Equation of state for two-flavor QCD with an improved Wilson quark action at non-zero chemical potential

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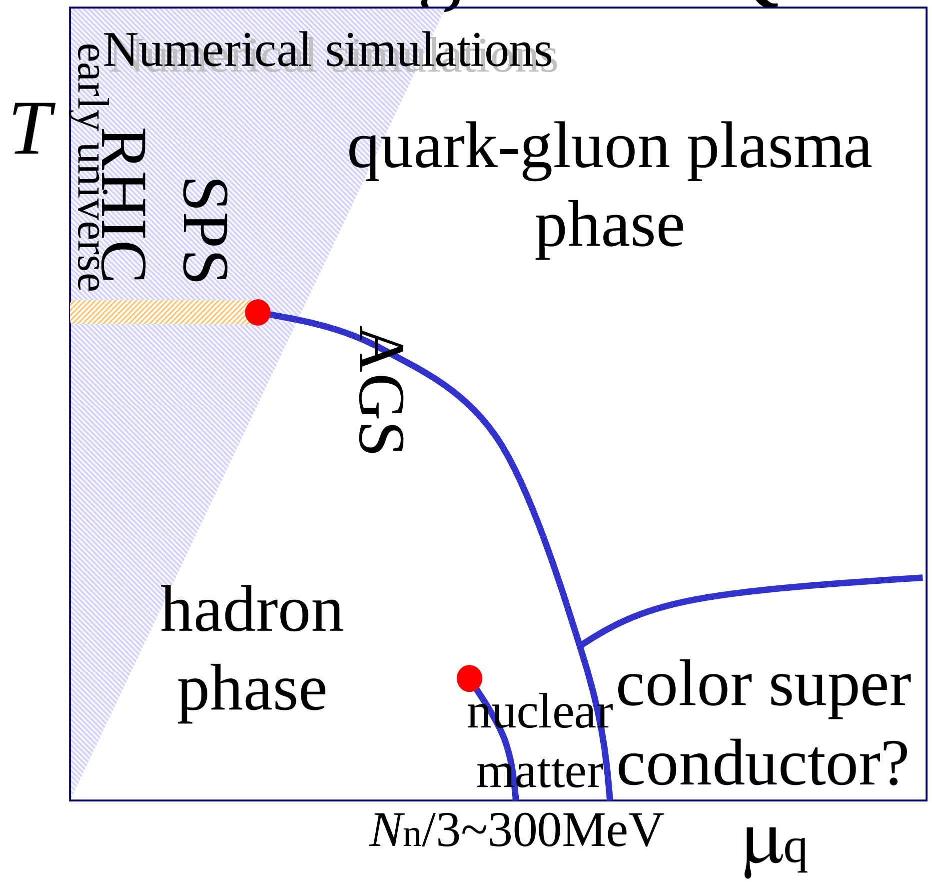
Abstract

We discuss the equation of state for QCD in the low density regime. The studies of the equation of state can provide basic input for the analysis of the experimental signatures for QGP formation and are very important. We perform simulations of two-flavor QCD with an improved Wilson quark action and calculate the Taylor expansion coefficients of the thermodynamic grand partition function in terms of chemical potential up to fourth order. These enable us to estimate the pressure, the quark number density and its susceptibility as a function of the chemical potential in the low density regime. In this poster, we report the current status of our project and show the preliminary results.

QCD Thermodynamics with Wilson type quarks

- Numerical study of full QCD at High temperature and density is very important.
- However, most of studies are performed using staggered-type quark action with the 4th root trick of the quark determinant.
- We study the QCD thermodynamics systematically using a Wilson type quark action, and want to obtain basic information for the analysis of the experimental data of heavy ion collisions, comparing the results by staggered-type quark actions.

Phase diagram of QCD



Iwasaki improved gauge action + Clover improved Wilson action

- Many studies have been done using this action [CPPACS,2000-2001].
  - $T=0$, $\mu_q=0$: light hadron spectrum, etc.
  - $T\neq0$, $\mu_q=0$: phase structure, Tc, O(4) scaling, equation of state, etc.
  - Useful information for performing simulations has been already obtained.

- Partition function

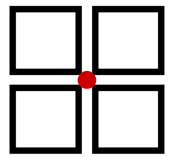
$$Z = \int \prod_{x,\mu} dU_\mu(x) \left(\det M\right)^{N_f} e^{-S_g},$$

μ_q: quark chemical potential

$$S_g = -\beta \left\{ c_0 \sum_{x,\mu>\nu} W^{1\times1}_{\mu\nu}(x) + c_1 \sum_{x,\mu\neq\nu} W^{1\times2}_{\mu\nu}(x) \right\}$$

$W^{n\times m}_{\mu\nu}$: nxm wilson loop

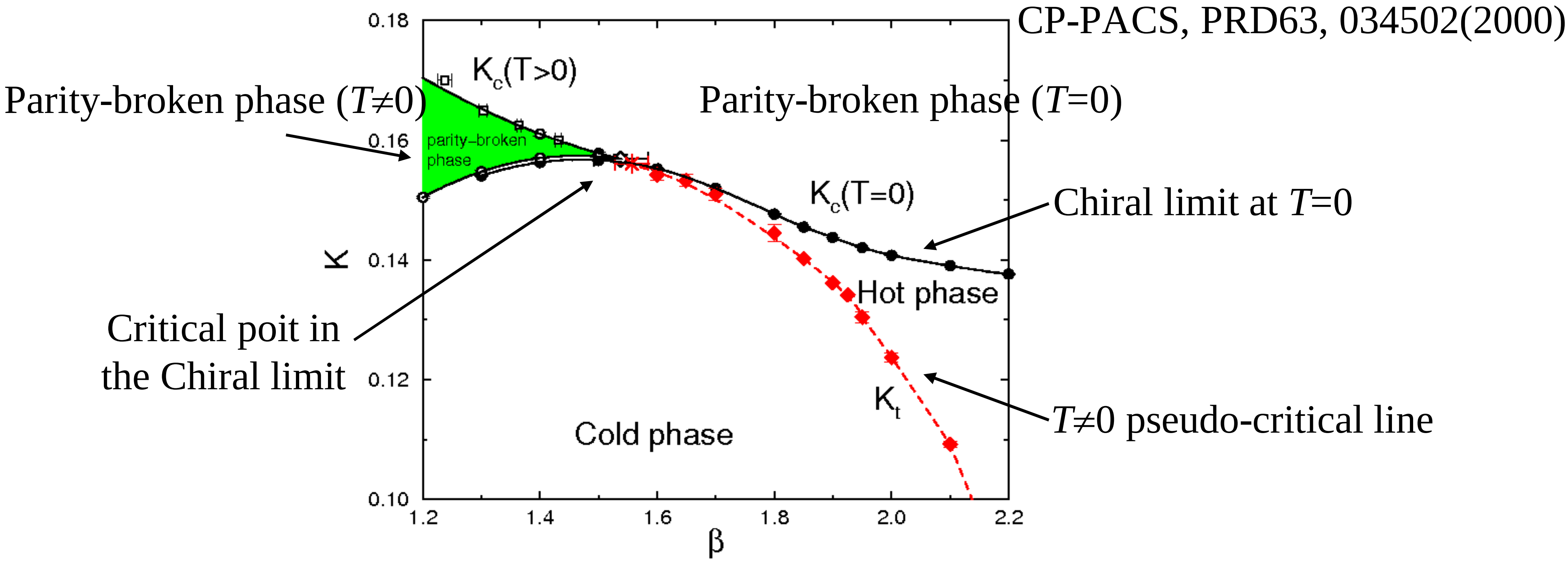
$$M_{x,y} = \delta_{x,y} - K\sum_{i=1}^{3}\left[(1-\gamma_i)U_i\delta_{x+\hat{i},y} + (1+\gamma_i)U_i^{+}\delta_{x-\hat{i},y}\right]$$
$$-K\left[\underline{e^{\mu_q a}}(1-\gamma_4)U_4\delta_{x+\hat{4},y} + \underline{e^{-\mu_q a}}(1+\gamma_4)U_4^{+}\delta_{x-\hat{4},y}\right] + \underline{\delta_{x,y}c_{SW}K\sum_{\mu<\nu}\sigma_{\mu\nu}F_{\mu\nu}}$$



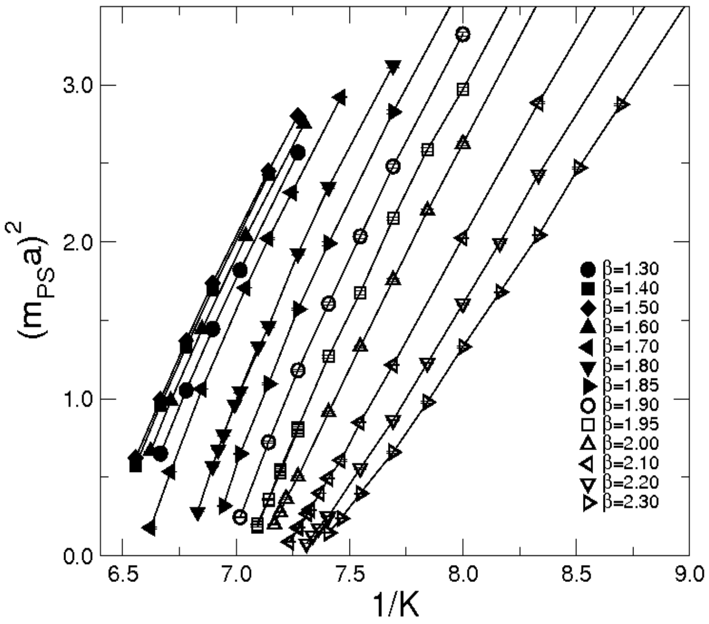
Clover term

$$\beta = 6/g^2, \quad c_1 = -0.331, \quad c_0 = 1-8c_1, \qquad c_{SW} = (1-0.8412\beta^{-1})^{-3/4}$$

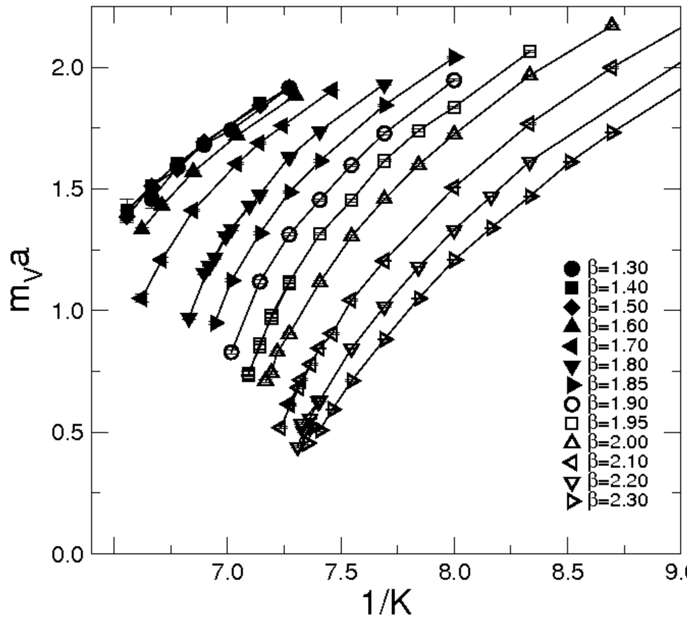
Phase structure of QCD with Wilson-type quarks



Pion mass (m_{PS}) at T=0



ρ meson mass (m_V) at T=0

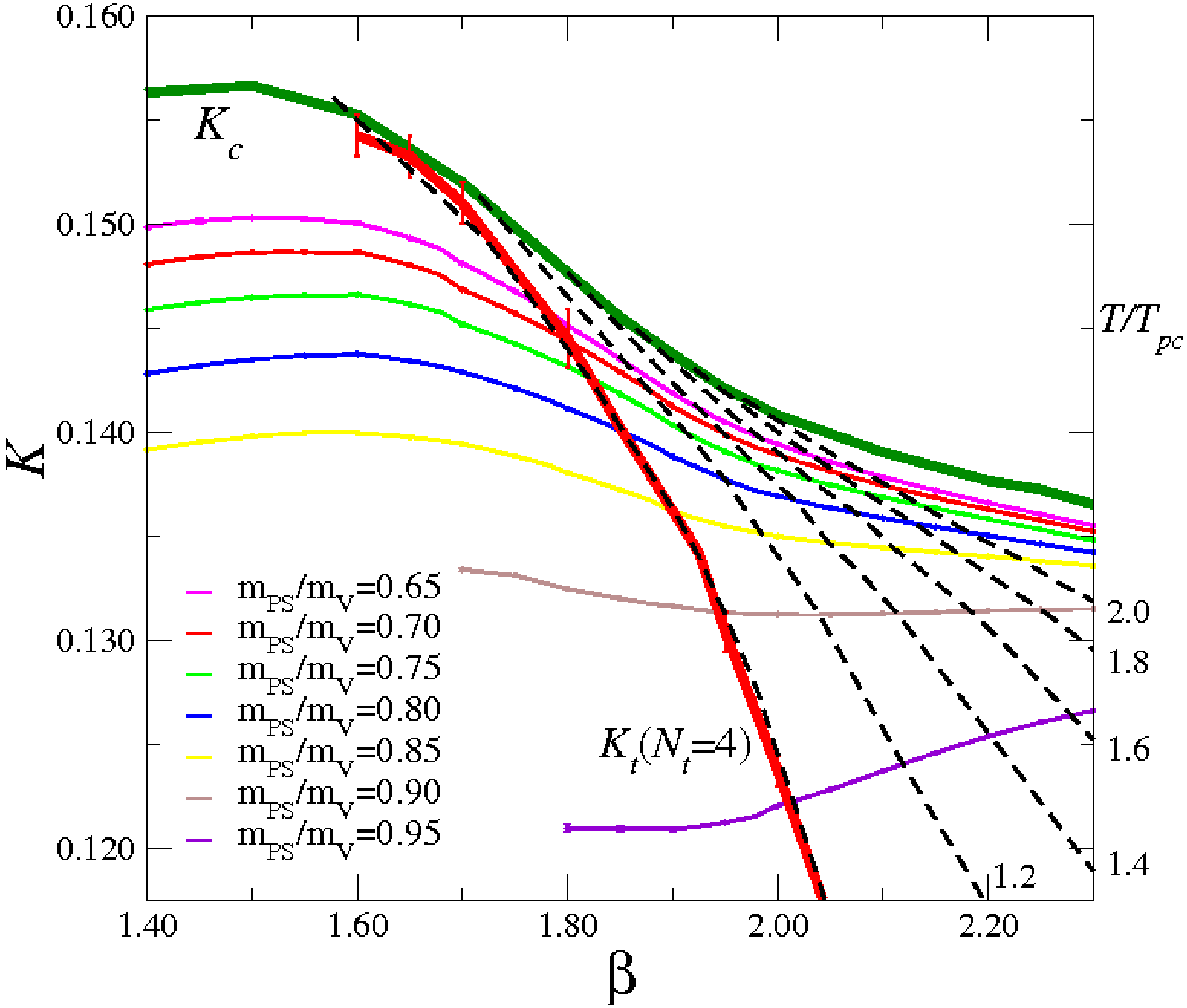


CP-PACS Collab.,
PRL85, 4674 (2000);
PRD63, 034502 (2000);
PRD64, 074510 (2001);
PRD65, 054505 (2001)

Lines of constant physics (LCP) and temperature (T) in the (β,K) plane

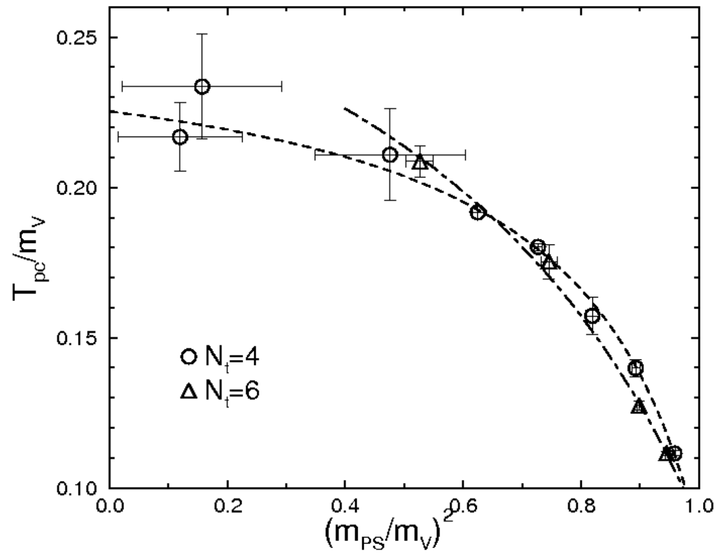
- Interpolating the data of m_{PS} and m_V, we determine lines of constant m_{PS}/m_V as lines of constant physics.
- Temperature is estimated by ρ meson mass m_V and normalized by T_{pc}/m_V for each LCP.

$$T/m_V = (N_t m_V a)^{-1}$$



Pseudo-critical temperature as a function of m_{PS}/m_V

(CP-PACS, PRD64, 07510 (2001))



- Colored lines: line of constant m_{PS}/m_V
- Green line (K_c): chiral limit, line of $m_{PS}=0$
- Red line (K_t): finite temperature pseudo-critical line
- Dashed lines: lines of constant T/T_{pc}

Equation of state at $\mu\neq 0$

Basic thermodynamic quantities at $\mu\neq 0$

- Pressure: $p = \frac{T}{V}\ln Z$ (Z: grand partition function)
- Quark number density: $n_{u,d} = \frac{T}{V}\frac{\partial \ln Z}{\partial \mu_{u,d}} = \frac{\partial p}{\partial \mu_{u,d}}$
- Quark (Baryon) number susceptibility:

$$\chi_q = 9\chi_B = \left(\frac{\partial}{\partial \mu_u} + \frac{\partial}{\partial \mu_d}\right)(n_u + n_d) = \frac{\partial^2 p}{\partial \mu_q^2} \qquad \left(\mu_q = (\mu_u + \mu_d)/2\right)$$

- Isospin susceptibility:

$$\chi_I = \left(\frac{\partial}{\partial \mu_u} - \frac{\partial}{\partial \mu_d}\right)(n_u - n_d) = \frac{\partial^2 p}{\partial \mu_I^2} \qquad \left(\mu_I = (\mu_u - \mu_d)/2\right)$$

- Charge susceptibility: important for experiments.

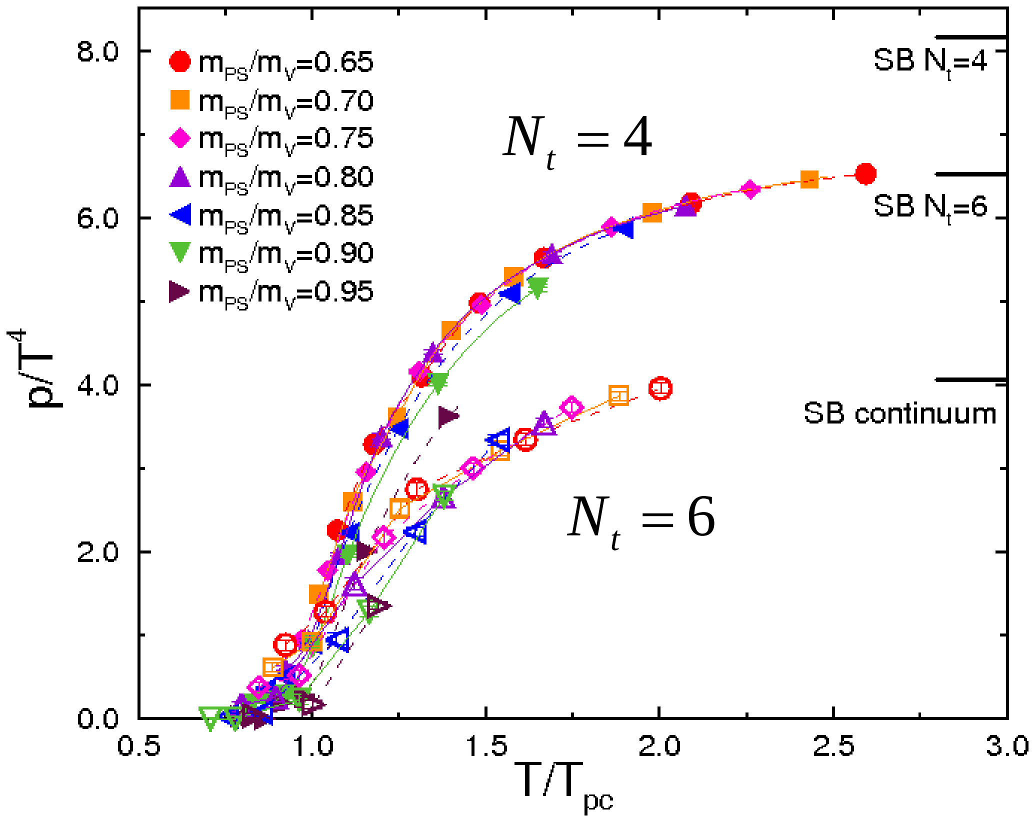
$$\chi_C = \left(\frac{2}{3}\frac{\partial}{\partial \mu_u} - \frac{1}{3}\frac{\partial}{\partial \mu_d}\right)\left(\frac{2}{3}n_u - \frac{1}{3}n_d\right)$$

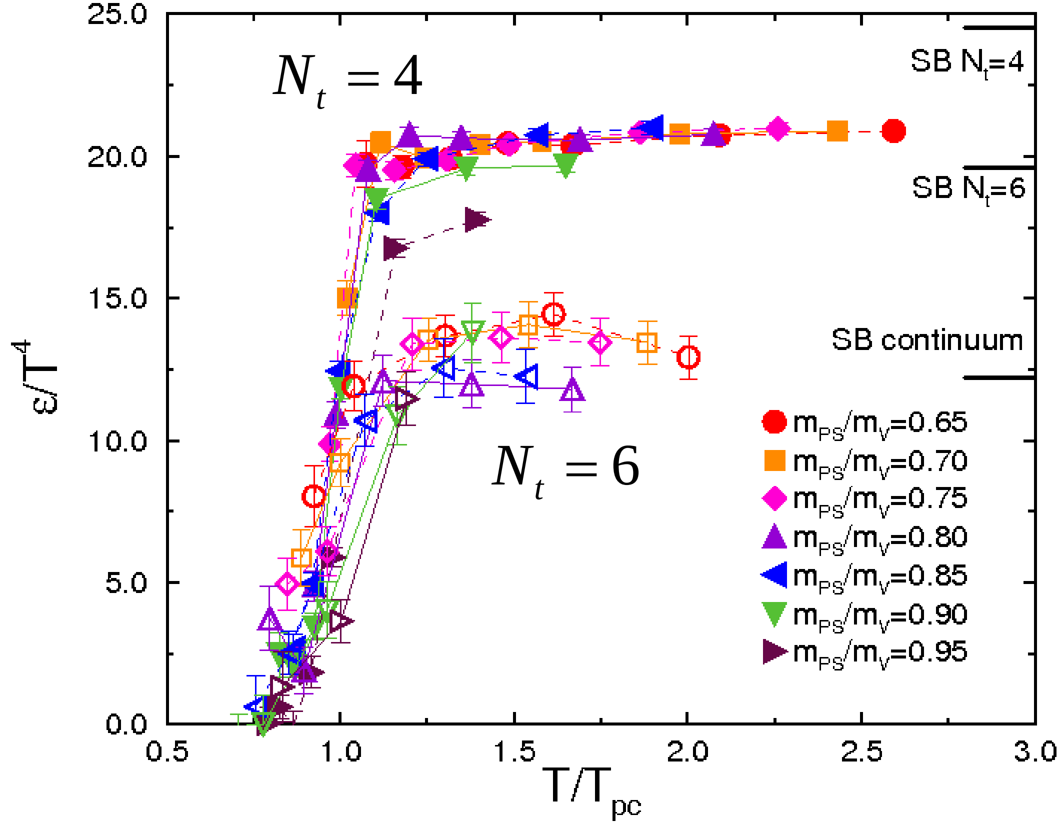
For the case: $\mu_u = \mu_d \equiv \mu_q$

$$\frac{\chi_C}{T^2} = \frac{1}{36}\frac{\chi_q}{T^2} + \frac{1}{4}\frac{\chi_I}{T^2}$$

Pressure and Energy density at μ=0

(CP-PACS, PRD64, 074510 (2001))





- Pressure is computed by the integral method for μ=0.
- We extend this study into finite μ.
- We compute the Taylor expansion coefficients of pressure at μ=0 by Monte Carlo simulations and extrapolate into finite μ.

$$\frac{p}{T^4}(\mu)=\frac{p}{T^4}(0)+c_2\left(\frac{\mu_q}{T}\right)^2+c_4\left(\frac{\mu_q}{T}\right)^4+c_6\left(\frac{\mu_q}{T}\right)^6+\cdots+c_2^I\left(\frac{\mu_I}{T}\right)^2+6c_4^I\left(\frac{\mu_q}{T}\right)^2\left(\frac{\mu_I}{T}\right)^2+\cdots$$

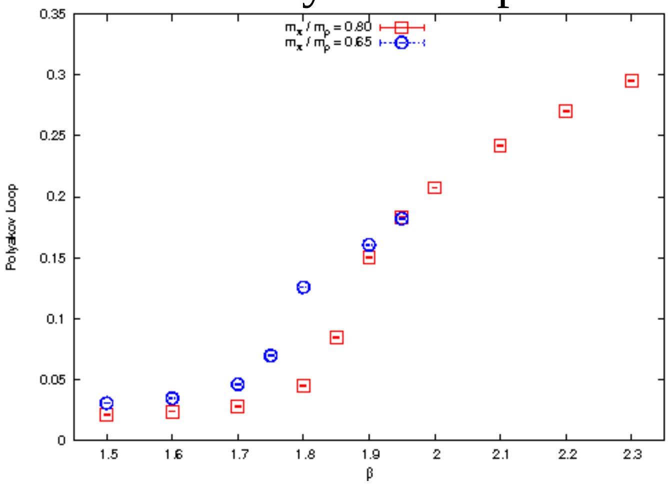
Simulation parameters

- We perform simulations for Nf=2 at m_{PS}/m_{V}=0.65 and 0.80 and investigate T dependence of Taylor expansion coefficients.

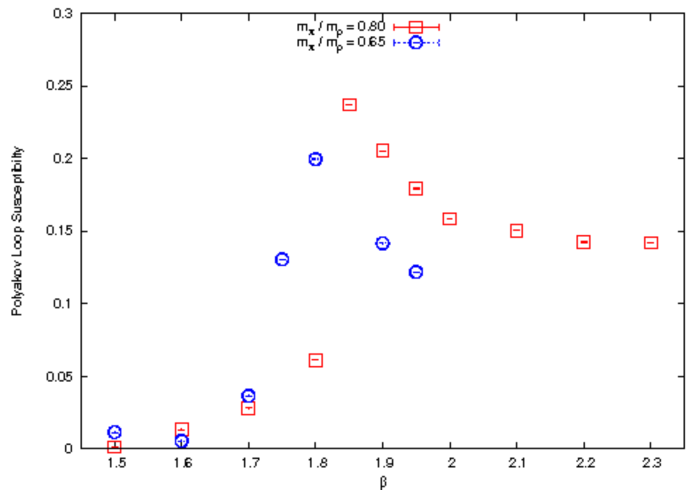
$$c_2 = \frac{N_t^3}{2N_s^3}\frac{\partial^2 \ln Z}{\partial(\mu_q/T)^2},\quad c_4 = \frac{N_t^3}{4!N_s^3}\frac{\partial^4 \ln Z}{\partial(\mu_q/T)^4},\quad c_2^I = \frac{N_t^3}{2N_s^3}\frac{\partial^2 \ln Z}{\partial(\mu_I/T)^2},\quad c_4^I = \frac{N_t^3}{4!N_s^3}\frac{\partial^4 \ln Z}{\partial(\mu_I/T)^2\partial(\mu_q/T)^2}$$

- Iwasaki improved gauge action and clover-improved Wilson fermion action are used.
- Lattice size: $N_{\rm site} = N_s^3 \times N_t = 16^3 \times 4$
- 500 ~ 600 configurations (5000 ~ 6000 trajectories) for each T (β).
- We use the random noise method with Nnoise=10. (see Appendix)

Polyakov loop



Polyakov loop susceptibility



Derivatives of grand partition function

$$c_2 = \frac{N_t}{2! N_s^3} \frac{\partial^2 \ln Z}{\partial (\mu_q a)^2} = \frac{N_t}{2! N_s^3} \mathrm{A}_2, \quad c_4 = \frac{1}{4! N_s^3 N_t} \frac{\partial^4 \ln Z}{\partial \mu^4} = \frac{1}{4! N_s^3 N_t} \left(\mathrm{A}_4 - 3\mathrm{A}_2^2\right),$$

$$c_2^I = \frac{N_t}{2! N_s^3} \frac{\partial^2 \ln Z}{\partial (\mu_I a)^2} = \frac{N_t}{2! N_s^3} B_2, \quad c_4^I = \frac{1}{4! N_s^3 N_t} \frac{\partial^4 \ln Z}{\partial (\mu_I a)^2 \partial (\mu_q a)^2} = \frac{1}{4! N_s^3 N_t} \left(B_4 - B_2 \mathrm{A}_2\right).$$

$$\left(\mu \equiv \mu_q a = (\mu_q / T)/N_t\right)$$

$$\mathrm{A}_2 = \left\langle N_f \frac{\partial^2 \ln\det M}{\partial \mu^2} \right\rangle + \left\langle \left(N_f \frac{\partial \ln\det M}{\partial \mu} \right)^2 \right\rangle,$$

$$\mathrm{A}_4 = \left\langle N_f \frac{\partial^4 \ln\det M}{\partial \mu^4} \right\rangle + 4\left\langle N_f^2 \frac{\partial^3 \ln\det M}{\partial \mu^3} \frac{\partial \ln\det M}{\partial \mu} \right\rangle + 3\left\langle \left(N_f \frac{\partial^2 \ln\det M}{\partial \mu^2} \right)^2 \right\rangle + 6\left\langle N_f^3 \frac{\partial^2 \ln\det M}{\partial \mu^2} \left(\frac{\partial \ln\det M}{\partial \mu} \right)^2 \right\rangle + \left\langle \left(N_f \frac{\partial \ln\det M}{\partial \mu} \right)^4 \right\rangle,$$

$$B_2 = \left\langle N_f \frac{\partial^2 \ln\det M}{\partial \mu^2} \right\rangle, \quad B_4 = \left\langle N_f \frac{\partial^4 \ln\det M}{\partial \mu^4} \right\rangle + 2\left\langle N_f^2 \frac{\partial^3 \ln\det M}{\partial \mu^3} \frac{\partial \ln\det M}{\partial \mu} \right\rangle + \left\langle \left(N_f \frac{\partial^2 \ln\det M}{\partial \mu^2} \right)^2 \right\rangle + \left\langle N_f^3 \frac{\partial^2 \ln\det M}{\partial \mu^2} \left(\frac{\partial \ln\det M}{\partial \mu} \right)^2 \right\rangle$$

$$\frac{\partial \ln\det M}{\partial \mu} = \mathrm{Tr}\left(M^{-1} \frac{\partial M}{\partial \mu} \right), \qquad \frac{\partial^2 \ln\det M}{\partial \mu^2} = \mathrm{Tr}\left(M^{-1} \frac{\partial^2 M}{\partial \mu^2} \right) - \mathrm{Tr}\left(M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} \right),$$

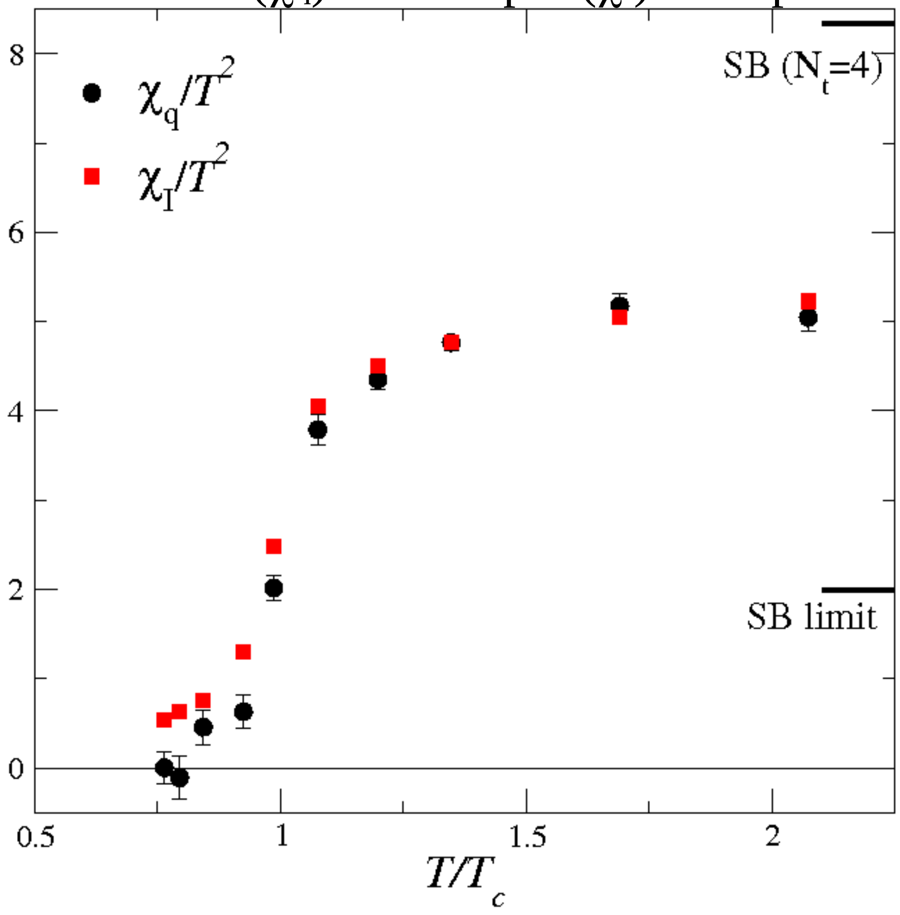
$$\frac{\partial^3 \ln\det M}{\partial \mu^3} = \mathrm{Tr}\left(M^{-1} \frac{\partial^3 M}{\partial \mu^3} \right) - 3\mathrm{Tr}\left(M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^2 M}{\partial \mu^2} \right) + 2\mathrm{Tr}\left(M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} \right), \cdots$$

$$\left.\frac{\partial^n M}{\partial \mu^n}\right|_{x,y} = \begin{cases} -K\left[(1-\gamma_4)U_4 \delta_{x+\hat{4},y} - (1+\gamma_4)U_4^+ \delta_{x-\hat{4},y}\right] & \text{for } n:\text{odd} \\ -K\left[(1-\gamma_4)U_4 \delta_{x+\hat{4},y} + (1+\gamma_4)U_4^+ \delta_{x-\hat{4},y}\right] & \text{for } n:\text{even} \end{cases}$$

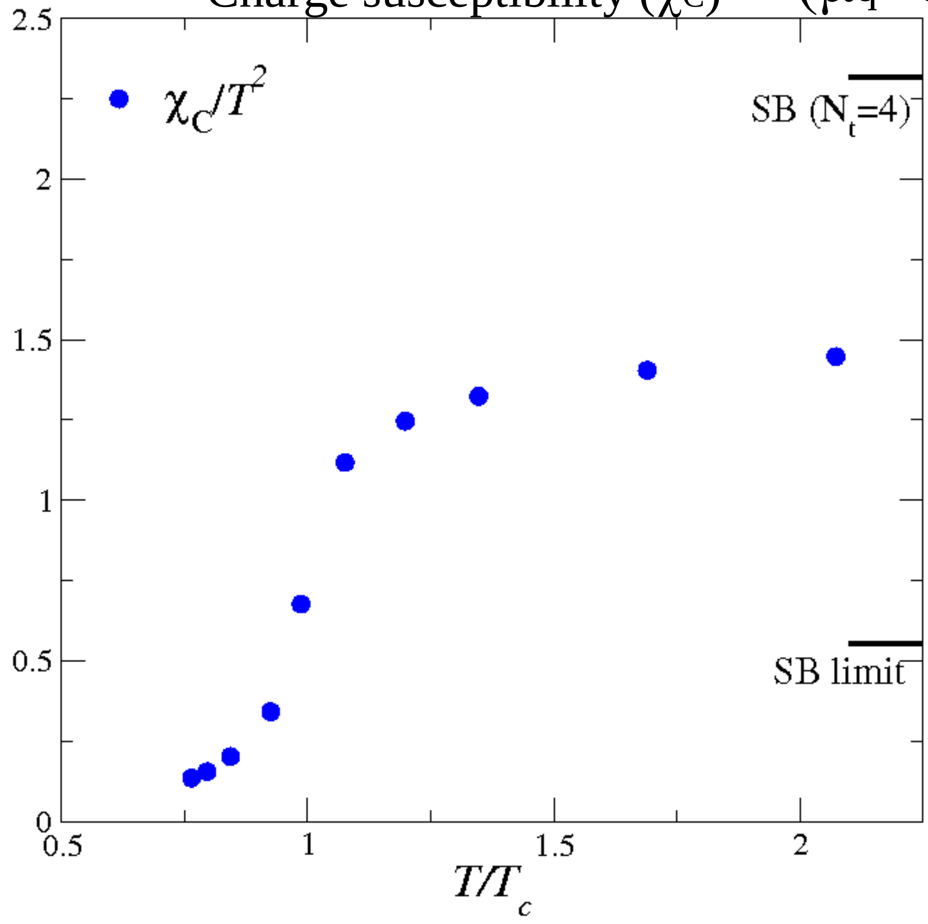
- These operators can be calculated by the random noise method.

Preliminary results of susceptibilities (m_{PS}/m_V=0.8)

Quark number(χ_q) and Isospin (χ_I) susceptibilities



Charge susceptibility (χ_C) (μ_q=0)



- We also measured the second derivatives of these susceptibilities (4th derivatives of pressure) with respect to μ_q and μ_I, however the statistical errors are too large at present. (200-500 configurations are used for this calculation.)
- The choice of the number of noise vectors may be important (see Appendix).

Lines of constant pressure

- It is interesting to compare the line of constant pressure (or energy density) to the chemical freeze out points.
- We estimate the line of constant p near μ_q=0.

Along this line

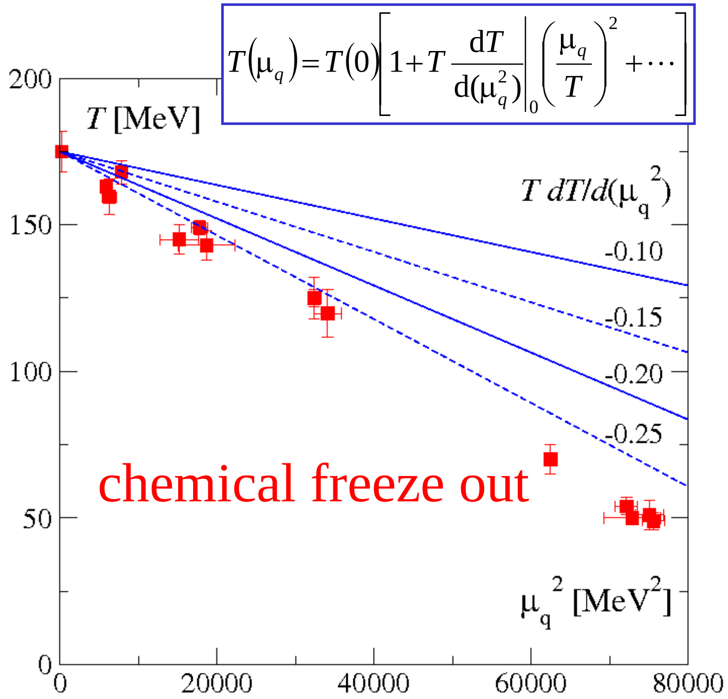
$$\Delta p = \frac{\partial p}{\partial T}\Delta T + \frac{\mathrm{d}p}{\mathrm{d}(\mu_q^2)}\Delta(\mu_q^2) = 0$$

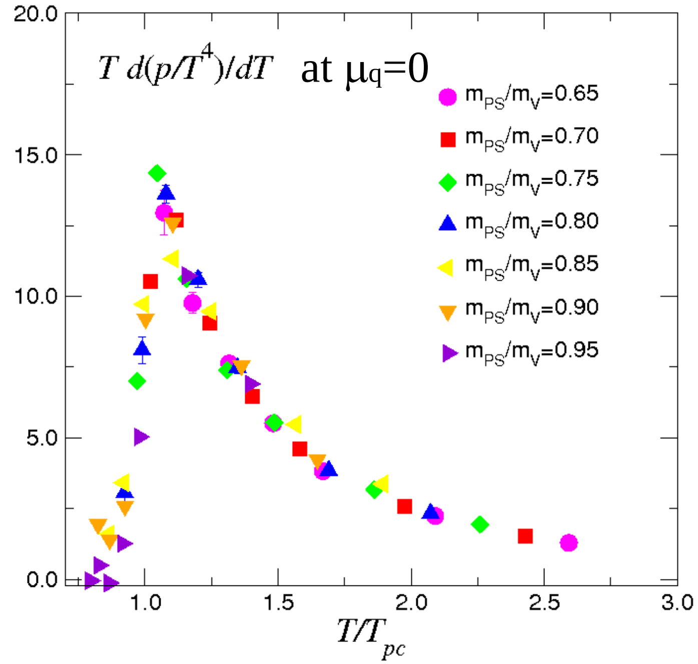
- The slope of constant p line in the (T, μ_q) plane is given by

$$T\frac{\mathrm{d}T}{\mathrm{d}(\mu_q^2)} = -\frac{\partial\left(p/T^4\right)}{\partial(\mu_q/T)^2}\Bigg/\left(T\frac{\partial\left(p/T^4\right)}{\partial T} + \frac{4p}{T^4}\right)$$

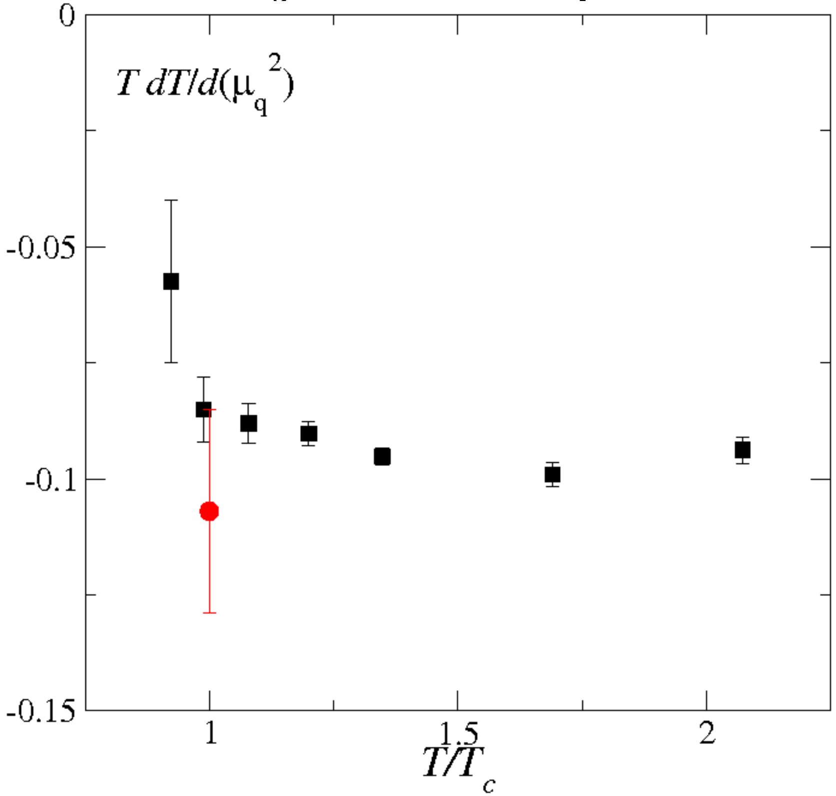
$$T\frac{\partial\left(p/T^4\right)}{\partial T} = -a\frac{\partial\beta}{\partial a}\frac{\partial\left(p/T^4\right)}{\partial\beta} - a\frac{\partial K}{\partial a}\frac{\partial\left(p/T^4\right)}{\partial K} = \frac{\varepsilon-3p}{T^4}$$

(Data in CP-PACS, PRD64, 074510 (2001))





Preliminary results at μ_q=0 for m_{PS}/m_V=0.8



- The slope at μ_q=0 is about -0.1. This is roughly consistent with the previous results by an improved staggered (Red dot: Bielefeld-Swansea, PRD66, 074507(2002), (mPS/mV=0.7)).
- Further studies at small quark mass and large N_t are necessary to compare with the experimental results.

Summary

- We report the current status of our study of QCD thermodynamics with a Wilson-type quark action.
- Lines of constant physics, i.e. constant m_{PS}/m_V, in the (β,K) plane are investigated and determined the relation between the parameters (β,K) and $(T, m_{PS}/m_V)$.
- Derivatives of pressure with respect to μ_q and μ_I up to 2^{th} order are computed and the preliminary results are obtained.
- Fluctuations of Quark number density, Isospin density and charge density are discussed.
- The line of constant pressure in the (T,μ) plane is also discussed.
- For the calculation of 4^{th} order derivatives, the choice of the number of noise vector (N_{noise}) is important. We find that $N_{noise}=10$ is not enough.
- Further studies with large N_t is also important.

Appendix: Remarks on the noise method

- Calculation of $\mathrm{Tr}\left(\frac{\partial M}{\partial \mu} M^{-1}\right), \mathrm{Tr}\left(\frac{\partial^2 M}{\partial \mu^2} M^{-1}\right), \mathrm{Tr}\left(\frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1}\right), \cdots$
- Random noise method is applied. (η: random noise vector)

$$\mathrm{Tr}\left(AM^{-1}\right) \approx \frac{1}{N_{\mathrm{noise}}} \sum_{n=1}^{N_{\mathrm{noise}}} \eta_n^\dagger A M^{-1} \eta_n$$

where η satisfies $$\frac{1}{N_{\mathrm{noise}}} \sum_{n=1}^{N_{\mathrm{noise}}} \eta_{n\,i}^{*} \eta_{n\,j} \approx \delta_{i,j}$$

e.g. $$\eta_{n\,j} = \left\{ e^{i\theta} ; \theta = [0, 2\pi] \right\},$$

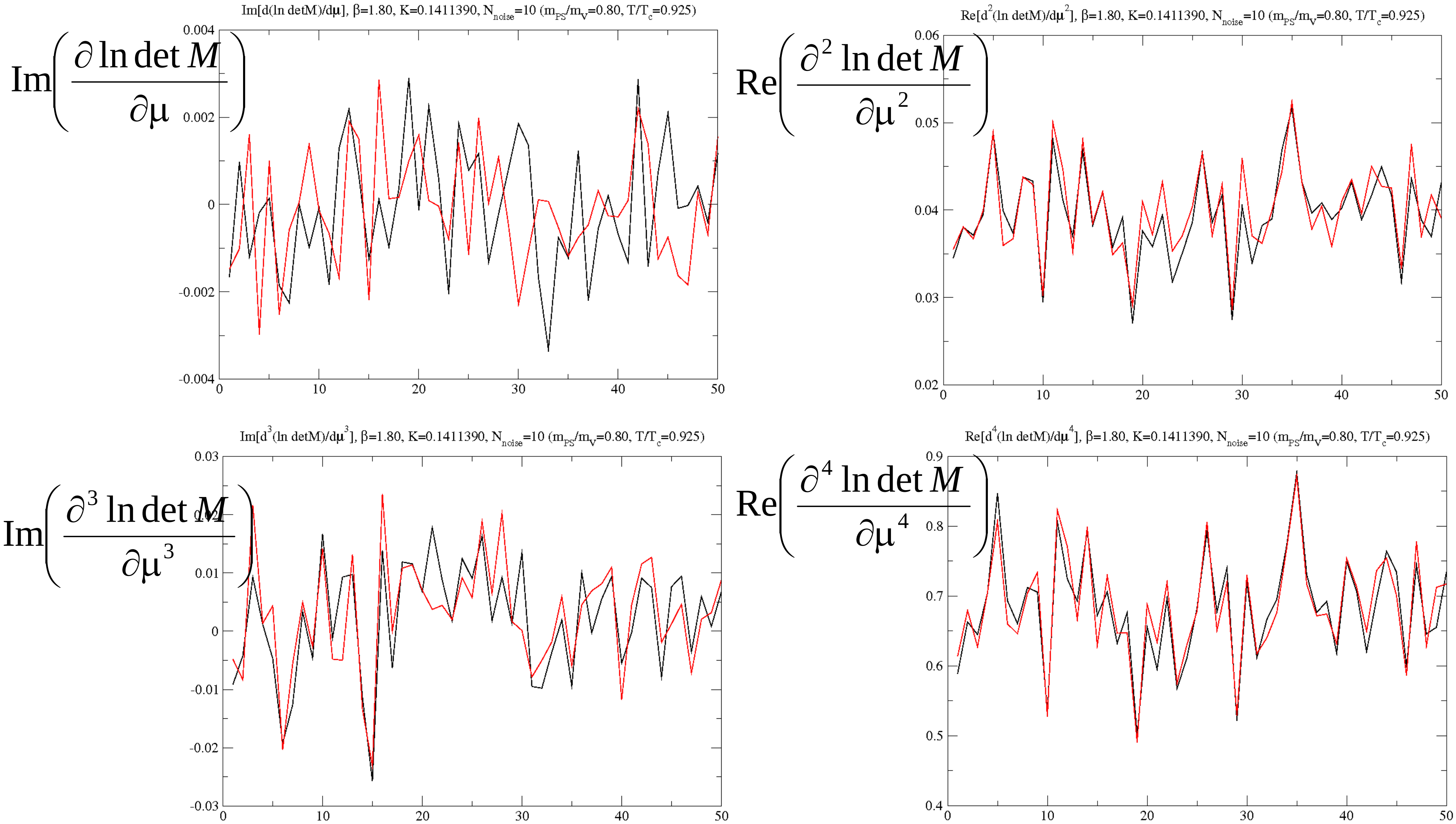
– Calculation of $M^{-1}\eta$ is easier than M^{-1} itself.

Solve $$\eta_i = M_{ij} x_j \to x_j = M_{ji}^{-1} \eta_i$$

– We can reduce the CPU time. ➡ large lattices and high statistics

– The number of noise vectors dependence: must be checked.

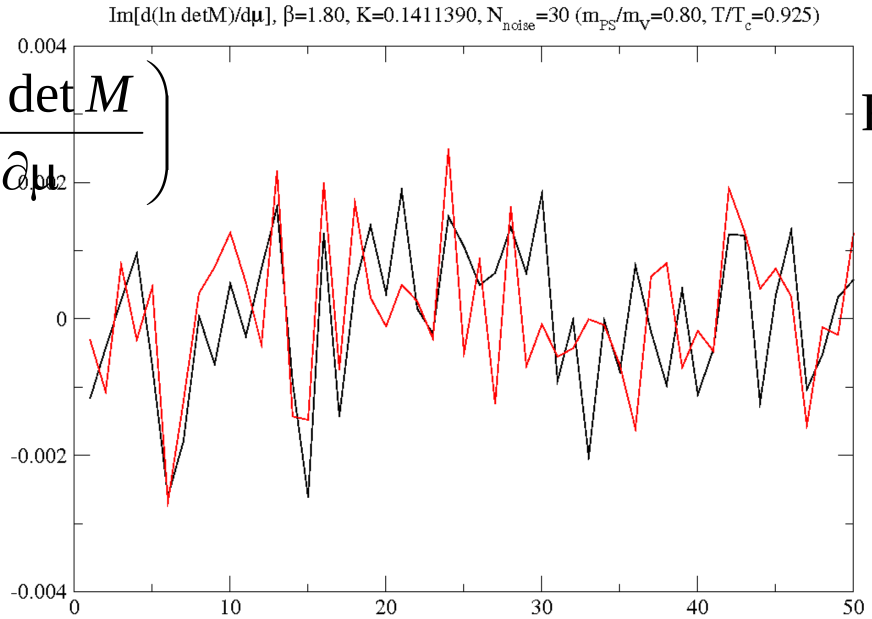
- To estimate the error form the noise method, we calculate operators using 2 independent sets of noise vectors with N_{noise}=10 and plot the time histories.



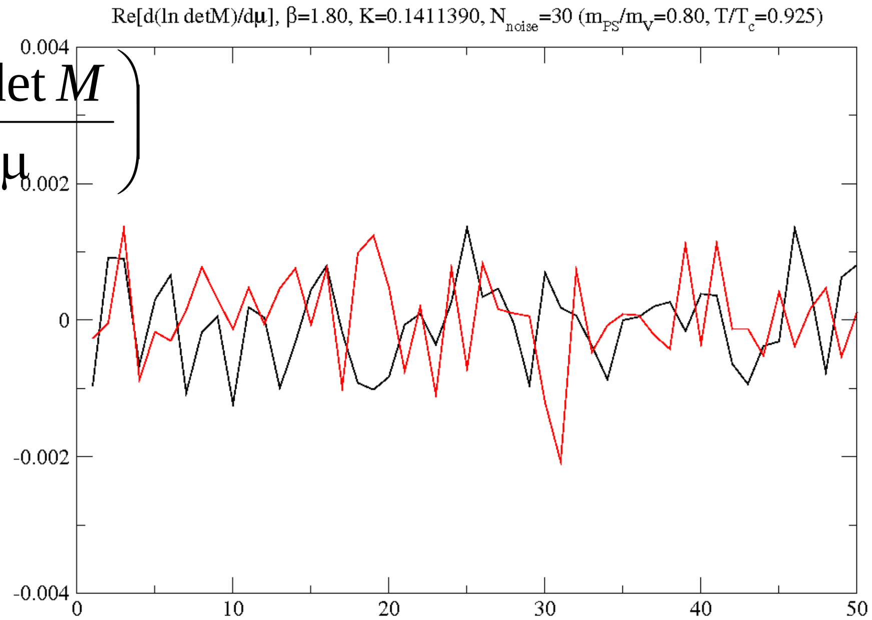
The results by different noise sets are almost consistent for the even order derivative terms, however the difference is visible for the odd order terms, especially for the 1st order term.

We increase the number of noise vectors up to $N_{\mathrm{noise}}=30$.

$\mathrm{Im}\left(\frac{\partial \ln \det M}{\partial \mu}\right)$



$\mathrm{Re}\left(\frac{\partial \ln \det M}{\partial \mu}\right)$



- The result of $\mathrm{Im}(d \ln \det M / d\mu)$ is getting better. The choice of N_{noise} is important for the calculation of the odd derivative term.
- Because $d \ln \det M / d\mu$ is purely imaginary, there are no correlation between the results of the real part obtained by two independent noise sets.
- The best choice of N_{noise} depends on the type of operators.

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