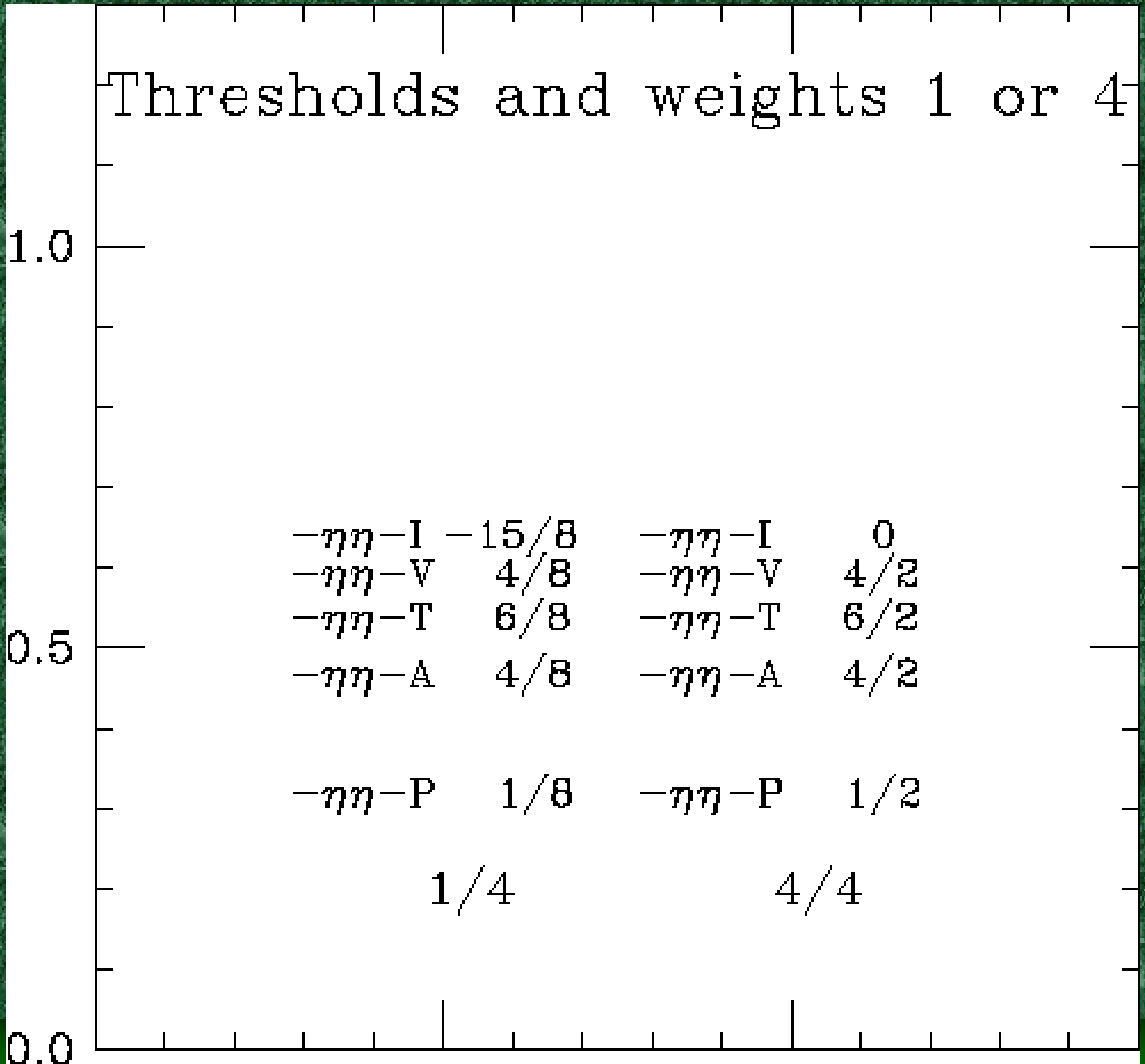
Rooted 2 + 1 flavor Theory



Unrooted 2x4 + 4 Theory



Single flavor



Single flavor from rooting

- All Goldstone-mode thresholds cancel in the continuum limit.

Fitting function

Simultaneous fit to a0 (5 momenta) and f0 (5 momenta)

$$
\begin{aligned}
C_{a0}(p,t) &= f_{\text{meson},a0}(p,t) + f_{\text{bubble},a0}(p,t) \\
C_{f0}(p,t) &= f_{\text{meson},f0}(p,t) + f_{\text{bubble},f0}(p,t)
\end{aligned}
$$

$$
\begin{aligned}
f_{\text{meson},a0}(p,t) &= b_{a0}(p)\exp[-E_{a0}(p)t] + b_{\pi,A}(p)(-)^t\exp[-E_{\pi,A}(p)t] + (N_t - t) \\
f_{\text{meson},f0}(p,t) &= c_0(p) + b_{f0}(p)\exp[-E_{f0}(p)t] + b_{\eta,A}(p)(-)^t\exp[-E_{\eta,A}(p)t] + (N_t - t)
\end{aligned}
$$

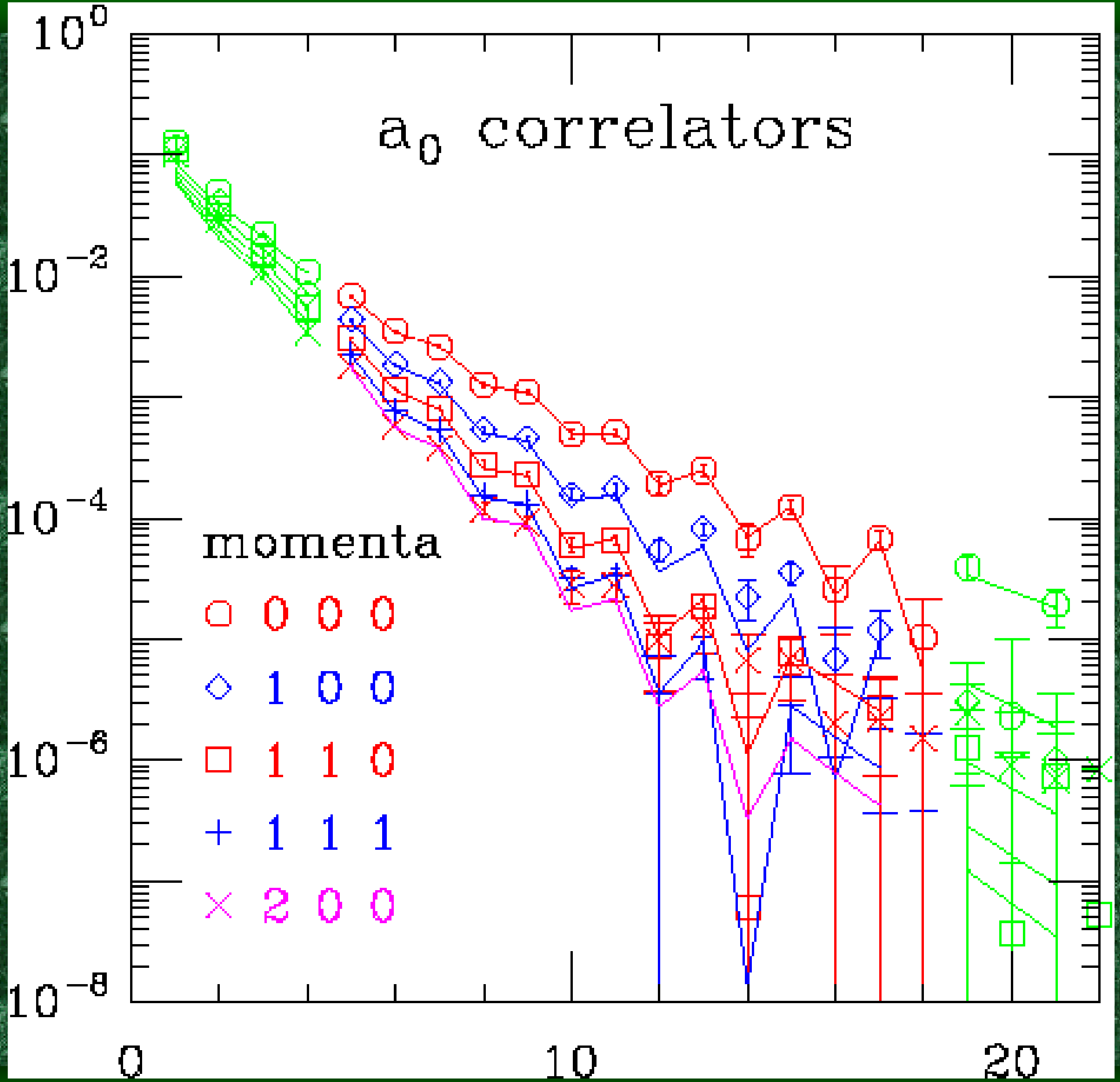
Meson terms: 13 params

Bubble terms: 3 params

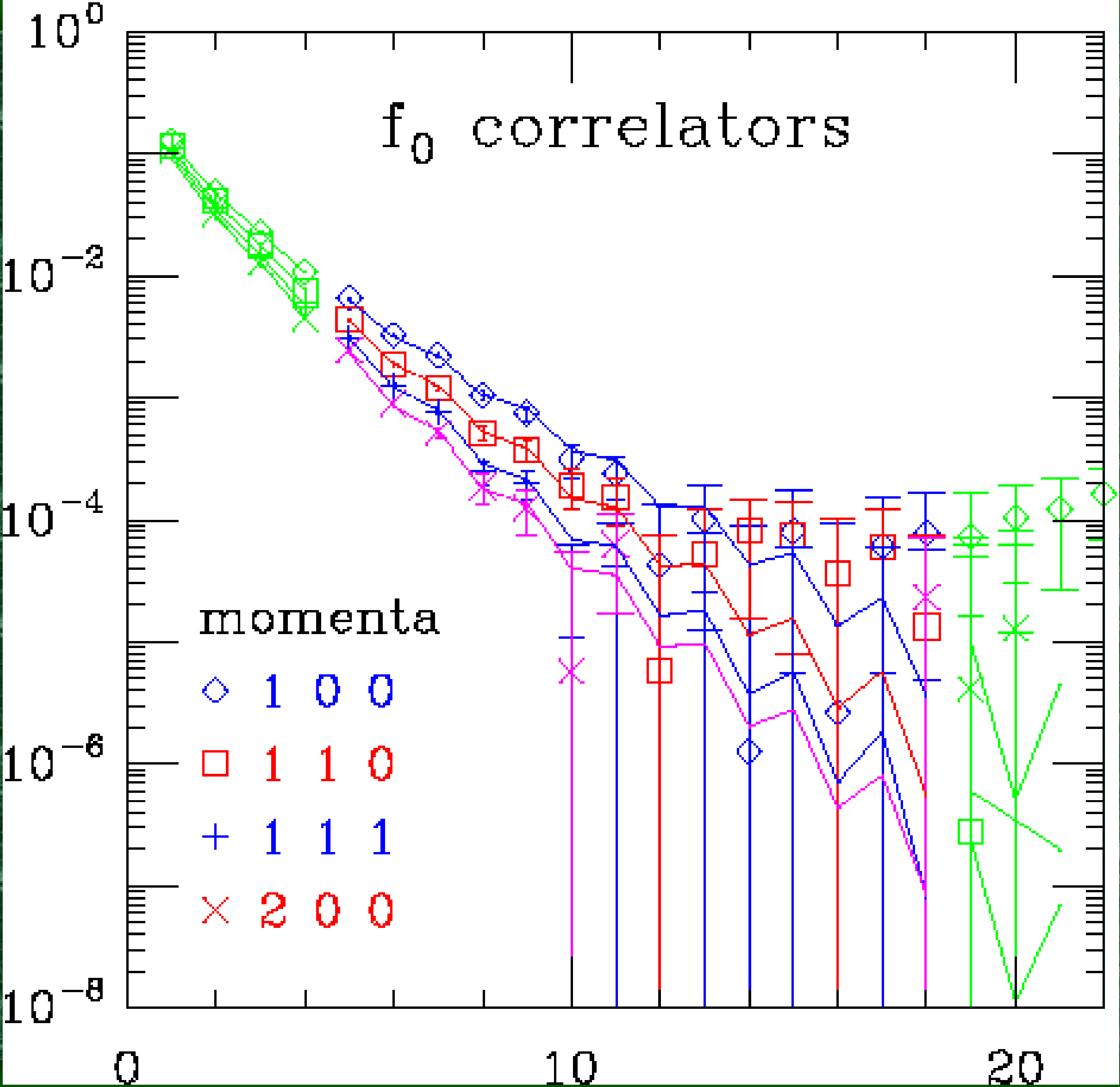
$$\mu = m_\pi^2/(2m_{u,d}), \quad \delta_A = a^4\delta'_A, \quad \delta_V = a^4\delta'_A$$

Ensemble

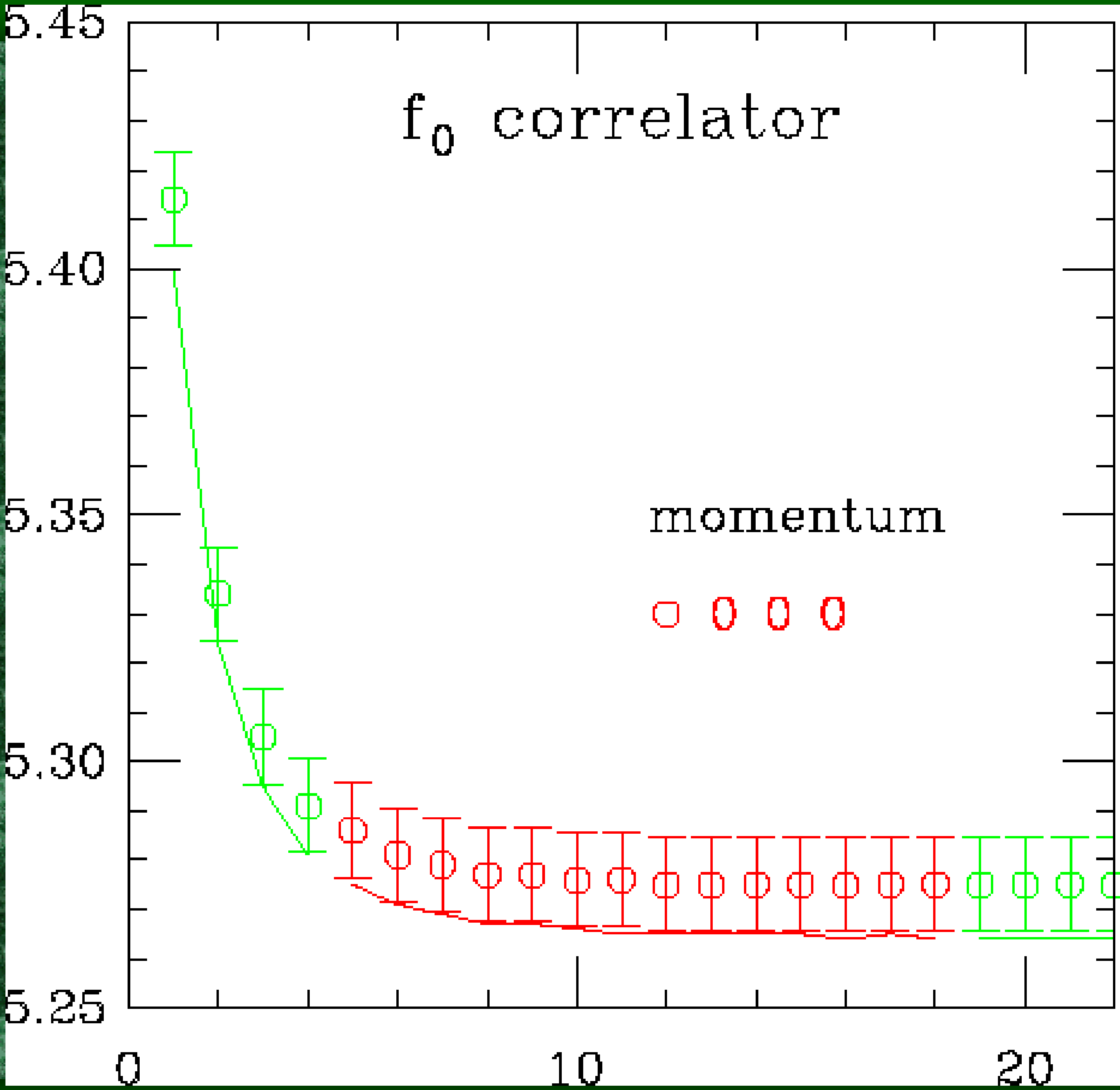
- MILC Asqtad   a = 0.125 fm   24^3 x 64
- a m_ud = 0.005   a m_s = 0.05
- m_pi/m_rho = 0.303
- 510 configs   8 time slices 200 rand sources



a_0 correlators
momenta
○ 0 0 0
◇ 1 0 0
□ 1 1 0
+ 1 1 1
× 2 0 0
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
10
20



f0 correlators
momenta
◇ 1 0 0
□ 1 1 0
+ 1 1 1
× 2 0 0
10^0
10^-2
10^-4
10^-6
10^-8
0
10
20


f0 correlator
momentum
0 0 0
5.45
5.40
5.35
5.30
5.25
0
10
20

Fit Result (Preliminary)

- Compare with LEC's from meson masses and decays

	Our fit	meson mass, decay
$r_1 m_\pi^2/(2m_{u,d})$	8.2(1.1)	6.7
δ_V	(prior)	-0.016(23)
δ_A	-0.053(7)	-0.040(6)

- Results are reasonably consistent
- a0 mass (lattice units) 0.71(3) f0 mass 0.47(7)
- ainv = 1.642 MeV

Conclusions

- Taste symmetry breaking complicates two-particle thresholds
- For scalar mesons the bubble term is completely determined from chiral low energy constants.
- Fit is reasonably consistent with LEC’s from meson masses and decays.
- Unitarity violations and nonlocality are similar to what occurs with partial quenching.
- There are no spurious Goldstone modes in the continuum limit