Analytical continuation from imaginary to real chemical potential in 2-color QCD under scrutiny

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Introduction (1)

- It is known that the fermion determinant in QCD becomes complex in presence of a non-zero chemical potential, thus preventing us from performing standard Monte Carlo simulations.

- To circumvent this problem one may perform numerical simulations with ***imaginary chemical potential*** and then obtain physical results by ***analytical continuation*** to ***real chemical potential.***

- It is very important to have some control on the method of ***analytical continuation.***

  To this purpose is useful to consider the case of **2-color QCD** where, for any value of the chemical potential, ***the fermion matrix is real***.

Our aim is to perform a **new test** of the ***method of analytical continuation*** in finite temperature **SU(2) gauge theory** with staggered fermions (N_f=8) and non-zero baryon chemical potential, both imaginary and real.

Introduction (2)

- The method of analytical continuation [Alford-Kapustin-Wilczek, PRD59 (1999)054502; Lombardo, NPB(Proc.Suppl.)83(2000)375] has been applied in:

  - **SU(3), N_f=4** [D'Elia-Lombardo, PRD67(2003)014505]
  - **SU(3), N_f=2** [deForcrand-Philipsen, NPB642(2002)290]
  - **SU(3), N_f=3** [deForcrand-Philipsen, NPB673(2003)170]

  and tested in:

  - **SU(3) + adj. Higgs model in 3-d** [Hart-Laine-Philipsen, PLB505(2001)141]
  - **SU(2), N_f=8** [Giudice-Papa, PRD69(2004)094509]
  - **3-d Z(3) Potts model** [Kim et al., Lattice2005]

- ***Until now***, the method of analytical continuation has been applied using **truncated Taylor series**

→ **Here we propose and test an approach based on the use of ratio of polynomials**

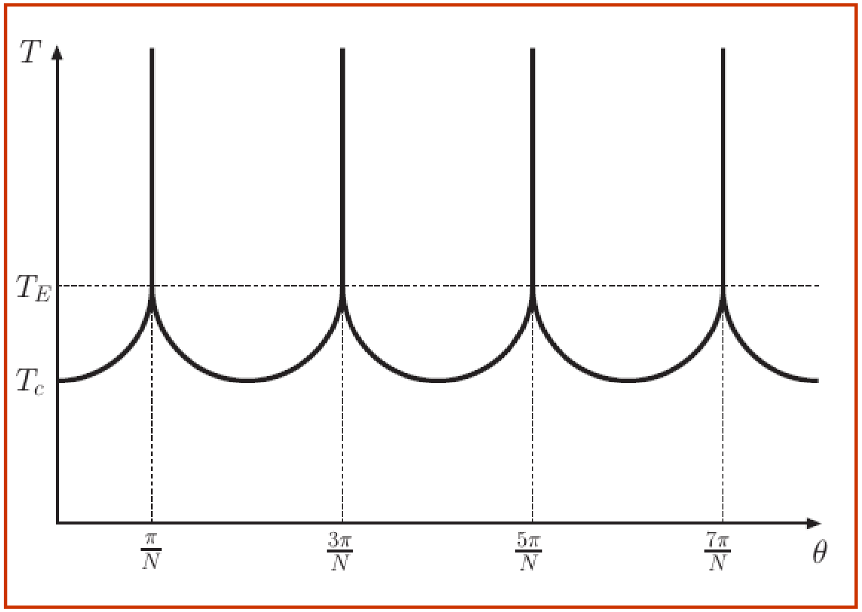
Theoretical background

- Roberge and Weiss [NPB275(1986)734] showed that the partition function of any SU(N) gauge theory is periodic in the parameter $\theta = \mu_I / T$, where T is the temperature and μ_I is the imaginary part of the chemical potential.

Above a certain temperature T_E physical observables exhibit discontinuities for

$$\theta = 2\pi(k+1/2)/N\,, \quad k = 0, 1, \ldots$$

Therefore, the window in imaginary chemical potential where to perform numerical simulations is limited to the interval **[0, π T/N]**, thus strongly limiting the region useful for the analytical continuation.



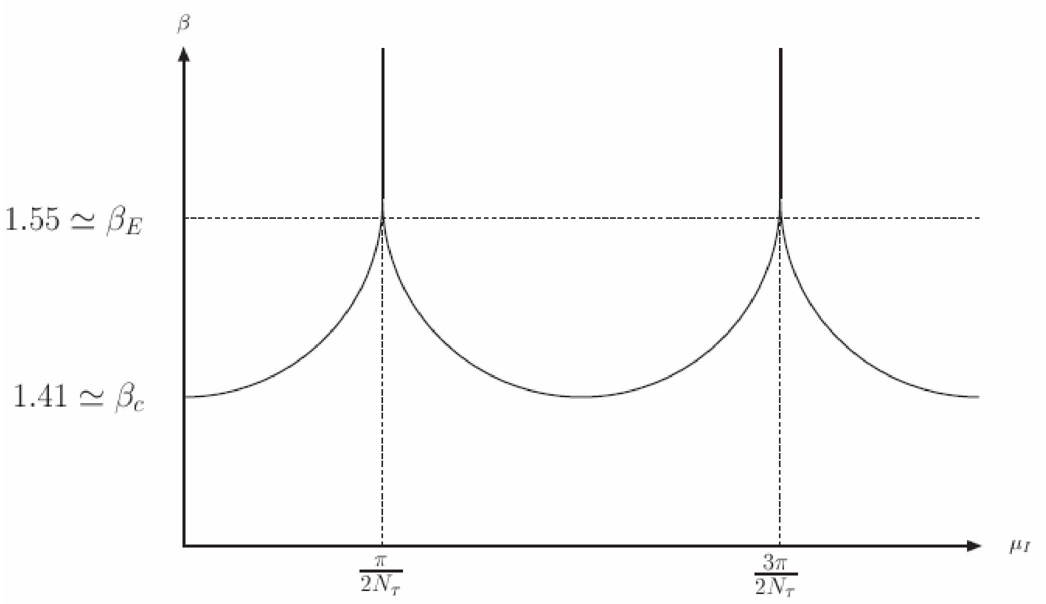
Expected phase diagram for SU(N) gauge theory

- vertical lines above T_E are first order
- curved lines represent first/second/crossover transition depending on the model and on its parameters

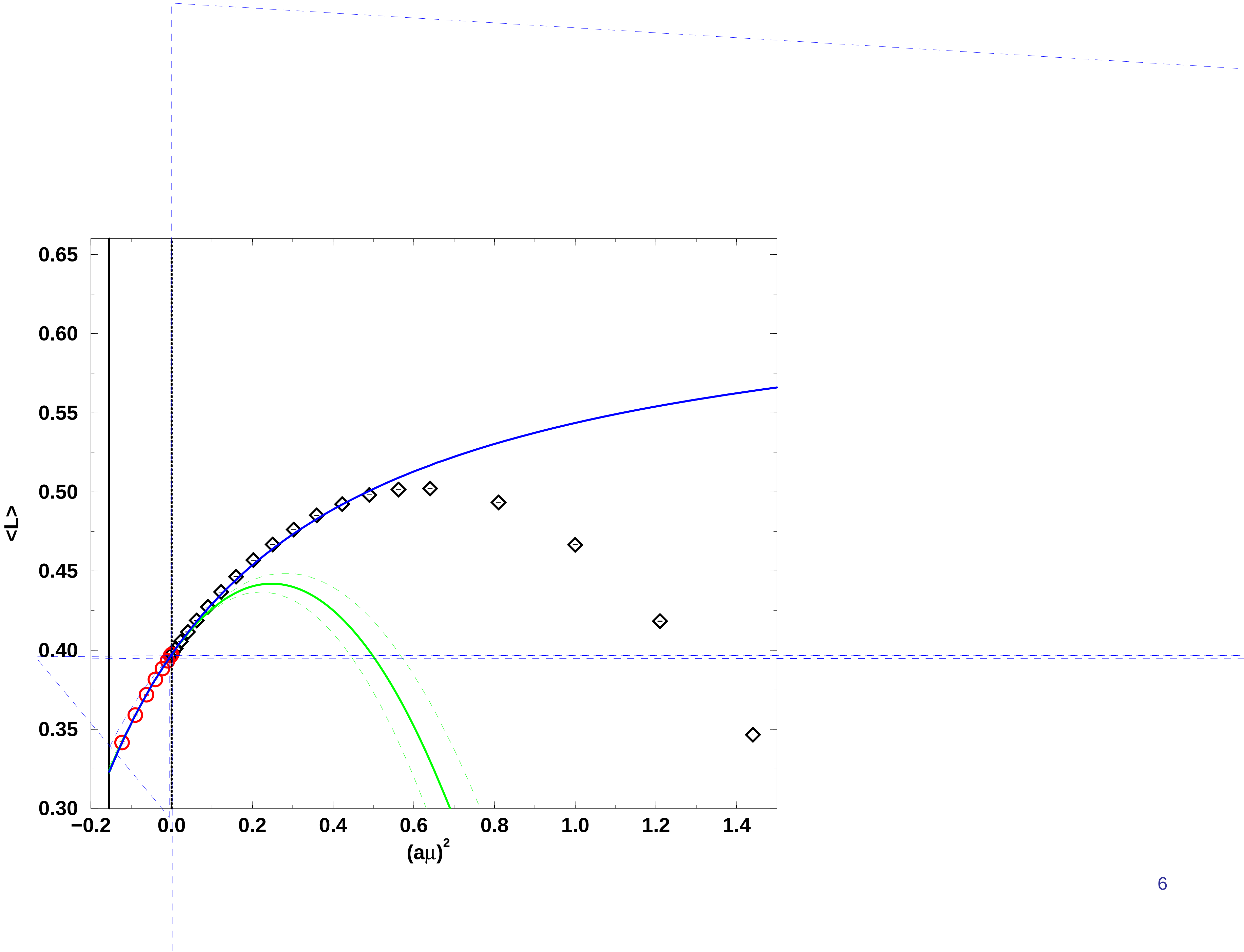
Lattice simulations

- We consider **SU(2)** gauge theory with $N_f=8$ staggered fermions, $am=0.07$ on a $16^3 \times 4$ lattice
- Numerical simulations have been performed using hybrid Monte Carlo with $dt=0.01$
- We measure the following observables: chiral condensate, Polyakov loop, and baryon density (statistics: 1000-5000 trajectories)

Simulations have been done using **APEmille crate in Bari** and the recently installed computer facilities at **INFN apeNEXT Computing Center**

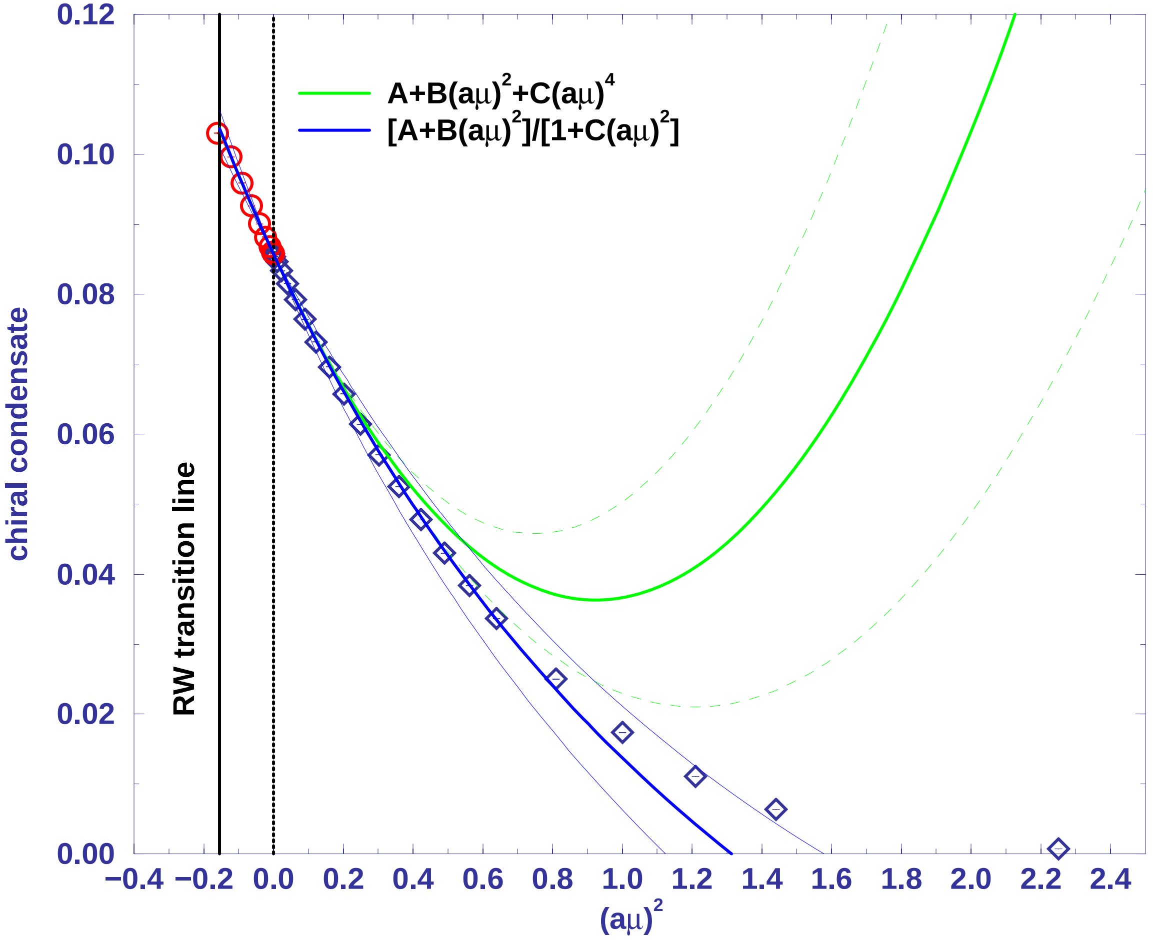


We have considered ***preliminarily*** $\beta=1.90$ for which the window in μ_I free of non-analyticities is the largest possible.



0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
<L>
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
(aμ)²

Results: chiral condensate

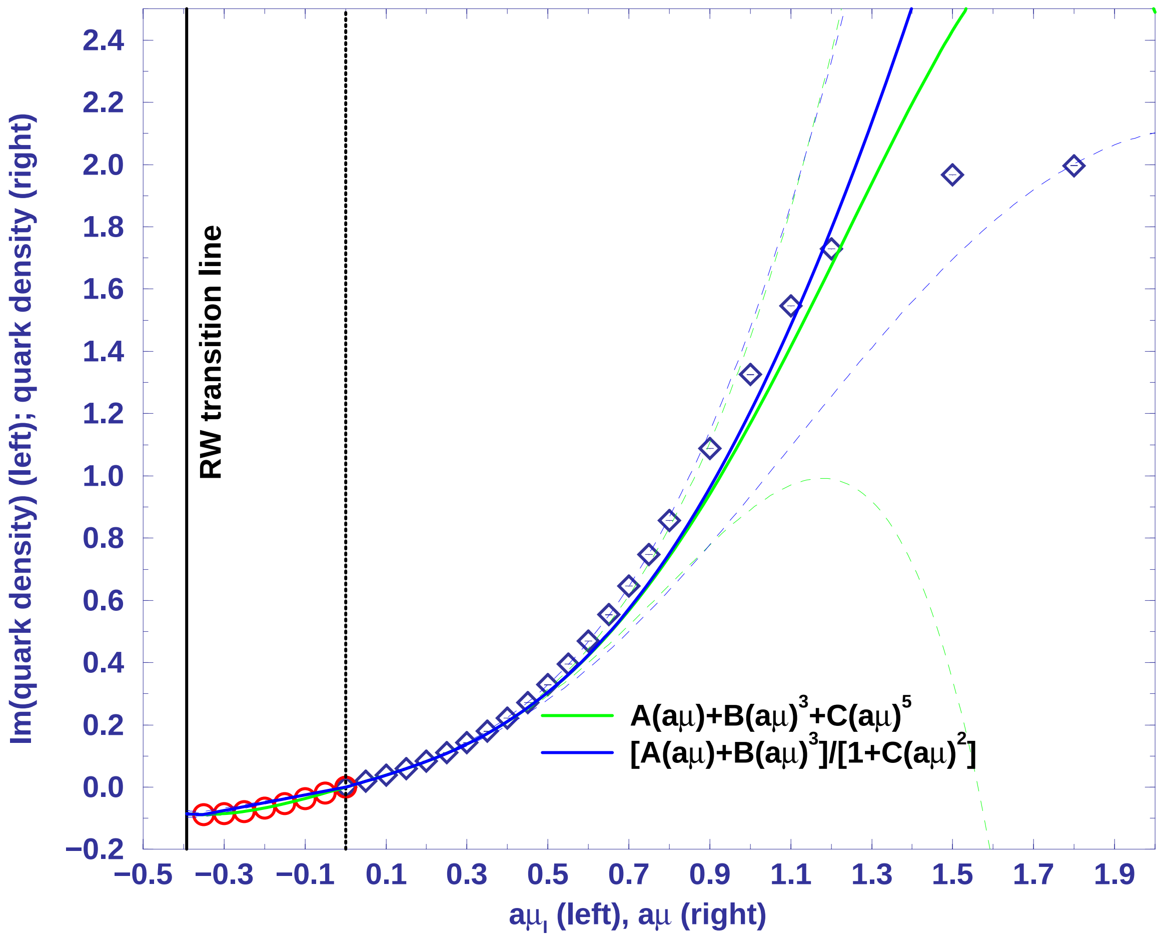


- **Interpolation** for $(a\mu)^2 \leq 0$
- **Extrapolation** to $(a\mu)^2 > 0$

(1) $A + B\hat{\mu}^2 + C\hat{\mu}^4$

(2) $\dfrac{A + B\hat{\mu}^2}{1 + C\hat{\mu}^2}$

Results: baryon density



- **Interpolation** for imaginary chemical potential
- **Extrapolation** to real chemical potential

(1) $A\hat{\mu} + B\hat{\mu}^3 + C\hat{\mu}^5$

(2) $$\frac{A\hat{\mu} + B\hat{\mu}^3}{1 + C\hat{\mu}^2}$$

Conclusions

- **We have verified by comparison with direct Monte Carlo determinations at real chemical potential in 2-color QCD that the method of analytical continuation considerably improves if the ratio of polynomials is used as interpolating function instead of truncated Taylor series.**

- **In the case of the *Polyakov loop* and of the *chiral condensate* an interpolation of numerical data at imaginary chemical potential over the window permitted by Roberge-Weiss singularities using the ratio of two polynomials of first order in μ^2 allows an extrapolation to real values of the chemical potential over a much larger region.**
  ***Deviations* at very large values of the chemical potential could be due to unphysical saturation of the baryon number (*"Pauli blocking"*).**

Conclusions (cont'd)

- Our results clarify that *the presence of the RW transition has no influence on the analyticity of the partition function at real values of* μ.

- Our method looks very promising in view of applications to *real QCD*.

Work in progress

- We are extending our test to other values of the gauge coupling, in particular to those values for which non-analyticities (different from the first-order Roberge-Weiss lines) are expected for real and/or imaginary chemical potential.