Light baryon spectrum using improved interpolating operators

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Lattice 2006, Tucson, Arizona, July 24, 2006

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Goal: Determine baryon spectra in lattice QCD.

How?: Need a set of baryon operators that couple to states of interest.

N^* state	J^P	
N(939)	$1/2^+$	****
N(1440)	$1/2^+$	****
N(1520)	$3/2^-$	****
N(1535)	$1/2^-$	****
N(1650)	$1/2^-$	****
N(1675)	$5/2^-$	****
N(1680)	$5/2^+$	****
N(1700)	$3/2^-$	***
N(1710)	$1/2^+$	***
N(1720)	$3/2^+$	****
N(1900)	$3/2^+$	**
N(1990)	$7/2^+$	**
N(2000)	$5/2^+$	**

- Discretization of space
- Must apply a subgroup of the continuum rotational group. The double-covered point group of cube is O^D.

	continuum	lattice
group	SU(2)	**octahedral group O^D**
irrep	$J = 1/2, 3/2, \cdots$	$\Lambda = G_1, G_2, H$
"row"	$-J \leq M \leq J$	$\lambda = \{1,2\}$ for G_1, G_2
		$\lambda = \{1,2,3,4\}$ for H

- Subduction of SU(2) to O^D

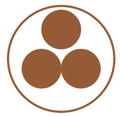
J	1/2	3/2	5/2	7/2	$\cdots$
Λ	G_1	H	$G_2 + H$	$G_1 + G_2 + H$	$\cdots$

Outline of Talk

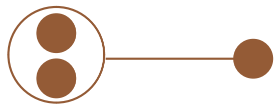
1. Baryon operator design for lattice QCD simulations

Phys. Rev. D72, 074501 (2005); Phys. Rev. D72, 094506 (2005).

- Local operators



- One-link operators



2. Lattice results of N^* and Δ^* spectra

- Observation of degeneracy patterns
- Volume dependence of masses

3. Summary and Future

Variational Method

A matrix of correlation functions is constructed using baryon interpolating operators $\overline{B}_m^{\Lambda \mathcal{P} \lambda}(\vec{x},t)$.

$$C_{lm}^{(\Lambda \mathcal{P})}(t) = \sum_{\vec{x}} \langle 0 | B_l^{\Lambda \mathcal{P} \lambda}(\vec{x},t) \overline{B}_m^{\Lambda \mathcal{P} \lambda}(\vec{0},0) | 0 \rangle$$

Solve the generalized eigenvalue equation

(M. Lüscher and U. Wolf, Nucl. Phys. B**339**, 222 (1990)).

$$\begin{aligned}
C_{lm}(t)\, v_m^{(n)}(t,t_0) &= \alpha^{(n)}(t,t_0)\, C_{lm}(t_0)\, v_m^{(n)}(t,t_0), \\
\alpha^{(n)}(t,t_0) &\simeq e^{-M_n(t-t_0)}, \\
M_n &= -ln[\alpha(t+1,t_0)/\alpha(t,t_0)].
\end{aligned}$$

Optimized operator $\rightarrow$ $\widetilde{\overline{B}}^{\Lambda \mathcal{P} \lambda (n)}(\vec{x},t) = \sum_m v_m^{(n)}(t,t_0)\, \overline{B}_m^{\Lambda \mathcal{P} \lambda}(\vec{x},t)$

Local baryon operators

$$\psi^{\Lambda}_{\lambda}(\vec{x},t) = c^{(\Lambda,\lambda)}_{\alpha\beta\gamma}\,\varepsilon_{abc}\,\phi_{ABC}\;q^{Aa}_{\alpha}(\vec{x},t)\,q^{Bb}_{\beta}(\vec{x},t)\,q^{Cc}_{\gamma}(\vec{x},t),$$

$c^{(\Lambda,\lambda)}_{\alpha\beta\gamma}$: coefficients tying Dirac indices to yield overall irrep Λ and row λ

ε_{abc} : color antisymmetry

ϕ_{ABC} : quark flavors

baryon	operator $(I = I_z)$	color	flavor	space	spin index	num.
N	$(u_\alpha d_\beta - d_\alpha u_\beta)u_\gamma$	A	MA	S	MA	20
Δ	$u_\alpha u_\beta u_\gamma$	A	S	S	S	20
Ω	$s_\alpha s_\beta s_\gamma$	A	S	S	S	20
Λ	$(u_\alpha d_\beta - d_\alpha u_\beta)s_\gamma$	A	MA,A	S	MA,A	24
Σ	$u_\alpha u_\beta s_\gamma$	A	S,MS	S	S,MS	40
Ξ	$s_\alpha s_\beta u_\gamma$	A	S,MS	S	S,MS	40

One-link baryon operators

Define a covariant displacement operator d_i acted on $\overline{\Psi}_\lambda^\Lambda(\vec{x},t)$ as

$$d_i\overline{\Psi}_\lambda^\Lambda(\vec{x}) \equiv c_{\alpha\beta\gamma}^{(\Lambda,\lambda)}\,\varepsilon_{abc}\,\phi_{ABC}\,\overline{q}_\alpha^{Aa}(\vec{x})\,\overline{q}_\beta^{Bb}(\vec{x})\,\overline{q}_\gamma^{Cc'}(\vec{x}+a\hat{e}_i)U_i^{\dagger c'c}(\vec{x}).$$

(3 spinors) $\times$ (1 spatial)

$\longrightarrow \{d_x, d_y, d_z, d_{-x}, d_{-y}, d_{-z}\}$ transform amongst themselves.

The spatial part is reduced into $A_1(1), T_1(3)$, and $E(2)$ (single-valued) irrep of the octahedral group. Define

$$\begin{pmatrix} \hat{A}_1\overline{\Psi} \\ \hat{D}_+\overline{\Psi} \\ \hat{D}_-\overline{\Psi} \\ \hat{D}_0\overline{\Psi} \\ \hat{E}_0\overline{\Psi} \\ \hat{E}_2\overline{\Psi} \end{pmatrix} \equiv \begin{pmatrix} \frac{1}{\sqrt{6}}(d_x\overline{\Psi}+d_y\overline{\Psi}+d_z\overline{\Psi}+d_{-x}\overline{\Psi}+d_{-y}\overline{\Psi}+d_{-z}\overline{\Psi}) \\ \frac{i}{2a}[(d_x\overline{\Psi}-d_{-x}\overline{\Psi})+i(d_y\overline{\Psi}-d_{-y}\overline{\Psi})] \\ -\frac{i}{2a}[(d_x\overline{\Psi}-d_{-x}\overline{\Psi})-i(d_y\overline{\Psi}-d_{-y}\overline{\Psi})] \\ -\frac{i}{\sqrt{2}a}(d_z\overline{\Psi}-d_{-z}\overline{\Psi}) \\ \frac{1}{2\sqrt{3}a^2}[2(d_z\overline{\Psi}+d_{-z}\overline{\Psi})-(d_x\overline{\Psi}+d_{-x}\overline{\Psi})-(d_y\overline{\Psi}+d_{-y}\overline{\Psi})] \\ \frac{1}{2a^2}[(d_x\overline{\Psi}+d_{-x}\overline{\Psi})-(d_y\overline{\Psi}+d_{-y}\overline{\Psi})] \end{pmatrix}.$$

basis operators	irrep (spatial)	spherical harmonics	wave
$\hat{A}_1\overline{\Psi}$	A_1	Y_{00}	S
$\hat{D}_\pm\overline{\Psi},\quad \hat{D}_0\overline{\Psi}$	T_1	$Y_{1\pm1},\quad Y_{10}$	P
$\hat{E}_0\overline{\Psi},\quad \hat{E}_2\overline{\Psi}$	E	$Y_{20},\quad (Y_{22}+Y_{2-2})$	D

One-link operators $\overline{B}_\lambda^\Lambda$ with irrep Λ and row λ are given by

$$\begin{aligned}
\text{“}\, J &= L+S \,\text{”} \\
\overline{B}_\lambda^\Lambda &= \hat{A}_1\overline{\Psi}_\lambda^\Lambda \\
\overline{B}_\lambda^\Lambda &= \sum_{r,\lambda'} C\begin{pmatrix}\Lambda & T_1 & \Lambda' \\ \lambda & r & \lambda'\end{pmatrix}\hat{D}_r\overline{\Psi}_{\lambda'}^{\Lambda'} \\
\overline{B}_\lambda^\Lambda &= \sum_{r,\lambda'} C\begin{pmatrix}\Lambda & E & \Lambda' \\ \lambda & r & \lambda'\end{pmatrix}\hat{E}_r\overline{\Psi}_{\lambda'}^{\Lambda'}
\end{aligned}$$

where $C\begin{pmatrix}\Lambda & \Lambda'' & \Lambda' \\ \lambda & \lambda'' & \lambda'\end{pmatrix}$ are the Clebsch-Gordan coefficients of the octahedral group.

T_1 one-link operators

$$\hat{D}_m \overline{\Psi}^{(\Lambda^{\mathcal{P}})}_{S,S_z} = \hat{D}_m \overline{N}^{(\Lambda^{\mathcal{P}})}_{S,S_z},\ \hat{D}_m \overline{\Delta}^{(\Lambda^{\mathcal{P}})}_{S,S_z},\ \hat{D}_m \overline{\Lambda}^{(\Lambda^{\mathcal{P}})}_{S,S_z},\ \cdots$$

G_1 sources ($s = 1/2, 7/2, \ldots$)	$\mathcal{P}$	wave	J_z
$-\sqrt{\frac{2}{3}}\hat{D}_+\overline{\Psi}^{(G_1^{\pm},k)}_{1/2,-1/2} + \frac{1}{\sqrt{3}}\hat{D}_0\overline{\Psi}^{(G_1^{\pm},k)}_{1/2,1/2}$	$\mp$	$p_{1/2}$	$+1/2$
$\sqrt{\frac{2}{3}}\hat{D}_-\overline{\Psi}^{(G_1^{\pm},k)}_{1/2,1/2} - \frac{1}{\sqrt{3}}\hat{D}_0\overline{\Psi}^{(G_1^{\pm},k)}_{1/2,-1/2}$	$\mp$	$p_{1/2}$	$-1/2$
$\frac{1}{\sqrt{2}}\hat{D}_-\overline{\Psi}^{(H^{\pm})}_{3/2,3/2} + \frac{1}{\sqrt{6}}\hat{D}_+\overline{\Psi}^{(H^{\pm})}_{3/2,-1/2} - \frac{1}{\sqrt{3}}\hat{D}_0\overline{\Psi}^{(H^{\pm})}_{3/2,1/2}$	$\mp$	$p_{1/2}$	$+1/2$
$-\frac{1}{\sqrt{2}}\hat{D}_+\overline{\Psi}^{(H^{\pm})}_{3/2,-3/2} - \frac{1}{\sqrt{6}}\hat{D}_-\overline{\Psi}^{(H^{\pm})}_{3/2,1/2} + \frac{1}{\sqrt{3}}\hat{D}_0\overline{\Psi}^{(H^{\pm})}_{3/2,-1/2}$	$\mp$	$p_{1/2}$	$-1/2$

G_2 sources ($s = 5/2, 7/2, \cdots$)	$\mathcal{P}$	wave	J_z
$-\frac{1}{\sqrt{6}}\hat{D}_+\overline{\Psi}^{(H^{\pm})}_{3/2,3/2} + \frac{1}{\sqrt{2}}\hat{D}_-\overline{\Psi}^{(H^{\pm})}_{3/2,-1/2} + \frac{1}{\sqrt{3}}\hat{D}_0\overline{\Psi}^{(H^{\pm})}_{3/2,-3/2}$	$\mp$		
$\frac{1}{\sqrt{6}}\hat{D}_-\overline{\Psi}^{(H^{\pm})}_{3/2,-3/2} - \frac{1}{\sqrt{2}}\hat{D}_+\overline{\Psi}^{(H^{\pm})}_{3/2,1/2} - \frac{1}{\sqrt{3}}\hat{D}_0\overline{\Psi}^{(H^{\pm})}_{3/2,3/2}$	$\mp$		

H sources ($s = 3/2, 5/2, 7/2, ..$)	$\mathcal{P}$	wave	J_z
$\hat{D}_+ \overline{\Psi}^{(G_1^\pm,k)}_{1/2,1/2}$	$\mp$	$p_{3/2}$	$+3/2$
$\frac{1}{\sqrt{3}} \hat{D}_+ \overline{\Psi}^{(G_1^\pm,k)}_{1/2,-1/2} + \sqrt{\frac{2}{3}} \hat{D}_0 \overline{\Psi}^{(G_1^\pm,k)}_{1/2,1/2}$	$\mp$	$p_{3/2}$	$+1/2$
$\frac{1}{\sqrt{3}} \hat{D}_- \overline{\Psi}^{(G_1^\pm,k)}_{1/2,1/2} + \sqrt{\frac{2}{3}} \hat{D}_0 \overline{\Psi}^{(G_1^\pm,k)}_{1/2,-1/2}$	$\mp$	$p_{3/2}$	$-1/2$
$\hat{D}_- \overline{\Psi}^{(G_1^\pm,k)}_{1/2,-1/2}$	$\mp$	$p_{3/2}$	$-3/2$
$-\sqrt{\frac{2}{5}} \hat{D}_+ \overline{\Psi}^{(H^\pm)}_{3/2,1/2} + \sqrt{\frac{3}{5}} \hat{D}_0 \overline{\Psi}^{(H^\pm)}_{3/2,3/2}$	$\mp$	$p_{3/2}$	$+3/2$
$-\sqrt{\frac{8}{15}} \hat{D}_+ \overline{\Psi}^{(H^\pm)}_{3/2,-1/2} + \sqrt{\frac{2}{5}} \hat{D}_- \overline{\Psi}^{(H^\pm)}_{3/2,3/2} + \frac{1}{\sqrt{15}} \hat{D}_0 \overline{\Psi}^{(H^\pm)}_{3/2,1/2}$	$\mp$	$p_{3/2}$	$+1/2$
$-\sqrt{\frac{2}{5}} \hat{D}_+ \overline{\Psi}^{(H^\pm)}_{3/2,-3/2} + \sqrt{\frac{8}{15}} \hat{D}_- \overline{\Psi}^{(H^\pm)}_{3/2,1/2} - \frac{1}{\sqrt{15}} \hat{D}_0 \overline{\Psi}^{(H^\pm)}_{3/2,-1/2}$	$\mp$	$p_{3/2}$	$-1/2$
$\sqrt{\frac{2}{5}} \hat{D}_- \overline{\Psi}^{(H^\pm)}_{3/2,-1/2} - \sqrt{\frac{3}{5}} \hat{D}_0 \overline{\Psi}^{(H^\pm)}_{3/2,-3/2}$	$\mp$	$p_{3/2}$	$-3/2$
$\frac{1}{\sqrt{10}} \hat{D}_+ \overline{\Psi}^{(H^\pm,k)}_{3/2,1/2} + \sqrt{\frac{5}{6}} \hat{D}_- \overline{\Psi}^{(H^\pm,k)}_{3/2,-3/2} + \frac{1}{\sqrt{15}} \hat{D}_0 \overline{\Psi}^{(H^\pm,k)}_{3/2,3/2}$	$\mp$		
$\sqrt{\frac{3}{10}} \hat{D}_+ \overline{\Psi}^{(H^\pm,k)}_{3/2,-1/2} + \frac{1}{\sqrt{10}} \hat{D}_- \overline{\Psi}^{(H^\pm,k)}_{3/2,3/2} + \sqrt{\frac{3}{5}} \hat{D}_0 \overline{\Psi}^{(H^\pm,k)}_{3/2,1/2}$	$\mp$		
$\frac{1}{\sqrt{10}} \hat{D}_+ \overline{\Psi}^{(H^\pm,k)}_{3/2,-3/2} + \sqrt{\frac{3}{10}} \hat{D}_- \overline{\Psi}^{(H^\pm)}_{3/2,1/2} + \sqrt{\frac{3}{5}} \hat{D}_0 \overline{\Psi}^{(H^\pm,k)}_{3/2,-1/2}$	$\mp$		
$\sqrt{\frac{5}{6}} \hat{D}_+ \overline{\Psi}^{(H^\pm,k)}_{3/2,3/2} + \frac{1}{\sqrt{10}} \hat{D}_- \overline{\Psi}^{(H^\pm,k)}_{3/2,-1/2} + \frac{1}{\sqrt{15}} \hat{D}_0 \overline{\Psi}^{(H^\pm,k)}_{3/2,-3/2}$	$\mp$		

Lattice results of

N & Δ baryon spectra

using local & one-link operators

Simulation Setup

- Quenched, anisotropic lattices using Wilson gauge action with $\beta = 6.1$.
- Lattice size $16^3 \times 64$ and $24^3 \times 64$ with renormalized anisotropy $\xi = 3.0$.
- $a_s^{-1} = 2.0$ GeV, $a_t^{-1} = 6.0$ GeV. $\Rightarrow$ spatial volume $\simeq (1.6\,\mathrm{fm})^3$; $\simeq (2.4\,\mathrm{fm})^3$.
- Anisotropic Wilson fermion action with $M_\pi \simeq 490$ MeV.
- Smeared quark fields are used to improve overlapping with low-lying states.
- Smeared gauge links are used in sinks and sources.
- Operators from local and one-link constructions are used:
  - N operators: 23 G_1, 28 H, and 7 G_2 for a given row and parity.
  - Δ operators: 12 G_1, 17 H, and 4 G_2 for a given row and parity.

Effective masses obtained by matrix diagonalizations

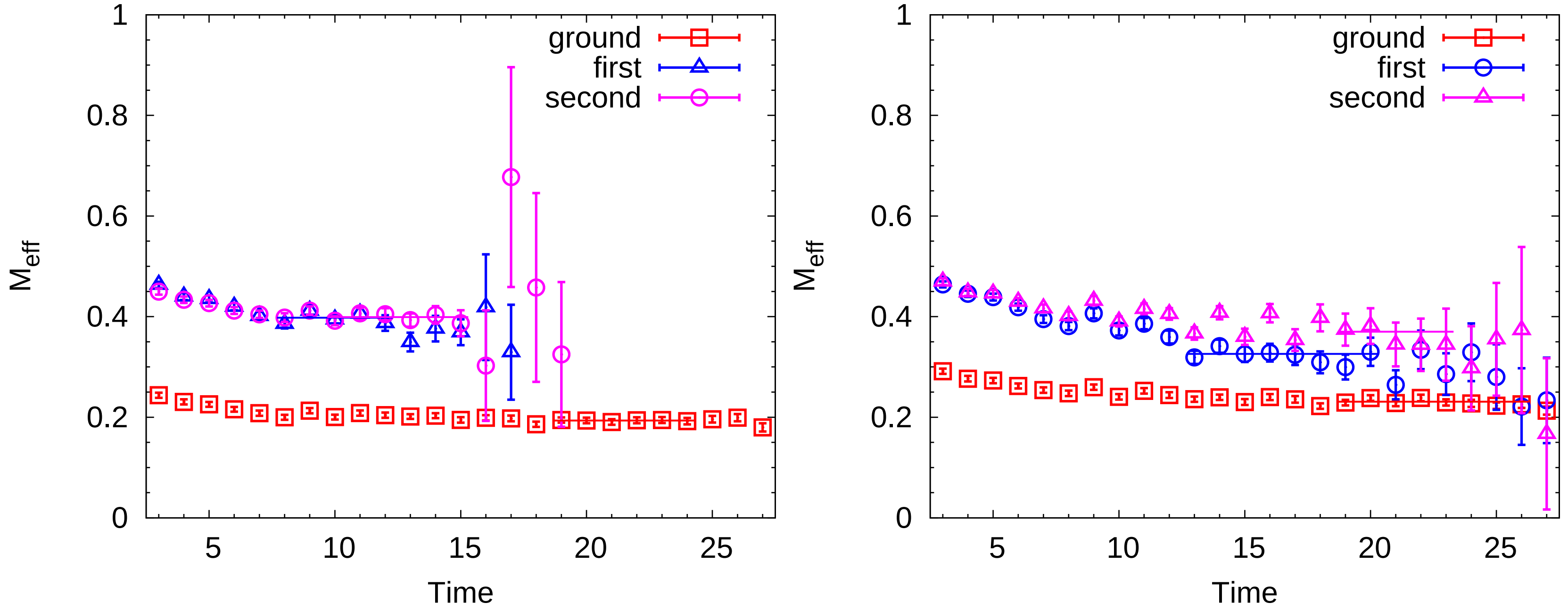
$$C_{lm}(t)v_m^{(n)}(t,t_0) = \alpha^{(n)}(t,t_0)C_{lm}(t_0)v_m^{(n)}(t,t_0), \quad \alpha^{(n)}(t,t_0) \simeq e^{-M_n(t-t_0)}$$

$I = 1/2$, G_1^+, 10×10 matrix

$J^P = 1/2^+, 7/2^+, \cdots$

$I = 3/2$, H^+, 4×4 matrix

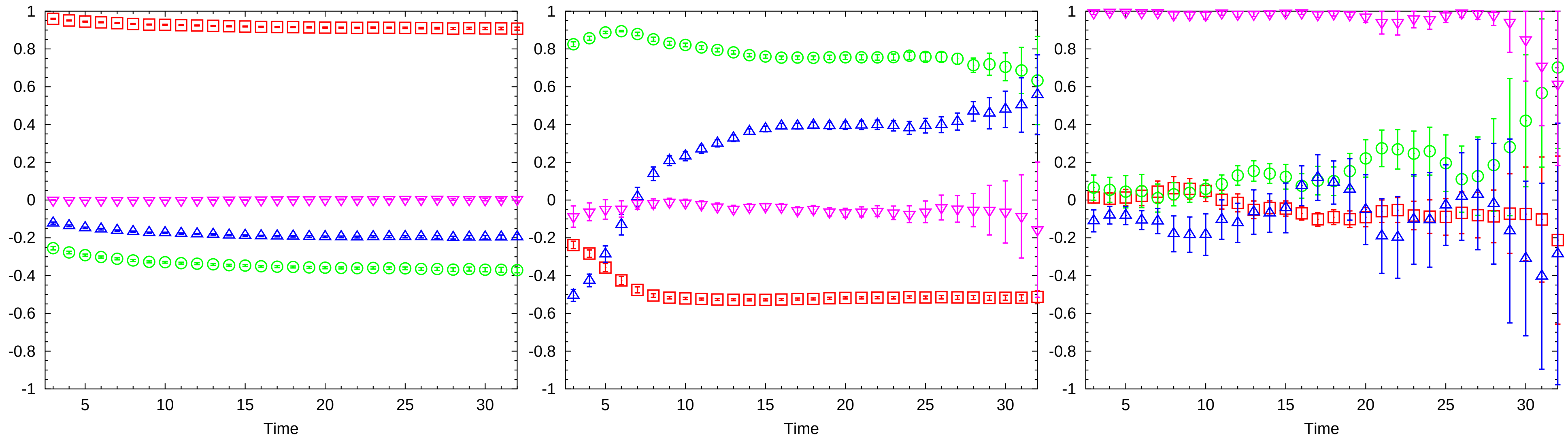
$J^P = 3/2^+, 5/2^+, 7/2^+, \cdots$



ground	0.193(3)	$19 \leq t \leq 24$
first	0.398(6)	$8 \leq t \leq 12$
second	0.399(7)	$10 \leq t \leq 15$

ground	0.231(3)	$19 \leq t \leq 26$
first	0.326(7)	$13 \leq t \leq 20$
second	0.370(25)	$19 \leq t \leq 23$

$I=3/2$, H^+ **eigenvectors:** $C_{lm}(t)v_m^{(n)}(t,t_0) = \alpha^{(n)}(t,t_0)C_{lm}(t_0)v_m^{(n)}(t,t_0)$



ground state | first excited state | second excited state

- Eigenvectors are stable for low-lying states.
- Important operators are identified.
- Less important operators can be eliminated without changing the spectrum (next page).

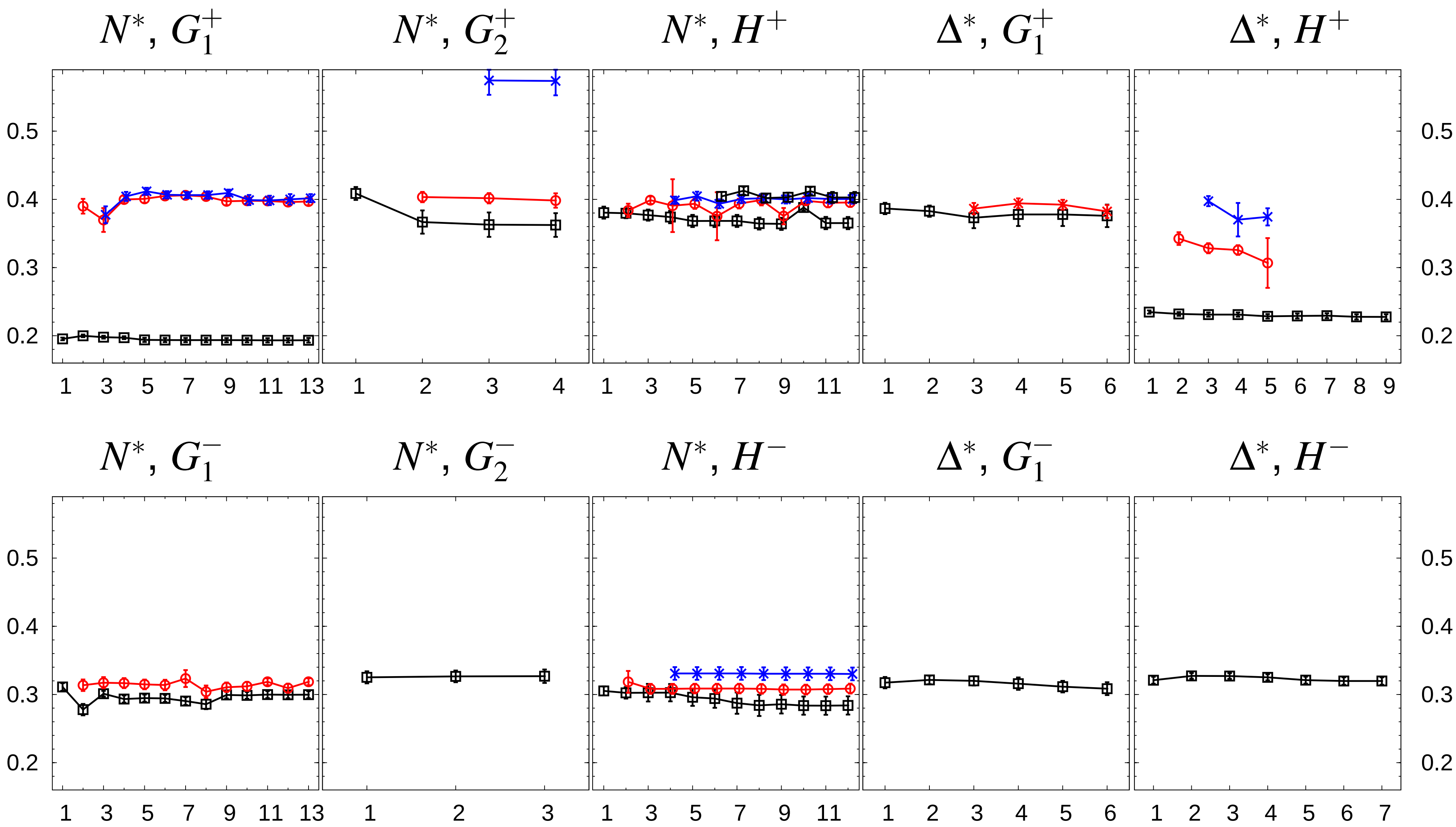
□ $\overline{\Delta}^{H_g,1}_{\frac{3}{2},\frac{3}{2}}$

○ $\sqrt{\frac{3}{5}}\hat{D}^-\overline{\Delta}^{H_u}_{\frac{3}{2},\frac{3}{2}} - \sqrt{\frac{2}{5}}\hat{D}^0\overline{\Delta}^{H_u}_{\frac{3}{2},\frac{1}{2}}$

△ $\hat{D}^+\overline{\Delta}^{G_{1u},1}_{\frac{1}{2},\frac{1}{2}}$

▽ $\frac{1}{\sqrt{10}}\hat{D}^+\overline{\Delta}^{H_u}_{\frac{3}{2},\frac{1}{2}} - \sqrt{\frac{5}{6}}\hat{D}^0\overline{\Delta}^{H_u}_{\frac{3}{2},-\frac{3}{2}} + \frac{1}{\sqrt{15}}\hat{D}^0\overline{\Delta}^{H_u}_{\frac{3}{2},\frac{3}{2}}$

Stability of $M_{\rm eff}$ −Diagonalization of matrices of different dimensions



No diagonalization on $\Delta^*, G_{2g/u}$ is performed.

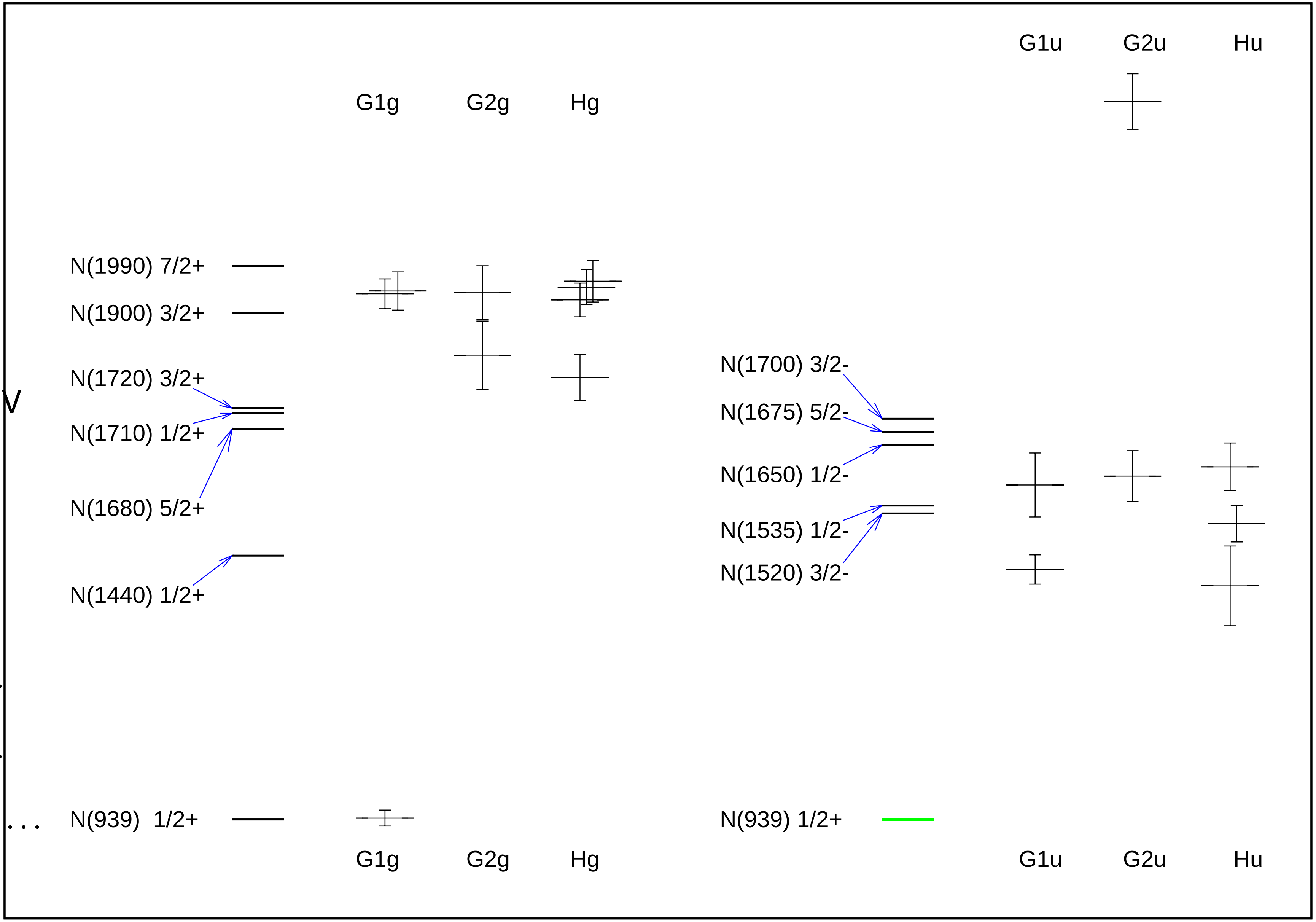
Nucleon spectra

$N(939)\,\frac{1}{2}^+$ and $N(G_1^+)$, 1182 MeV are drawn at the same level.

$G_{1g/u}: \frac{1}{2}^\pm, \frac{7}{2}^\pm, \cdots$

$G_{2g/u}: \frac{5}{2}^\pm, \frac{7}{2}^\pm, \cdots$

$H_{g/u}: \frac{3}{2}^\pm, \frac{5}{2}^\pm, \frac{7}{2}^\pm, \cdots$



Delta baryon spectra

$\Delta(1232)\,\frac{3}{2}^{+}$ and

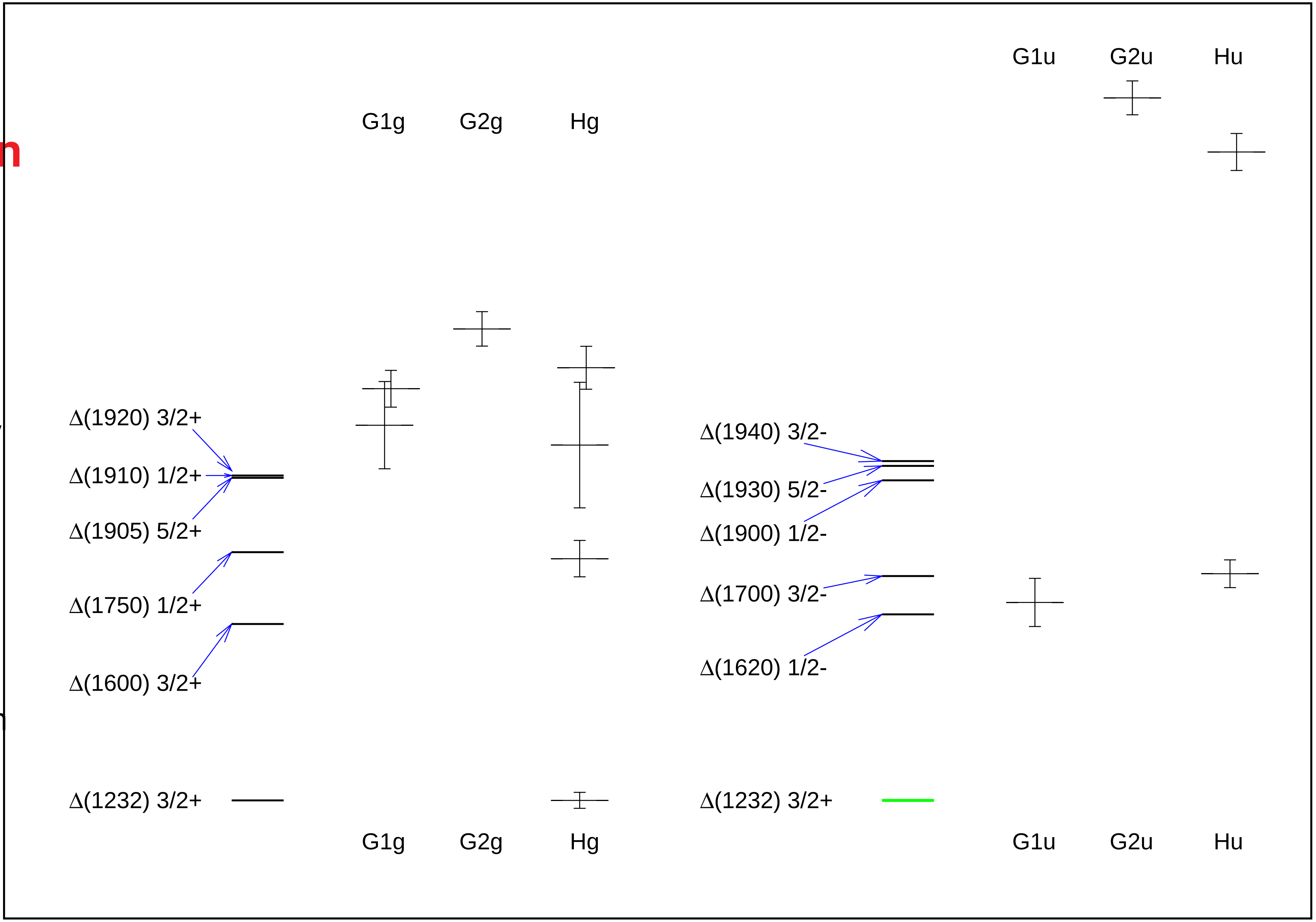
$\Delta(H^{+})$, 1416 MeV

are drawn at

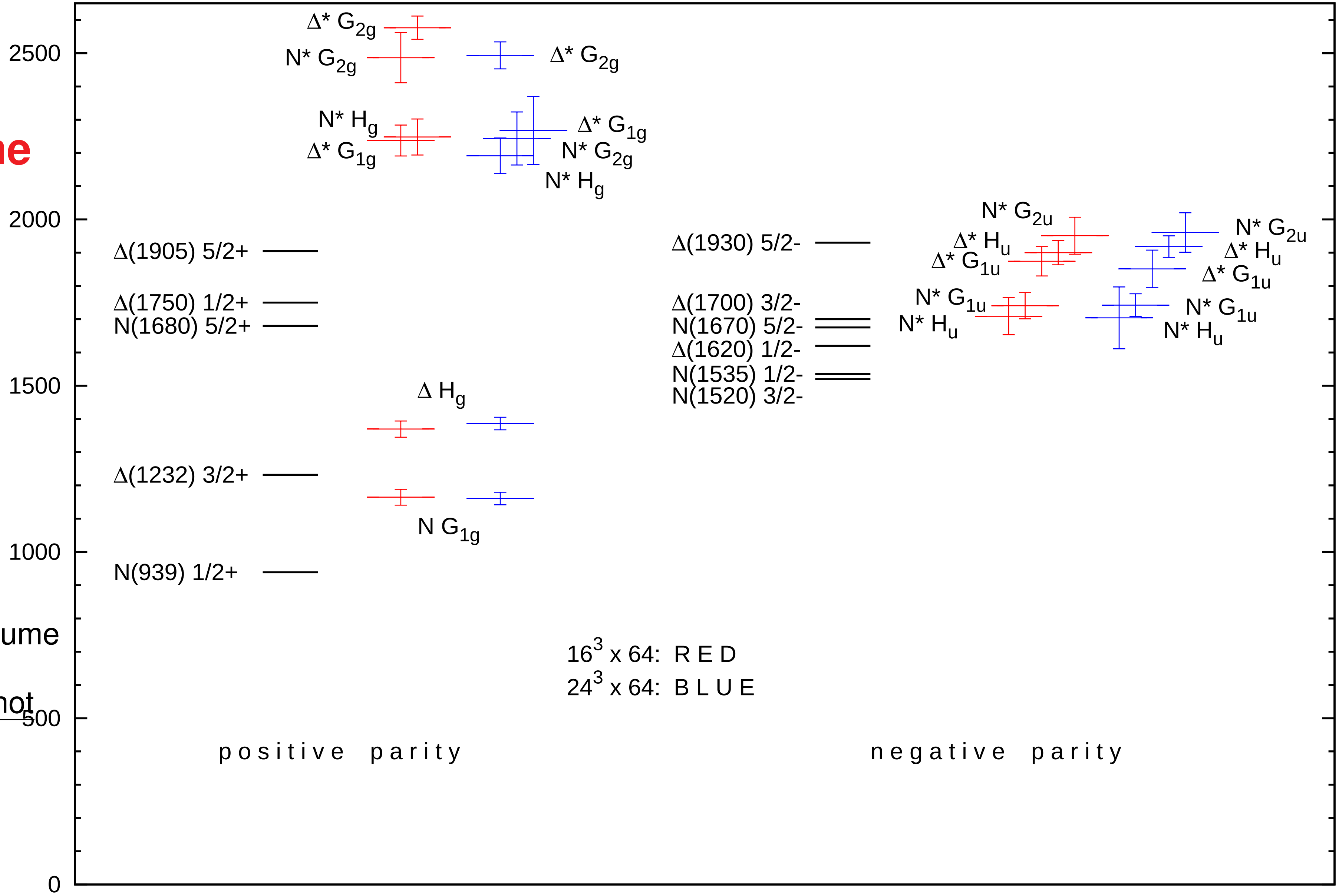
the same level.

No diagonalization

for $G_2^{\pm}$.



Finite volume effect

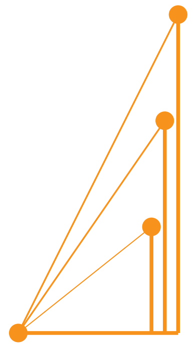


Finite volume effect is not strong at this M_π.

Summary and outlook

- Group theoretical constructions of baryon operators $(N, \Delta, \Lambda, \Sigma, \Xi, \Omega^-)$ are outlined.
  - Local operators are classified into two irreps: $G_1(2), H(4)$.
  - One-link operators are constructed by taking linear combinations of displacements and spins. Clebsch-Gordan coefficients of the group are used.
- Matrices of correlation functions are computed in each symmetry channel. A few low-lying masses are obtained by the variational technique.
- The *Ordering* of lowest-lying masses calculated in each irrep agrees with the empirical data within error.
- Baryon spectra are analyzed in two volumes: $(1.6\,\mathrm{fm})^3$ and $(2.4\,\mathrm{fm})^3$. Volume dependence of N and Δ baryon spectra is not strong at the pion mass we used (490 MeV), except the $N^* G_{2g}$.

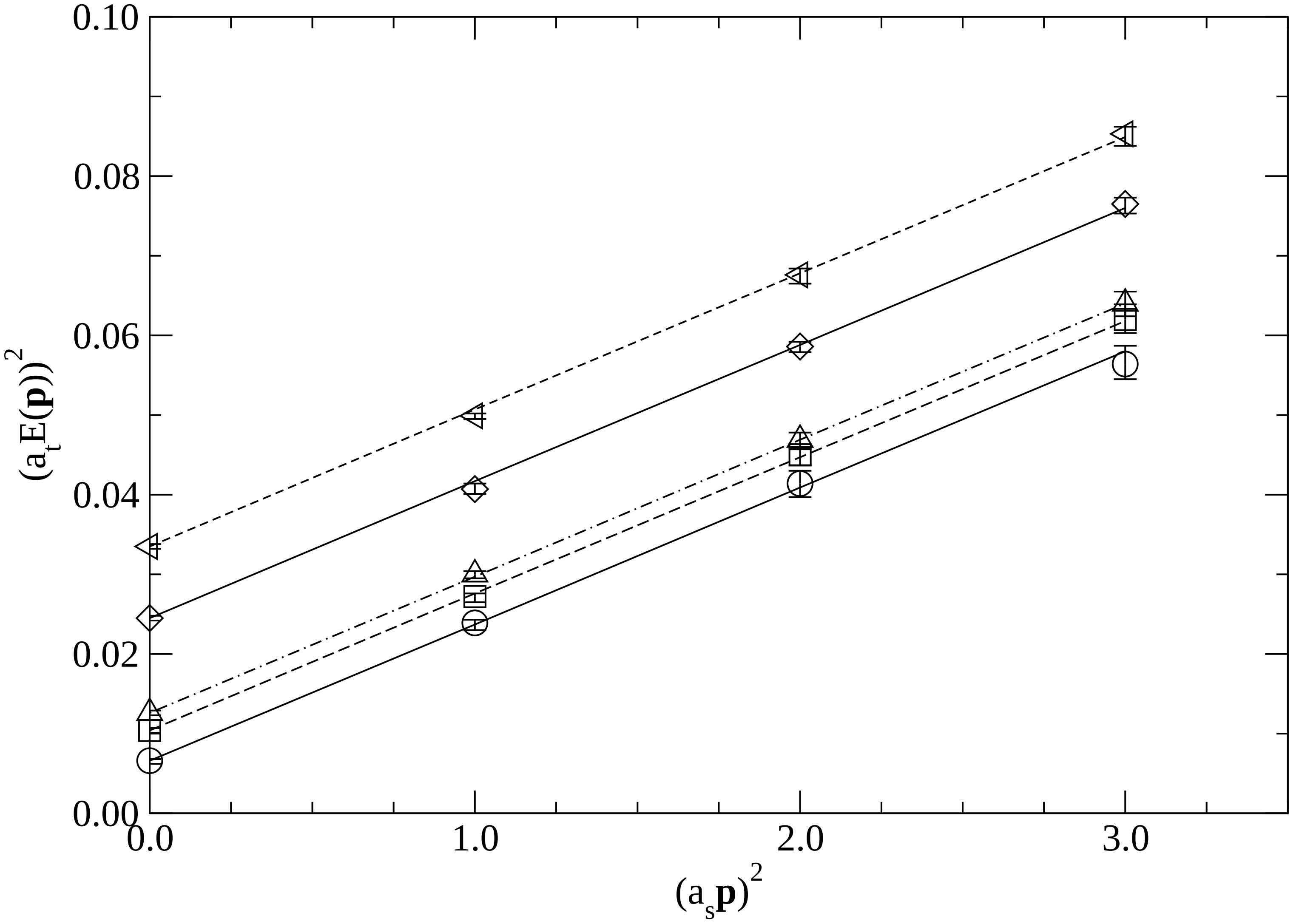
- More complicated spatial distributions of valence quarks need to be considered to have richer orbital structures.



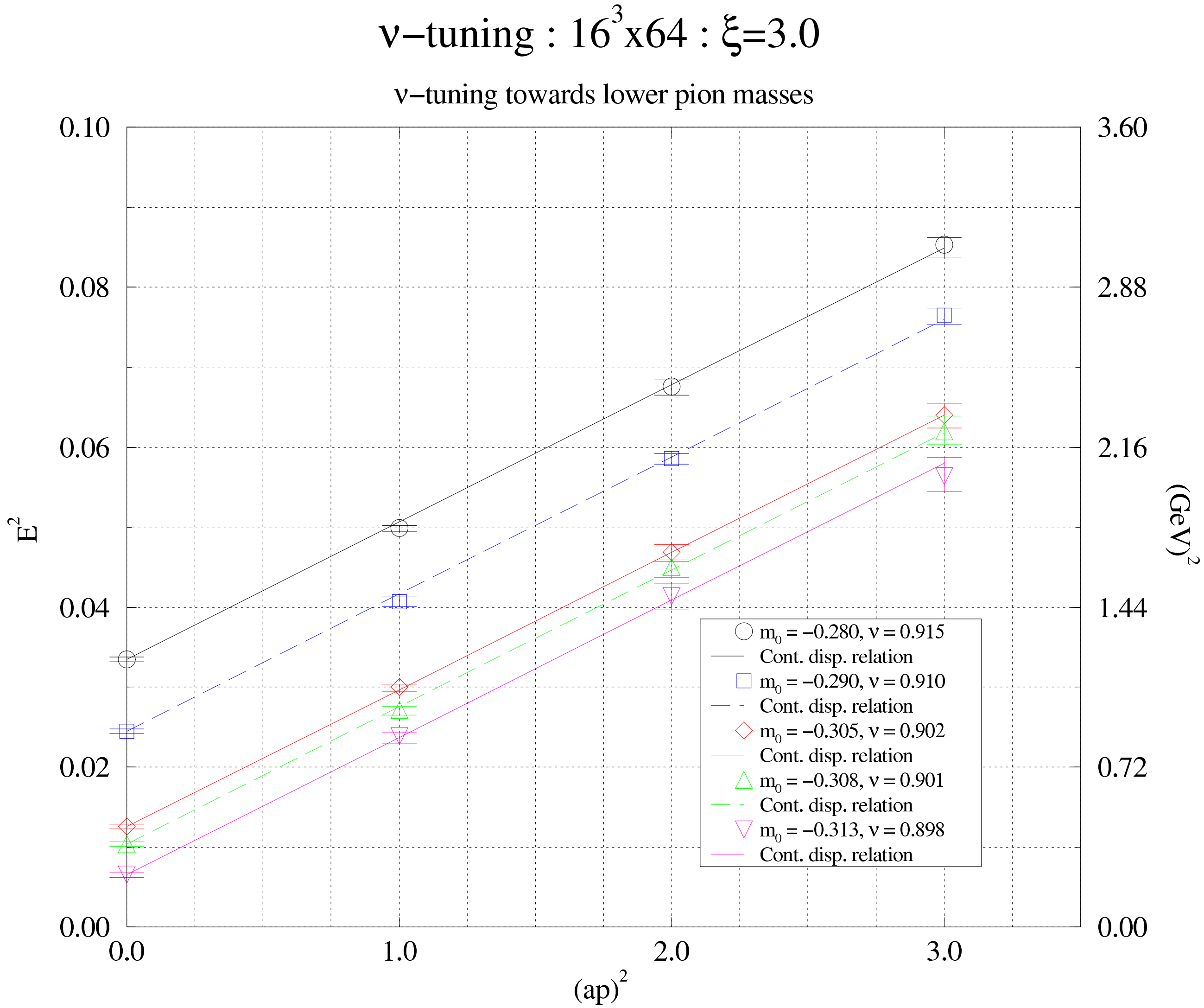
- Next LHPC project: Full QCD simulations with lower pion masses with tadpole-improved, anisotropic clover fermion action, using more sources.

- Applications of operators:

  - Hybrids by adding chromo-electric/-magnetic fields.
  - Multihadrons.
  - Hadron-hadron interactions.



0.10
0.08
0.06
0.04
0.02
0.00
0.0
1.0
2.0
3.0
$(a_t E(\mathbf{p}))^2$
$(a_s \mathbf{p})^2$



ν−tuning : 16³x64 : ξ=3.0
ν−tuning towards lower pion masses
0.10
0.08
0.06
0.04
0.02
0.00
3.60
2.88
2.16
1.44
0.72
0.00
E²
(GeV)²
0.0
1.0
2.0
3.0
(ap)²
m0 = −0.280, ν = 0.915
Cont. disp. relation
m0 = −0.290, ν = 0.910
Cont. disp. relation
m0 = −0.305, ν = 0.902
Cont. disp. relation
m0 = −0.308, ν = 0.901
Cont. disp. relation
m0 = −0.313, ν = 0.898
Cont. disp. relation