The Staggered Fourth Root in Chiral Perturbation Theory

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Motivation

Why do we need a staggered chiral perturbation theory (SχPT)?

- Chiral theory describes the lowest energy, longest distance sector, where problems from "fourth root trick" ought to show up.
    - Theory is inherently nonlocal at $a \neq 0$ [CB, Golterman, Shamir].
    - Theory has unitarity violations at $a \neq 0$ [Prelovsek; CB, DeTar, Fu, Prelovsek; CB; DeTar's talk].
- Once SχPT is confirmed as correct low energy theory for rooted staggered quarks, can use it to see how problems go away in $a \to 0$ limit.
- Crucial for controlling continuum and chiral extrapolations.

Ingredients of SχPT

- (1) Start with LO chiral theory for a single unrooted staggered field including a^2 taste violations. [Lee & Sharpe] *nomenclature: 1 staggered field = 1 flavor (4 tastes if unrooted; 1 taste if rooted)*.
- (2) Generalize to many unrooted flavors [Aubin & CB].
- (3) To take into account fourth root, Aubin & CB proposed locating sea-quark loops and multiplying each by $\frac{1}{4}$.
    - To find sea quark loops, I will use replica trick [introduced for partially quenched chiral theories by Damgaard and Splittorff].

Steps (1) & (2) are relatively straightforward and noncontroversial.

Step (3) is nontrivial.

- Question: Is it SχPT, defined this way, the correct low energy effective theory for rooted staggered quarks?

Replica trick

- Replicate the sea-quark degrees of freedom, replacing each field by n_R identical copies ($n_R =$ positive integer).
- Calculate order by order in corresponding (unrooted) chiral theory.
- Take $n_R \to 1/4$ at end.
  - Dependence on n_R is polynomial at any finite order in SχPT, so $n_R \to 1/4$ is well-defined.
  - Treat LECs as free parameters for each n_R — LECs are taken independent of n_R in this procedure.

Overview of argument

- We know (trivially) how fourth root works when we have 4 degenerate flavors: $\left(\sqrt[4]{\mathrm{Det}}\right)^4 = \mathrm{Det}$.
  - Get 1-flavor, unrooted theory.
  - Local lattice action & known chiral theory [Lee & Sharpe].
- To get non-degenerate 4-flavor theory, expand around degenerate, massive point.
- To get theory with $n_F < 4$ flavors, decouple one quark at a time.

Note: in this talk, I always take all quark masses real and positive.

(n_F, n_T, n_R) notation

- $(n_F, n_T, n_R)_{LQCD}$ is generating functional for lattice QCD theory with:
  - n_F flavors
  - n_T tastes
  - n_R replicas of each flavor

- $(n_F, n_T, n_R)_\chi$ is corresponding generating functional for chiral theory.

- Omit n_R if it is trivially equal to 1 (because replica trick not relevant).

(n_F, n_T, n_R) notation

Relevant theories:

- $(1,4)_{LQCD}$ and $(1,4)_\chi$
  - Single unrooted staggered field.
  - $(1,4)_\chi$ is SχPT of Lee & Sharpe.
  - No replica trick necessary.

- $(n_F, 4, n_R)_{LQCD}$ and $(n_F, 4, n_R)_\chi$
  - n_F staggered fields.
  - n_R indicated explicitly $\Rightarrow$ integer only.
  - $(n_F, 4, n_R)_\chi$ is SχPT of Aubin & CB for $n_R \cdot n_F$ sea-quark flavors (still no rooting).

(n_F, n_T, n_R) notation

Relevant theories (continued):

- $(n_F, \text{“1”})_{LQCD}$ and $(n_F, \text{“1”})_\chi$
  - n_F staggered fields with $\sqrt[4]{\mathrm{Det}}$ taken.
  - Quotes on “1” taste $\Rightarrow$ don’t assume fourth root works.
  - $(n_F, \text{“1”})_\chi$ is by definition the low energy theory generated by integrating out all higher modes in $(n_F, \text{“1”})_{LQCD}$.
  - As far as we know so far, $(n_F, \text{“1”})_\chi$ could be horribly nonlocal, nonunitary, or otherwise sick.
  - Want to find $(n_F, \text{“1”})_\chi$ unambiguously.
- $(n_F, 4, \frac{1}{4})_\chi$
  - Chiral theory $(n_F, 4, n_R)_\chi$ with the replica trick $n_R \to 1/4$.
  - Defines SχPT for rooted theory.
  - Does $(n_F, \text{“1”})_\chi$ = $(n_F, 4, \frac{1}{4})_\chi$?

$n_F = 4$ basics

- Want to show:

$$(4, \text{“}1\text{”})_\chi = (4, 4, \tfrac{1}{4})_\chi$$

- Start with degenerate 4-flavor theory: $\mathcal{M} = \bar{m}I$, where I is identity matrix in flavor space:

$$\begin{aligned}
(4, \text{“}1\text{”})_{LQCD}\Big|_{\mathcal{M}=\bar{m}I} &= (1,4)_{LQCD}\Big|_{\bar{m}} \\
(4, \text{“}1\text{”})_{\chi}\Big|_{\mathcal{M}=\bar{m}I} &= (1,4)_{\chi}\Big|_{\bar{m}} = (4, 4, \tfrac{1}{4})_\chi\Big|_{\mathcal{M}=\bar{m}I}
\end{aligned}$$

- Last equivalence manifest order by order in SχPT.
  - Taking $4n_R$ degenerate flavors and then putting $n_R = 1/4$ $\Longleftrightarrow$ one-flavor theory.

$n_F = 4$: expansion around degenerate point

- To move away from degenerate limit, add taste-singlet scalar sources for sea-quark fields:

$$\mathcal{L}_{(4,\text{``1''})} \;=\; \cdots + \bar m\, \bar\Psi_i(x)\Psi_i(x) + \bar\Psi_i(x)\, s^{ij}(x)\, \Psi_j(x) + \ldots$$

$$\mathcal{L}_{(4,4,n_R)} \;=\; \cdots + \bar m \bar\Psi_i^r(x)\Psi_i^r(x) + \bar\Psi_i^r(x)\, s^{ij}(x)\, \Psi_j^r(x) + \ldots$$

[sum over i, j (flavor indices) and r (replica index)]

- When $s \neq 0$, we don't yet know that $(4, 4, \frac{1}{4})_\chi$ is right chiral theory
- Define $V[s]$ as amount of mismatch:

$$(4, \text{``1''};\, s)_\chi = (4, 4, \tfrac{1}{4};\, s)_\chi + V[s]$$

- $V[s] = 0$ when $s = 0$ or whenever flavor symmetry is exact

$n_F = 4$: expansion around degenerate point

- Claim:

$$\prod_n \frac{\partial}{\partial s^{i_n j_n}(x_n)} (4, \text{“1”};\ s)_\chi \Big|_{s=0} = \prod_n \frac{\partial}{\partial s^{i_n j_n}(x_n)} (4, 4, \tfrac{1}{4};\ s)_\chi \Big|_{s=0}$$

$$\Rightarrow \qquad \prod_n \left(\frac{\partial}{\partial s^{i_n j_n}(x_n)}\, V[s] \right) \Bigg|_{s=0} = 0$$

- Prove [CB, PRD 73 (2006), 114503 (hep-lat/0603011)] by relating sea Green’s functions to valence Green’s functions in partially quenched theory.
- Then can keep $s = 0$, where equivalence is known.
- Requires existence of (standard) PQχPT for unrooted theories.
    - Although theoretical basis for PQχPT is not nearly as secure as that for ordinary χPT, we have lots of numerical evidence for PQχPT and QχPT for ordinary lattice theories (and not just from staggered quarks).

$n_F = 4$: conclusion

- So all derivatives of $V[s]$ vanish at $s = 0$.
- If $V[s]$ analytic in s — up to possible isolated singularities — it vanishes everywhere.

$$\Rightarrow \qquad \begin{aligned} (4, \text{``1''};\, s)_\chi &= (4, 4, \tfrac{1}{4};\, s)_\chi \\ \therefore\, (4, \text{``1''})_\chi\Big|_{\mathcal{M}} &= (4, 4, \tfrac{1}{4})_\chi\Big|_{\mathcal{M}} \end{aligned}$$

where $\mathcal{M}$ is an arbitrary mass matrix.

- So SχPT (i.e. $(4, 4, \frac{1}{4})_\chi$) is the right chiral theory for arbitrary masses (as long as they're small enough that χPT applies).

$n_F = 3$: decoupling

- Try to get to $n_F = 3$ by taking one mass ("charm") large.
- Take m_c large as possible w/o leaving region where SχPT applies.
  - Nominally, say $m_c \sim 2m_s^{\rm phys}$.
- Integrate out m_c from $(4, 4, \frac{1}{4})_\chi$.
  - Should get $(3, 4, \frac{1}{4})_\chi$.
  - Since perturbative, there is little doubt here.
  - Explicit check in progress [CB & X. Du].
- So charm has decoupled from low energy physics when $m_c \sim 2m_s^{\rm phys}$.
- Assume it remains decoupled from low energy physics (up to usual renormalizations of relevant & marginal operators) as m_c increases to $\gg 1/a$.

$n_F = 3$: decoupling

- When $m_c \gg 1/a$, it is much larger than all eigenvalues of D.
  - $\sqrt[4]{\mathrm{Det}(D + m_c)}$ independent of gauge field.
  - charm decouples from $(4, \text{“}1\text{”})_{LQCD}$, leaving $(3, \text{“}1\text{”})_{LQCD}$.

$\Rightarrow$

$$(3, \text{“}1\text{”})_\chi = (3, 4, \tfrac{1}{4})_\chi$$

- Can repeat to argue $(2, \text{“}1\text{”})_\chi = (2, 4, \frac{1}{4})_\chi$ and $(1, \text{“}1\text{”})_\chi = (1, 4, \frac{1}{4})_\chi$.

Taste paradoxes [Creutz]

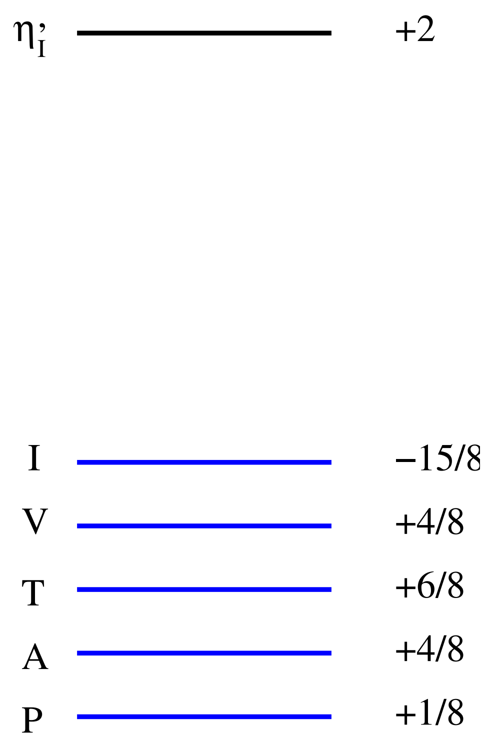
- General feature, but seen most clearly in $n_F = 1$ case.
- Theory with 1 flavor should have only heavy pseudoscalar, η', no light pseudo-Goldstone bosons.
- Theory with 1 rooted-staggered flavor has 16 pseudoscalars ("pions"); only the taste-singlet is heavy.
- Different weightings in rooted case compared to unrooted case, but otherwise similar.
- For consistency, light pions must cancel against each other in, or decouple from, pure-glue correlation functions when $a \to 0$.

Taste paradoxes

- NB: these are paradoxes about the continuum limit; so should be able to see issues and resolution in this limit, where taste symmetry is exact.
  - You can! [CB, Golterman, Shamir, Sharpe, hep-lat/0603027, v2; Golterman's talk]
  - With exact taste symmetry, $\sqrt[4]{\mathrm{Det}(\tilde{D} \otimes I)} = \mathrm{Det}(\tilde{D})$, so it must work.
- SχPT allows us to see approach to continuum limit.
  - Work by CB, DeTar, Fu, Prelovsek; more details in DeTar's talk.

One flavor paradox at $a \neq 0$

- Weights of various pseudoscalar pairs in scalar-scalar correlator when $n_R = 1/4$ (from SχPT):



- Light pion pairs of various tastes (in blue), become degenerate in continuum limit, and their weights cancel.
- Only heavy pseudoscalar (taste singlet η'_I) contributes. $\surd$

Consequences: health of rooted theory

- I've argued that SχPT is correct starting from chiral theory we know to be correct (degenerate 4-flavor case).
- But when $a \to 0$, $(n_F, 4, n_R)_\chi$ becomes ordinary χPT for $4n_F \cdot n_R$ "flavors."
- Taking $n_R \to 1/4$ order by order produces standard, continuum χPT for n_F flavors (for $m > 0$).
- $\Rightarrow$ In continuum limit, low energy sector of n_F-flavor lattice QCD with rooted staggered quarks becomes indistinguishable in structure from ordinary n_F-flavor QCD.
  - No violations of unitarity.
  - No unphysical nonlocal scales.
- Of course, this says nothing about sectors not described by χPT.

Consequences: mixed theory?

- It has been suggested (e.g. by Kennedy) that rooted staggered sea + staggered valence is a “mixed” theory (acts like it has different lattice actions for valence and sea sectors).
- But not hard to show that perturbative renormalization of sea and valence masses are the same.
- Also does not look like a mixed theory non-perturbatively, at least in context of SχPT.
  - $(n_F, 4, \frac{1}{4})_\chi$ obtained order by order from $(n_F, 4, n_R)_\chi$.
  - $(n_F, 4, n_R)_\chi$ has unbroken symmetries (on the lattice) interchanging valence and sea quarks.
- $\Rightarrow$ theory of rooted staggered sea quarks + staggered valence quarks behaves like a partially quenched theory, not a mixed theory.

Work in progress

- Deriving SχPT directly from rooted staggered lattice theory [CB, Golterman, Shamir].
- Note: replica trick makes sense order by order in SχPT because dependence on n_R is polynomial.
- Problem: at QCD level, straightforward replica trick fails:
  - $(n_F, \text{“}1\text{”})_{LQCD}$ cannot be obtained by continuuing $(n_F, 4, n_R)_{LQCD}$ to $n_R = 1/4$.
  - Dependence on n_R is non-polynomial; analytic continuation from integers is not unique.
  - Non-polynomial dependence of LQCD shows up as non-polynomial dependence of LECs — hidden from the chiral theory.

Work in progress

- Tentative solution: Use Shamir's reweighted theory as an intermediate step.
- It is local, so finding chiral theory is straightforward.
- Rooted staggered can be obtained by a convergent Taylor expansion around reweighted theory.
    - Can find a representation at the LQCD level for which the n_R dependence is polynomial (to a given order in taste-violating terms).
    - Gives (we hope) SχPT as the chiral theory.

APPENDIX A: Possible term in $V[s]$

- Example of possible term in $V[s]$:

$$V_1 = \bar{m}^2 \int d^4x\, d^4y \left(\frac{1}{\Box + M^2}\right)_{x,y} \left(\mathrm{Tr}\left[\mathrm{s(x)s(y)}\right] - \frac{1}{4}\mathrm{Tr}\left[\mathrm{s(x)}\right]\ \mathrm{Tr}\left[\mathrm{s(y)}\right]\right)$$

with $1/M$ a distance scale that might not vanish when $a \to 0$

- Claim:

$$\prod_n \frac{\partial}{\partial s^{i_n j_n}(x_n)} (4, \text{``}1\text{''};\ s)_\chi \Big|_{s=0} = \prod_n \frac{\partial}{\partial s^{i_n j_n}(x_n)} (4, 4, \tfrac{1}{4};\ s)_\chi \Big|_{s=0}$$

$$\Longrightarrow \qquad \prod_n \left(\frac{\partial}{\partial s^{i_n j_n}(x_n)} V[s]\right)\Bigg|_{s=0} = 0$$

- Prove by relating sea Green's functions to valence Green's functions in partially quenched theory
- Then can keep $s = 0$, where equivalence is known

APPENDIX B: partial quenching argument

- Add n_V staggered valence fields with sources $\sigma^{\alpha\beta}$ to all LQCD theories

$$cL = \cdots + \bar{m}\bar{q}_\alpha(x)q_\alpha(x) + \bar{q}_\alpha(x)\,\sigma^{\alpha\beta}(x)\,q_\beta(x) + \ldots$$

- n_V ghost fields also added, but not coupled to $\sigma^{\alpha\beta}$: cancel valence Det when $\sigma = 0$

$$(4, \text{“}1\text{”};\ s=0, \sigma)_{LQCD} \;=\; (1, 4;\ s=0, \sigma)_{LQCD}$$

$$\Longrightarrow (4, \text{“}1\text{”};\ s=0, \sigma)_\chi \;=\; (1, 4;\ s=0, \sigma)_\chi \;=\; (4, 4, \tfrac{1}{4};\ s=0, \sigma)_\chi$$

- Last equivalence again manifest order by order in SχPT.
  - Should be safe from any subtlety of type discussed by Golterman, Sharpe & Singleton.
  - e.g. non-trivial saddle point for ghost mesons.

APPENDIX B: partial quenching argument

- Relate derivatives w.r.t. s to derivatives w.r.t. σ
- Derivatives w.r.t. s in rooted theory bring down factors of $1/4$ from
$$\sqrt[4]{\mathrm{Det}(D+\bar{m}+s)} = \exp\frac{1}{4}\mathrm{tr}\ln(\mathrm{D}+\bar{\mathrm{m}}+\mathrm{s})$$
- Different terms ($\equiv$ different contractions) associated with different powers of $1/4$
  - power of $1/4$ is just the number of quark loops implied by corresponding contractions
- Derivatives w.r.t. s in replicated theory produce corresponding powers of n_R from sea-quark counting
- But with arbitrary n_V, can always adjust valence flavor indices on σ derivatives so only one contraction possible

APPENDIX B: partial quenching argument

- Examples ($i \neq j$, $\alpha \neq \beta$, no sums):

$$\frac{\partial}{\partial s^{ij}(x)}\frac{\partial}{\partial s^{ji}(y)}\;(4,\text{“1”};\;s,\sigma\!=\!0)_{LQCD}\Big|_{s=0} \;=\; \frac{1}{4}\,\langle \mathrm{tr}\Big(\mathrm{G_j(x,y)G_i(y,x)}\Big)\rangle$$

$$= \frac{1}{4}\;\frac{\partial}{\partial \sigma^{\alpha\beta}(x)}\frac{\partial}{\partial \sigma^{\beta\alpha}(y)}\;(4,\text{“1”};\;s\!=\!0,\sigma)_{LQCD}\Big|_{\sigma=0}$$

$$\frac{\partial}{\partial s^{ii}(x)}\frac{\partial}{\partial s^{ii}(x)}(4,\text{“1”};\;s,\sigma\!=\!0)_{LQCD}\Big|_{s=0} =$$

$$= \frac{1}{4}\langle \mathrm{tr}\Big(\mathrm{G_i(x,y)G_i(y,x)}\Big)\rangle + \left(\frac{1}{4}\right)^2 \langle \mathrm{tr}\Big(\mathrm{G_i(x,x)}\Big)\mathrm{tr}\Big(\mathrm{G_i(y,y)}\Big)\rangle$$

$$= \left[\frac{1}{4}\frac{\partial}{\partial \sigma^{\alpha\beta}(x)}\frac{\partial}{\partial \sigma^{\beta\alpha}(y)} + \left(\frac{1}{4}\right)^2 \frac{\partial}{\partial \sigma^{\alpha\alpha}(x)}\frac{\partial}{\partial \sigma^{\beta\beta}(y)}\right](4,\text{“1”};\;s\!=\!0,\sigma)_{LQCD}\Big|_{\sigma=0}$$

- For $(4,4,n_R)$ theory, just replace $1/4 \to n_R$

APPENDIX B: partial quenching argument

- Can therefore write:

$$\prod_n \frac{\partial}{\partial s^{i_n j_n}(x_n)} (4, \text{“}1\text{”};\ s, \sigma = 0)_{LQCD}\Big|_{s=0} =$$

$$= \sum_C \left(\tfrac{1}{4}\right)^{L_C} \prod_n \frac{\partial}{\partial \sigma^{\alpha_n^C \beta_n^C}(x_n)} (4, \text{“}1\text{”}; s = 0, \sigma)_{LQCD}\Big|_{\sigma=0}$$

$$\prod_n \frac{\partial}{\partial s^{i_n j_n}(x_n)} (4, 4, n_R;\ s, \sigma = 0)_{LQCD}\Big|_{s=0} =$$

$$= \sum_C (n_R)^{L_C} \prod_n \frac{\partial}{\partial \sigma^{\alpha_n^C \beta_n^C}(x_n)} (4, 4, n_R;\ s = 0, \sigma)_{LQCD}\Big|_{\sigma=0}$$

- C labels a contraction with L_C valence quark loops
- Valence indices α_n^C, β_n^C adjusted so only one contraction
- Same arrangements of valence flavor indices & powers L_C work in both cases

APPENDIX B: partial quenching argument

- Pass to corresponding chiral theories:

$$\prod_n \frac{\partial}{\partial s^{i_n j_n}(x_n)} (4, \text{“}1\text{”};\ s, \sigma\!=\!0)_\chi \Big|_{s=0} =$$

$$= \sum_C \left(\tfrac{1}{4}\right)^{L_C} \prod_n \frac{\partial}{\partial \sigma^{\alpha_n^C \beta_n^C}(x_n)} (4, \text{“}1\text{”}; s\!=\!0, \sigma)_\chi \Big|_{\sigma=0}$$

$$\prod_n \frac{\partial}{\partial s^{i_n j_n}(x_n)} (4, 4, n_R;\ s, \sigma\!=\!0)_\chi \Big|_{s=0} =$$

$$= \sum_C (n_R)^{L_C} \prod_n \frac{\partial}{\partial \sigma^{\alpha_n^C \beta_n^C}(x_n)} (4, 4, n_R;\ s\!=\!0, \sigma)_\chi \Big|_{\sigma=0}$$

- At any finite order in chiral perturbation theory both sides of last eqn are polynomial in n_R. Can take $n_R \to 1/4$

APPENDIX B: partial quenching argument

- After $n_R \to 1/4$ in second eqn:

$$\prod_n \frac{\partial}{\partial s^{i_n j_n}(x_n)} (4, \text{“}1\text{”};\ s, \sigma = 0)_\chi \Big|_{s=0} =$$

$$= \sum_C \left(\tfrac{1}{4}\right)^{L_C} \prod_n \frac{\partial}{\partial \sigma^{\alpha_n^C \beta_n^C}(x_n)} (4, \text{“}1\text{”}; s = 0, \sigma)_\chi \Big|_{\sigma=0}$$

$$\prod_n \frac{\partial}{\partial s^{i_n j_n}(x_n)} (4, 4, \tfrac{1}{4};\ s, \sigma = 0)_\chi \Big|_{s=0} =$$

$$= \sum_C \left(\tfrac{1}{4}\right)^{L_C} \prod_n \frac{\partial}{\partial \sigma^{\alpha_n^C \beta_n^C}(x_n)} (4, 4, \tfrac{1}{4};\ s = 0, \sigma)_\chi \Big|_{\sigma=0}$$

- Right sides equal since $(4, \text{“}1\text{”};\ s = 0, \sigma)_\chi = (4, 4, \frac{1}{4};\ s = 0, \sigma)_\chi$
- So left sides equal, which is what we wanted to show

APPENDIX C: analyticity assumptions

- Is assumption of analyticity obviously too strong?
  - "Don't expect convergent expansions in QFT."
  - Factorial growth of large orders in perturbation theory: expansion at best asymptotic.
  - But here every order is zero!

- How could analyticity go wrong?
  - Line of singularities, domain boundary.
    - Ground state for $(4, \text{“}1\text{”})_\chi$ changes discontinuously from state assumed by $(4, 4, \frac{1}{4})_\chi$.
    - Inside the range of m & a studied by MILC, such a singularity would have probably been detected.
    - No evidence outside MILC range, though.

APPENDIX C: analyticity assumptions

- How could analyticity go wrong? (continued)
  - Essential singularity at $s = 0$?
    - No reason to expect it (no obvious IR problem; expanding around massive theory).
  - NB: Not assuming that $(4, \text{“}1\text{”})_\chi$ and $(4, 4, \frac{1}{4})_\chi$ are separately analytic, only that difference is.
  - If $V[s]$ not analytic, then SχPT is wrong.

APPENDIX D: One-flavor paradox

- Mock up the kind of pure-glue correlation function that can persist in continuum limit: add taste-singlet scalar source to rooted one-flavor theory:

$$\mathcal{L}_{\rm source} = s(z)\bar\Psi(z)\Psi(z)$$

$$(1,\text{“}1\text{”})_{LQCD} = \frac{\int \mathcal{D}A\ \exp\{-S_G(A) + \frac{1}{4}\,{\rm tr}\,({\rm ln}\,({\rm D}+{\rm m}+{\rm s}))\}}{\int \mathcal{D}A\ \exp\{-S_G(A) + \frac{1}{4}\,{\rm tr}\,({\rm ln}\,({\rm D}+{\rm m}))\}}$$

- Look at connected part of

$$G(x-y) = \left(\frac{\partial}{\partial s(x)}\frac{\partial}{\partial s(y)}\ (1,\text{“}1\text{”})_{LQCD}\right)_{s=0}$$

[“connected” $\Rightarrow$ subtract $\langle\bar\Psi\Psi\rangle^2$]

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- $G(x-y)$ has two contractions, with different factors of $\frac{1}{4}$ from $\sqrt[4]{\mathrm{Det}}$

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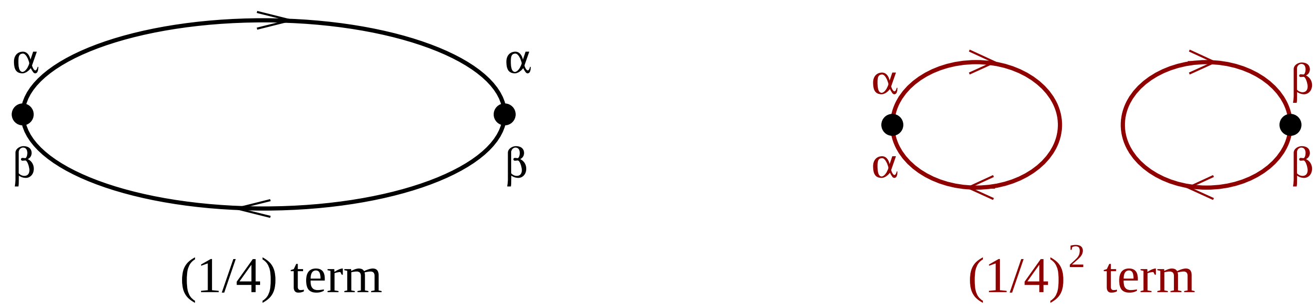

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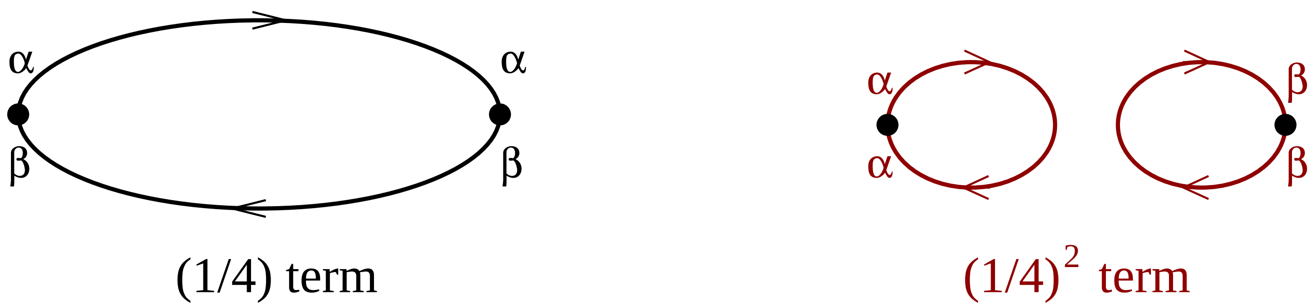

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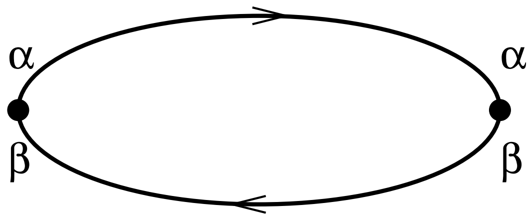
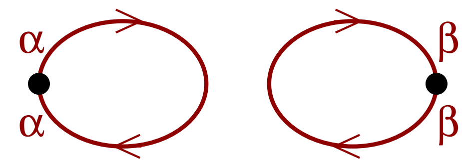


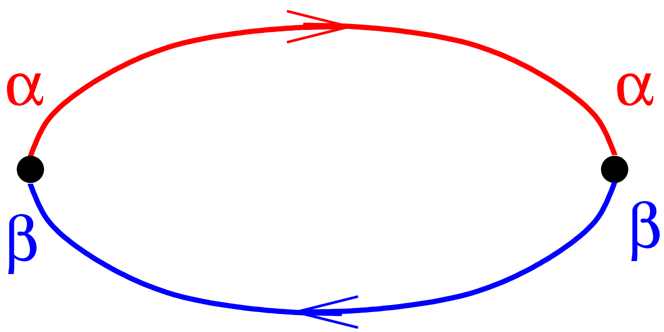
(1/4) term



$(1/4)^2$ term

- α, β are valence flavor indices, introduced to label the contractions uniquely
- Want to calculate G(x-y) for large $|x-y|$ in LO SχPT

APPENDIX D: One-flavor paradox: diagrams



QCD valence contraction
(term proportional to $\frac{1}{4}$)

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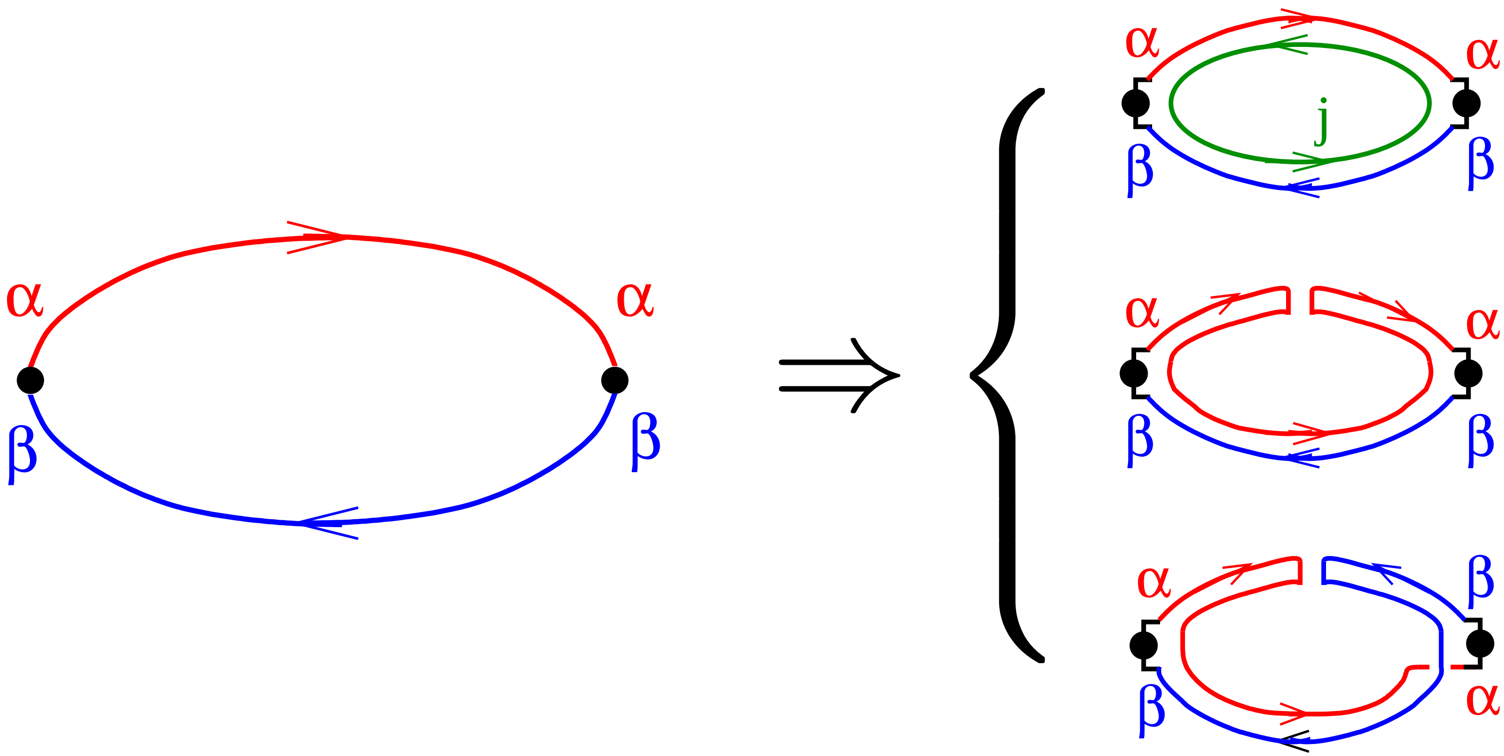

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chiral quark flow (note hairpins)

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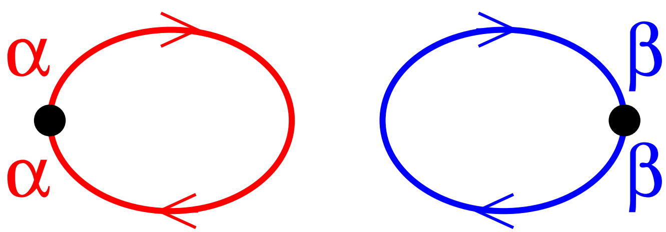

QCD valence contraction
(term proportional to $(\frac{1}{4})^2$)

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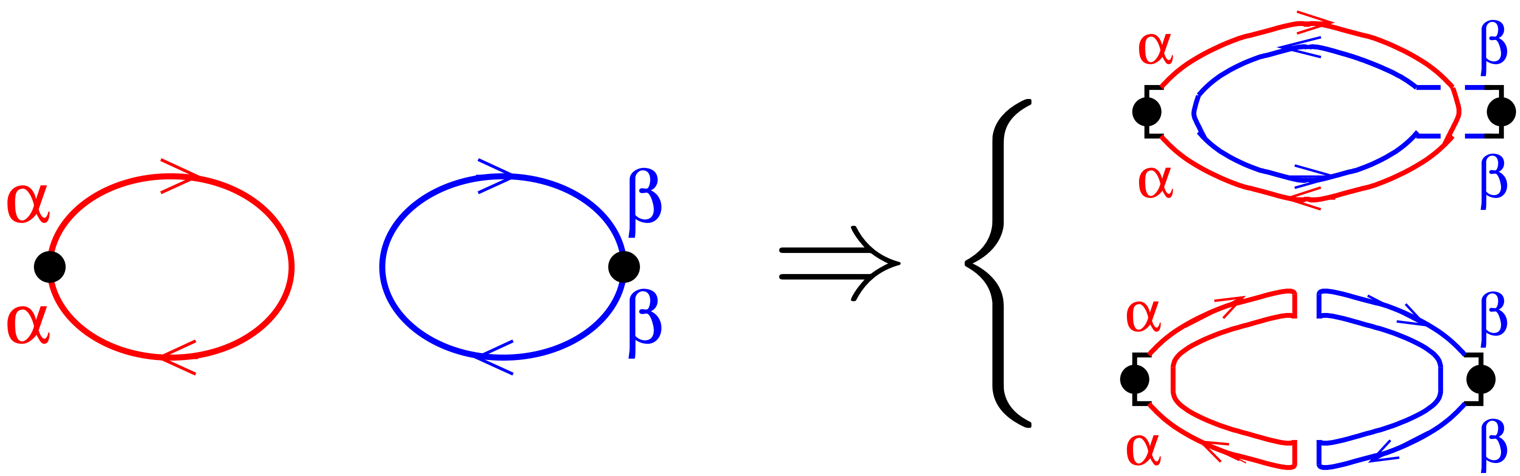

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chiral quark flow (note hairpins)

APPENDIX D: One-flavor paradox: resolution

$$\tilde{G}(q) = \mu^2 \int \frac{d^4p}{(2\pi)^4} \left\{ \frac{1}{8n_R^2} \frac{1}{\left(p^2 + M_{\eta_I'}^2\right)\left((p+q)^2 + M_{\eta_I'}^2\right)} + \right.$$

$$+\left(\frac{1}{4}n_R + \frac{1}{16}\right)\sum_{\Xi} \frac{1}{(p^2 + M_\Xi^2)\left((p+q)^2 + M_\Xi^2\right)} - \left(\frac{1}{n_R} - \frac{1}{8n_R^2}\right)\frac{1}{(p^2 + M_I^2)\left((p+q)^2 + M_I^2\right)}$$

$$\left. + \left(\frac{1}{2n_R} - \frac{1}{8n_R^2}\right)\left(\frac{1}{(p^2 + M_I^2)\left((p+q)^2 + M_{\eta_I'}^2\right)} + \frac{1}{\left(p^2 + M_{\eta_I'}^2\right)\left((p+q)^2 + M_I^2\right)}\right)\right\}$$

- $M_{\eta'}$ heavy.
- M_Ξ light (for all Ξ; including M_I).
- Setting $n_R = 1/4$, red terms vanish.
- When $a \to 0$, all 16 of M_Ξ degenerate $\Rightarrow$ blue terms vanish.
- So only η_I' left in intermediate state in continuum. $\surd$

APPENDIX E: Three-flavor paradox

- Creutz: Continuum QCD with $n_F = 3$ (or any odd n_F) is not even under $m \to -m$, but rooted staggered determinant is even
  - staggered D is anti-hermitian, eigenvalues of $D + m$ come in pairs $m \pm i\lambda$, so $\mathrm{Det}(D + m)$ is function of m^2
- In standard continuum χPT, mass term (take degenerate for simplicity) is

$$-m\mathrm{Tr}(\Sigma + \Sigma^\dagger)$$

  - For $n_F = 3$, $m \to -m$ cannot be rotated away by non-anomalous chiral transformation
  - for $m < 0$ ground state is $\Sigma = \exp(\pm 2\pi i/3)$ instead of $\Sigma = 1$
  - theory with $m < 0$ is physically different from $m > 0$
  - $m < 0$ violates parity
- In finite volume, expansion of QCD level theory around $m = 0$ must have odd powers of m as well as even

APPENDIX E: Three-flavor paradox

- In SχPT for $n_F = 3$, there are an even number of flavors $\times$ tastes for any integer n_R
  - Can rotate $-m \to m$ for each n_R
  - $(3, 4, \frac{1}{4})_\chi$ SχPT is a function of $|m|$ only
  - But, in continuum limit, $(3, 4, \frac{1}{4})_\chi$ reproduces continuum χPT correctly, as long as $m > 0$

- At LQCD level, $\sqrt[4]{\mathrm{Det}(D+m)}$ means that theory does not have to be analytic function of m around $m = 0$, even in finite volume
  - Can be function of $\sqrt[4]{m^4} = |m|$
  - Can be even under $m \to -m$, and yet not just depend on even powers of m
  - Perfectly possible that gives correct odd powers of m for $m > 0$ (as SχPT says it does) without getting the $m < 0$ case right $\surd$

APPENDIX F: health of rooted theory

- If SχPT is correct, there are implications for validity of rooted theory itself.
- When $a \to 0$, $(n_F, 4, n_R)_\chi$ becomes ordinary χPT for $4n_F \cdot n_R$ "flavors."
  - For given flavor combo, all 16 taste pions become degenerate in continuum (before including anomaly).
  - Anomaly affects only taste singlet, flavor singlet meson.
- Taking $n_R \to 1/4$ order by order produces standard, continuum χPT for n_F flavors.
  - NB: assumes vacuum of $(n_F, 4, n_R)_\chi$ ($\Sigma = 1$) is same as vacuum of continuum χPT.
  - True for $m > 0$, but not for $m < 0$ with n_F odd. To simulate latter case, need to do something different. [CB, Golterman, Shamir, Sharpe; beautiful treatment in Schwinger model by Dürr and Hoelbling, hep-lat/0604005].

APPENDIX F: health of rooted theory

- Since SχPT $\rightarrow$ χPT in continuum, low energy sector of n_F-flavor lattice QCD with rooted staggered quarks becomes indistinguishable in structure from ordinary n_F-flavor QCD.
  - No violations of unitarity.
  - No unphysical nonlocal scales.
- Of course, this says nothing about sectors not described by χPT.

APPENDIX G: mixed theory?

- “Mixed” theories have different lattice actions for sea and valence quarks
  - Sea and valence mass renormalizations different $\Rightarrow$ no simple way to enforce $m_S = m_V$
  - Continuum symmetries that rotate valence and sea quark into each other are violated by discretization effects
  - If quark masses adjusted to make meson masses $M_{SS} = M_{VV}$, then M_{SV} still differs by terms $\mathcal{O}(a^n)$
  - Such terms show up as new operators in mixed theory χPT (Bär, Rupak, Shoresh, . . .)

APPENDIX G: mixed theory?

- Some (e.g. Kennedy) have suggested that rooted staggered sea + staggered valence (“rooted staggered”) is a mixed theory
- But not hard to show that perturbative renormalization of sea and valence masses are the same
- Also does not look like a mixed theory non-perturbatively, at least in context of SχPT
  - $(n_F, 4, \frac{1}{4})_\chi$ obtained order by order from $(n_F, 4, n_R)_\chi$
  - $(n_F, 4, n_R)_\chi$ have symmetries interchanging valence and sea quarks
  - full symmetry group: $SU(4n_Rn_F\!+\!4n_V|4n_V)_L \times SU(4n_Rn_F\!+\!4n_V|4n_V)_R$.
  - Taste symmetries broken on lattice at $\mathcal{O}(a^2)$
  - But flavor subgroup (“residual chiral group”) $U(n_Rn_F\!+\!n_V|n_V)_\ell \times U(n_Rn_F\!+\!n_V|n_V)_r$ is exact (up to mass terms)

APPENDIX G: mixed theory?

- Chiral ops that split M_{SV} from M_{VV} & M_{SV} (when $m_V = m_S$) are forbidden by flavor subgroup in $(n_F, 4, n_R)_\chi$
- Corresponding sea-sea, valence-valence, and valence-sea mesons degenerate (when quark masses degenerate) in $(n_F, 4, n_R)_\chi$, and therefore in $(n_F, 4, \frac{1}{4})_\chi$
- Within SχPT, rooted staggered behaves like partially quenched theory, not like mixed theory
- NB: valence sector "richer" than sea sector
  - Valence sector includes particles in continuum limit whose sea-sector analogues have decoupled from physical theory
  - In normal partially quenched theory, can take more valence quarks than sea quarks & create valence states with no sea-quark analogues
  - Here, there's no choice: physical sea-quark states are always a proper subspace of valence states

APPENDIX H: Some final thoughts

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2) “There is something fascinating about science.
One gets such wholesale returns of conjecture out of such a trifling investment of fact.”

—Mark Twain