Diseases with rooted staggered quarks

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Summary

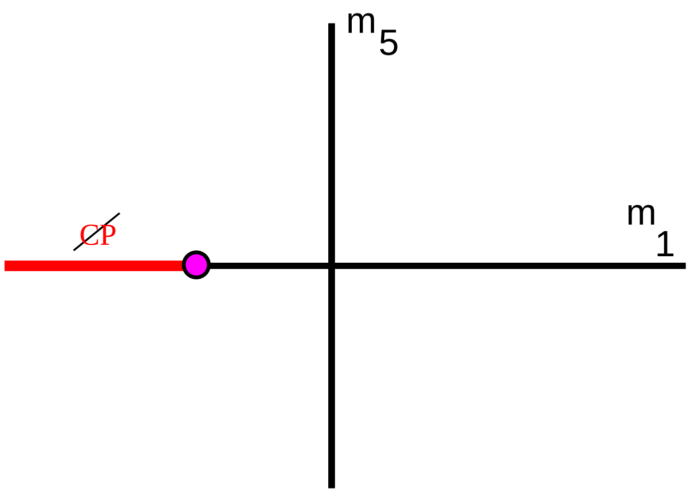
- Staggered quarks have an exact chiral symmetry
- This symmetry survives rooting
- More symmetry than allowed in the reduced theory

This presentation: http://latticeguy.net/root.pdf

Example: one flavor QCD

Generalized mass term $m_1 \; \overline{\psi}\psi + i \, m_5 \; \overline{\psi}\gamma_5\psi$

Phase diagram



Spontaneous CP violation at negative mass

Classical symmetry under $m \to -m$ broken by the anomaly

Gap between $m = 0$ and phase transition

- Physics analytic around vanishing mass

No chiral symmetry, parity doubling, Goldstone bosons

Rooted staggered quarks

Naive lattice fermions: 16 fold degeneracy (doublers)

Staggered quarks project one component on each site

- $\psi \to P\psi \qquad P = P^2 \qquad \mathrm{Tr}\ P = 1$

$$P = \frac{1}{4}\left(1 + i\gamma_1\gamma_2(-1)^{x_1+x_2} + i\gamma_3\gamma_4(-1)^{x_3+x_4} + \gamma_5(-1)^{x_1+x_2+x_3+x_4}\right)$$

Reduces 16 doublers down to 4

Rooting: replace $|D|$ with $|D|^{1/4}$ in the path integral

- $|D| \ =$ determinant of the staggered Dirac matrix
- Purportedly reducing 4 doublers to 1

Motivation: to save computer time

An exact symmetry

Generalize the staggered mass term to

$$(m_1 + iS(j)\, m_2)\; \overline{\psi}(j)\psi(j)$$

- $S(j)\;=\;$ parity of site $j\;=\;(-1)^{x_1+x_2+x_3+x_4}$

Determinant and path integral exactly invariant under

$$\begin{pmatrix} m_1 \\ m_2 \end{pmatrix} \longrightarrow \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}$$

Symmetry requires a Goldstone boson or parity doubling

For the unrooted theory this is OK

- A “taste” non-singlet symmetry

Rooting preserves this exact symmetry

Actions lie louder than words. -- Carolyn Wells

In finite volume on an even dimensional lattice

- $|D|$ is a polynomial in the combination $m_1^2 + m_2^2$
  - Proof: the hopping parameter expansion
- $|D|$ real and non-vanishing for real $m_1^2 + m_2^2 > 0$
  - Proof: eigenvalue analysis $|D| \geq (m_1^2 + m_2^2)^{N_c V/2}$
  - N_c = number of colors; V = number of lattice sites

Therefore roots of the determinant are analytic in m_1 and m_2

- in the vicinity of $m_1^2 + m_2^2 > 0$

Can analytically continue rooted path integral from m to $-m$

- $m_1 = m\cos(\theta), \qquad m_2 = m\sin(\theta), \qquad 0 \leq \theta \leq \pi$
- avoiding all singularities in $|D|^{1/4}$

All correlations at m_1 are identical to those at $-m_1$

Ward identities from $\frac{d^n}{d^n\theta} Z = 0$

- require a Goldstone boson or parity doubling

Contradicts known behavior of the one flavor theory

- analytic at $m = 0$
- m and $-m$ theories inequivalent
- no Goldstone bosons or parity doubling

Problems extend to $N_f > 1$ with non-degenerate quarks

At finite cutoff, rooted staggered fermions are qualitatively inconsistent with physical chiral behavior.

Can physics be saved in the continuum limit?

Bernard, Golterman, Shamir, Sharpe (hep-lat/0603027):

- non-commutation of $a \to 0$ and $m \to 0$ limits
- instantons introduce $\sqrt{m^2 + a^2}$ factors
  - break the $m \to -m$ symmetry if $a \to 0$ first
- extra Ward identities
  - satisfied by indefinite metric ‘‘taste’’ states
  - Goldstone bosons are there, but unphysical

Not proven

- ‘‘first principle’’ claims are at best premature

Plausible?

Infrared and ultraviolet cutoffs don't commute

- at finite physical volume?
- when there are no physical massless particles?

At finite a: chiral behavior is explicitly wrong

- unphysical non-analyticity at $|m| = O(a\Lambda_{qcd}^2)$?
- invalidates one original motivation for staggered?

Indefinite metric Goldstone bosons

- required for Ward identities?
- but no intermediate pairs in glueball propagators?
- must go beyond Osterwalder Seiler formalism?

Issues absent with other regulators

- Wilson, domain wall, overlap

Wanna buy a bridge?

References:

- MC, hep-lat/0603020, based on
  - Di Veccia and Veneziano, Nucl.Phys.B171:253,1980
  - MC, Rev.Mod.Phys.73:119-150,2001