Regulating QCD with rooted staggered fermions

Claude Bernard, Maarten Golterman, and Yigal Shamir

Lattice 2006, Tucson

Questions and answers:

1) Are rooted staggered fermions a regulator like any other, or not?
   No, they are non-local at $a \neq 0$. (this talk)

2) Can the continuum limit be taken, and is it in the correct universality class?
   Most likely: Yes! (Yigal's talk)

3) But: we work at $a \neq 0$, where the diseases are present!
   $\Rightarrow$ need EFT to parameterize the non-local effects.
   Likely candidate: SChPT + "replica trick" (for pions)

   arguments:
   - SChPT ok for unrooted + decoupling (Claude's talk)
   - direct derivation from RG framework (in progress)

Definition:

- separate staggered fields for each flavor
  single-site mass terms, masses m_u , m_d , m_s
  each flavors comes in four tastes

- continuum limit: 4 up, 4 down and 4 strange quarks
  with $U(4)_u \times U(4)_d \times U(4)_s$ symmetry (non-deg. masses)

- $\mathrm{Det}(D_{stag}) \sim \mathrm{Det}^4(D_{cont}) \Rightarrow$ take $\mathrm{Det}^{1/4}(D_{stag})$

- $\mathrm{Det}(D_{stag}) > 0$ (any m),
  $\mathrm{Det}(D_{cont}) > 0$ ($m > 0$) $\Rightarrow$ pick positive 4th root

Continuum limit — more detail:

$$Z_{cont}(J) = \int \mathcal{DU} \exp(-S_g) \prod_{i=1}^{N_f} \mathrm{Det}^{1/4}\Big((D + m_i) \times \mathbf{1}_{taste} + J\Big)$$

- project onto physical Hilbert space by taking $J = \tilde{J} \times \mathbf{1}_{taste}$
  $\Rightarrow$ correct correlation functions for QCD
  with all quark masses positive and any number of flavors!

- no “paradoxes” based on symmetries can arise!

- many unphysical states with non-trivial taste charges
  but can use $SU(4)_{taste}$ to relate (non-anomalous) charges,
  e.g. $\bar{u}\gamma_5 d \xrightarrow[SU(4)_{taste}]{} \bar{u}(\gamma_5 \times \Xi)d$

- mixing with gluonic states: must use taste-singlet operators.

(Bernard, MG, Shamir & Sharpe ‘06)

Non-locality of 4th-rooted staggered fermions:

Assume a local D exists such that (at $a \neq 0$)

$$\mathrm{Det}^{1/4}\,(D_{stag}) = \mathrm{Det}\,(D)\;\; \exp(-\,\delta S_{eff}\,/4)\;,$$

with δS_{eff} local (no long-distance effects). Take fourth power:

$$\mathrm{Det}(D_{stag}) = \mathrm{Det}\,(D_{4\mathrm{t}})\;\; \exp(-\,\delta S_{eff}), \qquad D_{4\mathrm{t}} = D \times \mathbf{1}\;;$$

$D_{4\mathrm{t}}$ describes a theory with exact $SU(4)$ taste symmetry.

Compare spectra at $a \neq 0$:

$D_{4\mathrm{t}}$: 15 degenerate pions in adjoint of $SU(4)$

D_{stag}: 15 pions are non-degenerate (only one "exact" pion)

$\Rightarrow$ δS_{eff} knows about long-distance effects!

Taste basis: $$D^{-1}_{taste} = \frac{1}{\alpha} + Q D^{-1}_{stag} Q^{\dagger}$$ (Shamir '04)

Q is a unitary matrix connecting one-component and taste bases; α is of order 1/*a* : just adds a contact term. We have

$$\mathrm{Det}(D_{stag}) = \mathrm{Det}((\alpha G)^{-1})\ \mathrm{Det}(D_{taste})$$

$$(\alpha G)^{-1} = \frac{1}{\alpha} D_{stag} + Q Q^{\dagger} = \frac{1}{\alpha} D_{stag} + \mathbf{1}$$

(Recover usual change of basis for $\alpha \rightarrow \infty$.)

D_{stag} and D_{taste} are completely equivalent.

Note: looks like starting point for RG blocking -- see Yigal's talk!

$U(1)_{\varepsilon}$ symm.: $\delta\chi(x) = i\,\epsilon(x)\ \chi(x) \longrightarrow \delta\psi = i\,\widehat{\gamma_5 \times \xi_5}\ \psi$

Non-locality and taste symmetry breaking:

Split
$$D_{taste} = D \otimes \mathbf{1} + \sum_A D_A \otimes \Xi_A$$

then
$$\log \mathrm{Det}(D_{taste}) = 4 \log \mathrm{Det}(D) + \log \mathrm{Det}\left(1 + \sum_A D^{-1} D_A \otimes \Xi_A\right)$$

D and D_A are local, but $\Sigma_A D^{-1} D_A \otimes \Xi_A$ is not!

I.e. taste breaking is local for action, but not for physics.

However, the taste-breaking D_A are irrelevant operators

$\Rightarrow$ conjecture: taste symmetry is restored in continuum limit
$\Rightarrow$ non-localities disappear in continuum limit.
(validity of 4th root is tied to validity of staggered fermions)

Free theory:

$$D_{taste} = \frac{\sum_\mu i(\gamma_\mu \otimes \mathbf{1})\bar{p}_\mu + (\mathbf{1} \otimes \mathbf{1})\left(m + \frac{1}{\alpha}(\hat{p}^2 + m^2)\right) + \frac{1}{2}\sum_\mu (\gamma_5 \otimes \xi_\mu \xi_5)\hat{p}_\mu^2}{1 + \frac{2m}{\alpha} + \frac{1}{\alpha^2}(\hat{p}^2 + m^2)}$$

$$\bar{p}_\mu \equiv \sin p_\mu \ , \qquad \hat{p}_\mu \equiv 2\sin(p_\mu/2) \ , \qquad \hat{p}^2 \equiv \sum_\mu \hat{p}_\mu^2$$

1) Note Wilson-like term: taste-invariant part has no doublers!
Use taste-inv. part as "comparison" theory in RG treatment.
(Yigal's talk)

2)

$$\delta S_{eff} = 2\sum_{p,\mu} \frac{a^2 p_\mu^4}{p^2 + m^2} + O(a^4)$$

clearly non-local

Comments:

- Non-locality comes from breaking of taste symmetry, which implies (e.g.) non-degeneracy of (too many) pions:

$$(m_\pi^A)^2 = (m_\pi^{GB})^2 + c^A\, a^2\Lambda^4_{QCD}$$

Two IR effects: quark mass m and splitting $(a\Lambda^2_{QCD})^2$,
related to splitting $a\Lambda^2_{QCD}$ of IR eigenvalues

$\Rightarrow$ remove unphysical IR scale first:

- take $a \to 0$ before taking $m \to 0$!

- Other masses split also, but pions lead to the most dramatic effect.

- Non-locality at $a \neq 0$ leads to unitarity violations:
(see Claude's and Carleton's talks)

- take $a \to 0$ before continuing to Minkowski space!

θ angle with rooted staggered

Det(*D*(*m*)) = Det(*D*(-*m*))

⇒ How does the 4th root theory handle the θ angle?

Just put in by hand:

$$\int \mathcal{DU}\, e^{-S_g}\, \mathrm{Det}^{1/4}(D_{stag}(m))\, e^{i\theta \int F\tilde{F}}$$

See recent work in Schwinger model by Dürr & Hoelbling