Twisted Mass QCD Thermodynamics: first results on apeNEXT

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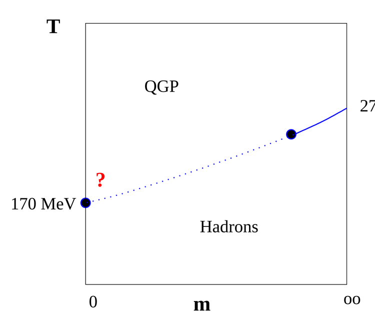
The motivations for simulating QCD thermodynamics with Wilson fermions and a twisted mass term are introduced. The twisted mass approach provides a natural infrared cutoff and O(a) improvement at maximal twist, and can be extended at finite temperature. Our strategy for exploring the QCD phase diagram at finite temperature in this setup, while taking advantage of the results at T=0, is explained. The first results for the order parameters and susceptibilities on a $16^3 \times 8$ lattice are presented. All dynamical simulations are carried out on the apeNEXT facility in Roma.

Motivations

Ultimately, we would like to study the finite temperature phase transition in QCD, and the properties of the plasma phase, with physical values of the quark masses , and in the continuum limit. The present project sets the stage for such a complete study.

The Phase Diagram of Two Flavor QCD is shown below: a (weakly) first order deconfinement transition line stemming from the infinite mass (quenched) transition turns into a crossover before ending up into a chiral transition, whose nature is still under investigation.



As a first step, we need to devise a strategy to cross these critical line or crossover on the lattice, while taking advantage of the properties of tmQCD: this means that we will have to explore a three-dimensional space : β, , μ.

tmQCD Thermodynamics

Wilson fermions have several advantages over staggered fermions but they have a more subtle chiral behaviour. In addition to that, the latter is far more expensive with respect to staggered fermions. The twisted mass approach improves over the standard Wilson behaviour in two ways:

- *It prevents the occurrence of exceptional configurations and should make it easier the approach to the chiral limit. It allows to reach mass-values of the light pseudoscalar mesons close to the physical pion mass*
- *Once the Wilson hoppping parameter is set to its critical value, the twisted mass term behaves as a conventional quark mass, and, at the same time, an O(a) improvement is authomatically guaranteed.*

In this first project we study the critical behaviour of the system as a function of the Wilson at a fixed β and a fixed twisted mass parameter.

Strategy and simulations

The simulations were performed on a $16^3 \times 8$ lattice with a HMC code with a tree level Symanzik improved gauge action.

- We choose to work at $\beta = 3.75$ and $\beta = 3.9$ so to take advantage of the $T = 0$ results. We then cross the (pseudo)critical line by tuning the quark mass, either varying k and/or μ in the diagonal part of the Fermion Action:

  $S[\chi, \bar{\chi}, U] = \sum_x \left\{ \bar{\chi}(x) \Big(1 + 2ia\mu\kappa\gamma_5\tau_3 \Big) \chi(x) \right\}$.

  as they both contribute to the bare mass: $m_q = \sqrt{(\mu^2 + (k - k_c)^2)}$
- For this strategy to be successful:

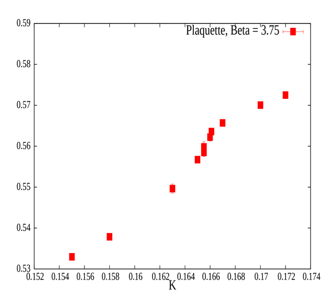
  $T_c^{chiral} < T_{simulation} = 1/(N_t a(\beta)) < T_c^{quenched}$
- We fixed $N_t = 8$ by taking into account:

  $a(3.75) \simeq 0.12$ fm ,$a(3.9) \simeq 0.095$ fm

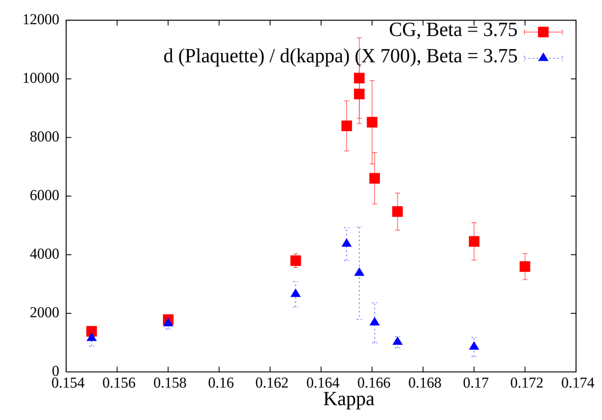
  $T_c^{chiral} \simeq 170$ MeV, $T_c^{quenched} \simeq 270$ MeV.
- In this first set of runs we fixed $\mu = 0.005$ and varied k. The next step shall be to scan in μ at fixed

  $k = k_c(T = 0)$

Results at $\beta = 3.75$



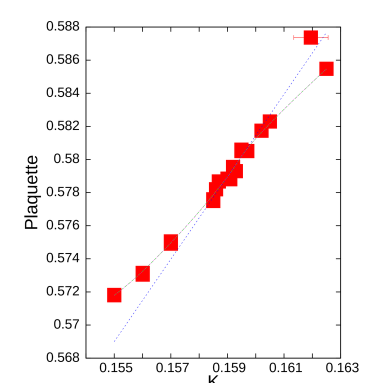
At $\beta = 3.75$ the $T = 0$ results show that the minimum pion mass which can be reached with our twisted mass parameter $\mu = 0.005$ is $m_\pi \simeq 400$ MeV, (extrapolating existing results at $\mu = 0.004$). This is the –relatively high – lower bound on the minimum mass at which the thermal transition could take place: our goal here was merely to check if a thermal phase trasition is located in the hadronic phase, $K_t < K_c$.

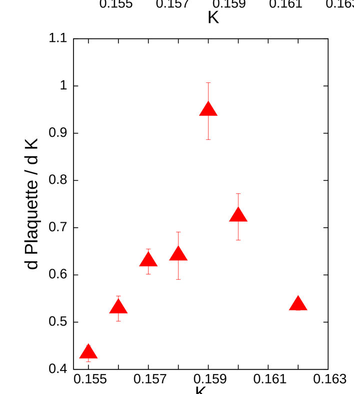


The steepest slope of the plaquette as well as the increase of the number of CG iterations needed for the inversion suggest a crossover or phase transition at K_t:

$$K_t(\beta = 3.75, \mu = 0.005) = 0.165(1) < K_c(T = 0) = 0.1669$$

Results at $\beta = 3.9$ - I





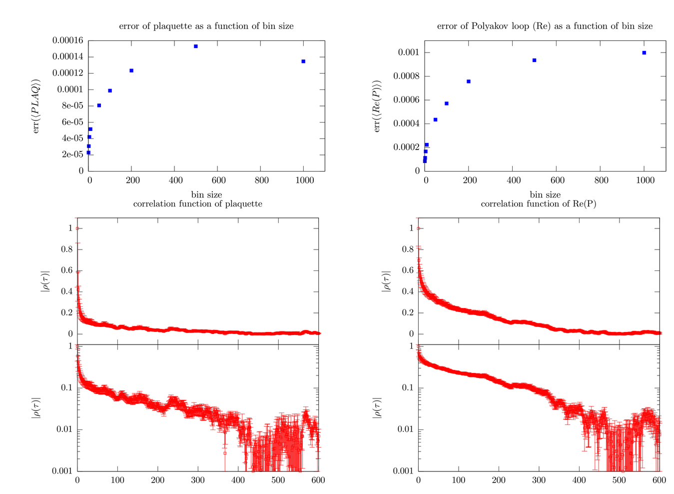
$\beta = 3.9$ was studied at $T = 0$ for $\mu = 0.0075$ and $\mu = 0.004$. The minimum T=0 pion mass for our $\mu = 0.005$ inferred from these results is about 350 MeV.

General features
After a first set of finite temperature runs of (1000) HMC sweeps each, the crossover (or transition) appeared much weaker that at $\beta = 3.75$, in agreement with previous observations. Hence, the critical behaviour was more challenging than in the previous case, and higher statistics runs in the critical region were required.

Plaquette
The upper diagram shows a collection of results for the plaquette. the errors are smaller than the symbol. The solid line is a bezier interpolation to guide the eye, while the straight line is the local fit with a maximum slope.
The derivative of the plaquette wrt to k (lower plot) is given by the slope of a straight line fit $f(k) = ak + b$ on subsequent intervals $\Delta(k) = 0.002$. The resulting a is then used as an estimator of the derivative in the central point of the interval.

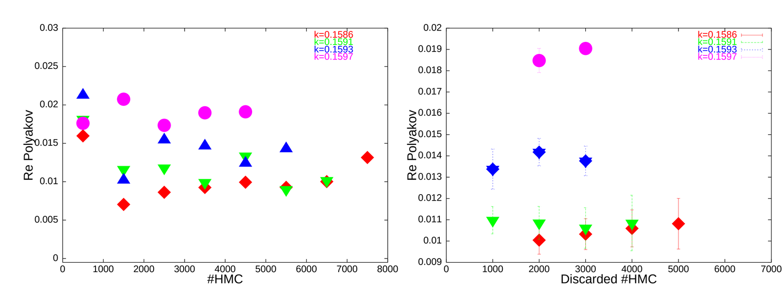
$\beta = 3.9$ II Large Statistics

Next, statistics was enhanced on a selected sample of points: $K = 0.1586, 0.1591, 0.1593, 0.1597$ in the candidate critical region at $\beta = 3.9$. We show here the error analysis for $K = 0.1586$

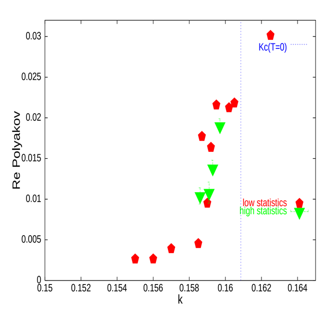


⋆ The Montecarlo evolution, with bins of 1000 iterations each, for the selected k in the critical region.

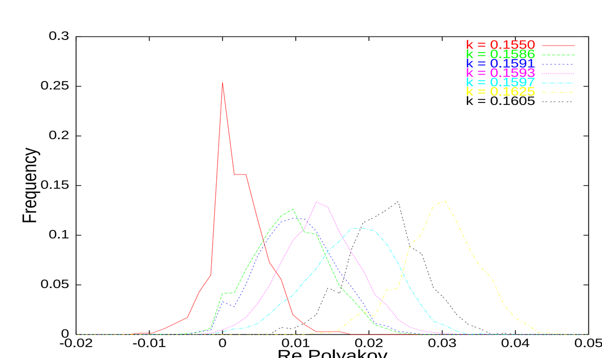
⋆ Averages and errors as a function of the number of discarded iterations: the results level off after two or three thousand iterations.



Results at $\beta = 3.9$ III



The Polyakov Loop
The Polyakov loops increases in the same k interval as the one observed for the plaquette. Using the results at $T = 0$ we can estimate the pion mass in the candidate critical region, ranging from $m_\pi \simeq 370$ ($k = 1.602$) to $m_\pi \simeq 380$ ($k = 1.592$). The main contribution to the quark mass here come from the twisted mass parameter: in the explored range the effect of k on the quark mass is on the order of a few percent.

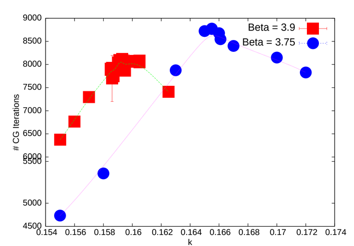


The Probability Distributions
The probability distributions, estimated from the frequency plot of the result, are narrow in the two pure phases, and broaden in the critical region, indicating an increase of the fluctuations. We see no sign of a flip-flop behaviour, but, as, usual, only FSSA can assess with confidence the nature of the critical behaviour.

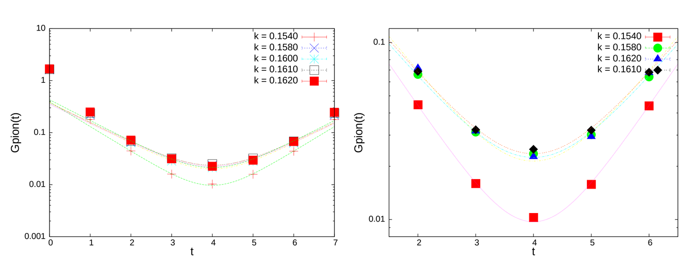
The steepest slope of the plaquette and of the Polyakov loop, as well as the broadening of the probability distributions suggest a crossover or phase transition at K_t:

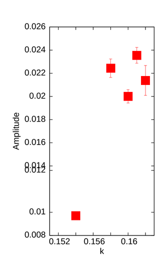
$$K_t(\beta = 3.9, \mu = 0.005) = 0.1597(5) < K_c(T = 0) = 0.160856$$

Dirac Inversion and Pion Propagator



⋆ **CG Iterations** The number of Conjugate Gradient iterations required for the inversion is an indicator of criticality. We show the number of CG iterations from both β's as a function of k. The solid line is a smooth interpolation to guide the eye.

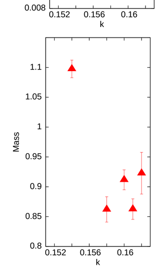




⋆ **Pion Propagators** The pion propagators were measured on a selected sample of couplings, and were fitted to a standard hyperbolic cosine form

$$G(t) = A \cosh M(t - T/2)$$

in the time interval [2:6]. We show the results, with the fits superimposed, in the diagrams above (note a different scale between the two).



⋆ **Results of the Fits** The resulting parameters of the fits A(mplitude) and M(ass) are collected in the plots at the left. They indicate that also at a finite temperature the pion mass decreases while approaching the thermal transition, while the amplitude increases, following the trend of the plaquette.

Summary and Outlook

- We have studied QCD with two flavor of dynamical Wilson fermions, and a twisted mass term on a $16^3 \times 8$ at two values of the temperature:
  - $\beta = 3.75$ corresponding to 205 MeV
  - $\beta = 3.9$ corresponding to 259 MeV.
- In either cases we have simulated O(10) values of K and observed a behaviour consistent with a crossover (and non excluding a real transition). The critical pion mass is well above its physical value.
- As a next step, we will set $K = K_c(\beta)$ and scan in mass by tuning the twisted mass parameter μ.

Results at T=0 used here were taken from the contribution to the Lattice05 proceedings *Dynamical twisted mass fermions* by F. Farchioni et al.

For a review of recent results on the QCD transition, see O. Philipsen at Lattice05, and Urs Heller at Lattice06.

Please see our writeup for a complete list of references.

It is a pleasure to warmly thank M. Creutz, R. Frezzotti, G.C. Rossi and C. Urbach for interesting and helpful discussions. We wish also to thank the APEnext Collaboration, and in particular Alessandro Lonardo,Davide Rossetti,Hubert Simma,Raffaele Tripiccione and Piero Vicini, for their crucial help and support, as well as Giampietro Tecchiolli for granting access to the Amaro machines.