Progress towards the phase diagram of QCD

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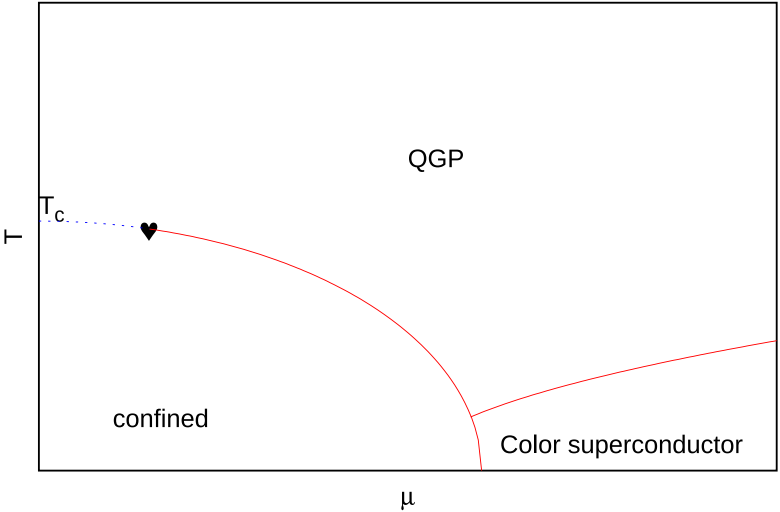
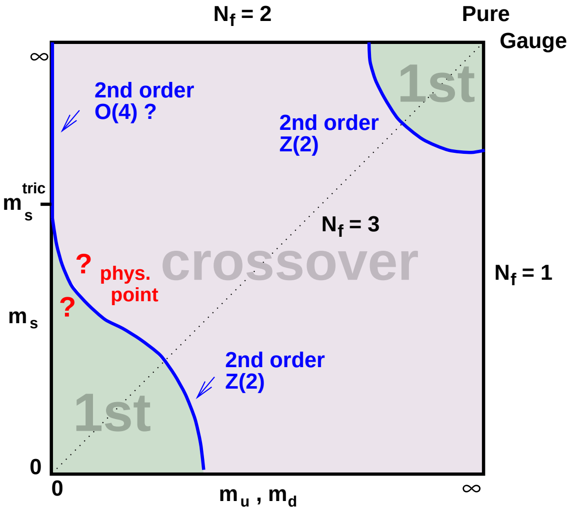
Phase diagram of QCD



Learn about QCD by generalizing to arbitrary $(m_{u,d}, m_s)$ quark masses

Phase diagram vs $(m_{u,d}, m_s)$, T and μ

$\mu = 0$



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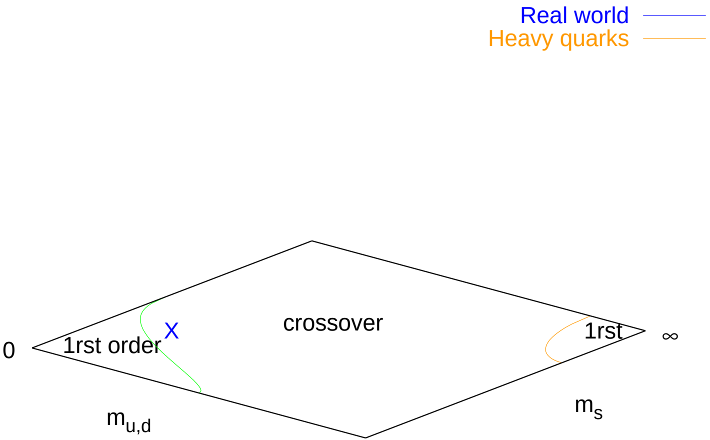
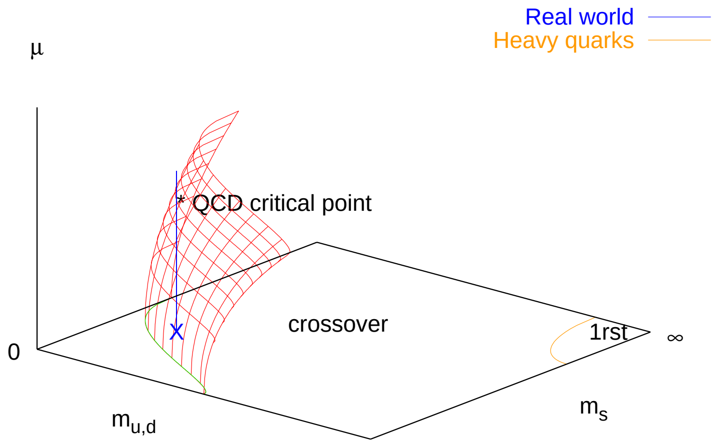


Now turn on μ

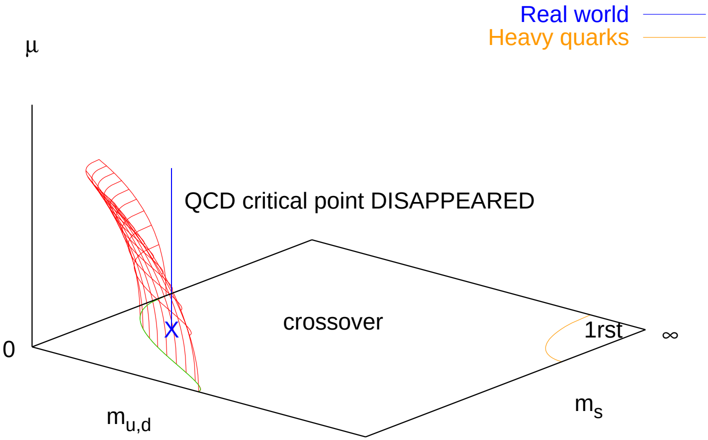
Phase diagram vs $(m_{u,d}, m_s)$, T and μ

$\mu \neq 0$



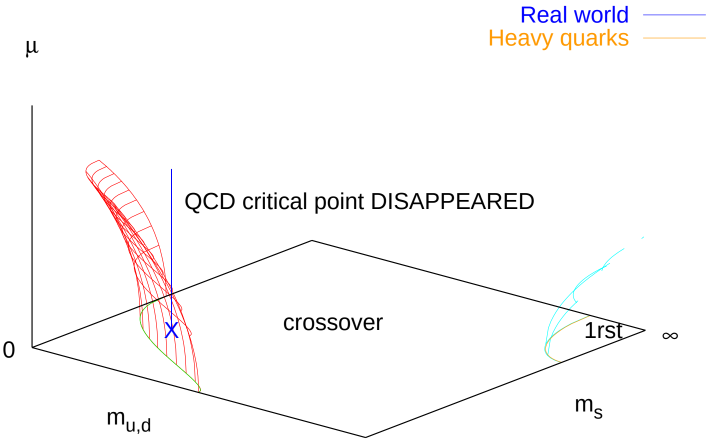
Conventional wisdom: first-order region expands with real $|\mu|$

Phase diagram vs $(m_{u,d}, m_s)$, T and μ



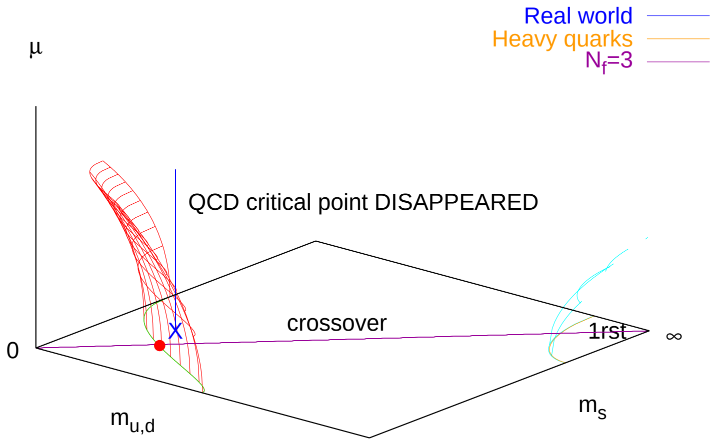
Exotic scenario: first-order region shrinks with real $|\mu|$

Phase diagram vs $(m_{u,d}, m_s)$, T and μ



For heavy quarks, first-order region shrinks (PdF, Kim & Takaishi)

Phase diagram vs $(m_{u,d}, m_s)$, T and μ

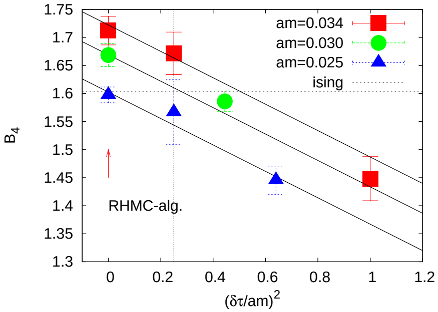


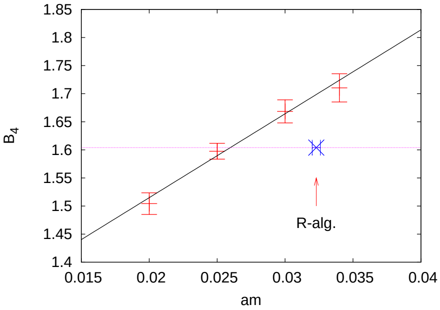
Detect 2nd order with Binder cumulant $B_4 \equiv \frac{\langle(\delta\bar{\psi}\psi)^4\rangle}{(\langle(\delta\bar{\psi}\psi)^2\rangle)^2}$. Check $N_f = 3$ first.

Eliminating the stepsize error: R $\longrightarrow$ RHMC

- Old R-alg. requires $\delta\tau \to 0$ extrapolation

$\delta\tau \neq 0 \to$ bias transition towards 1rst-order (Kogut & Sinclair)



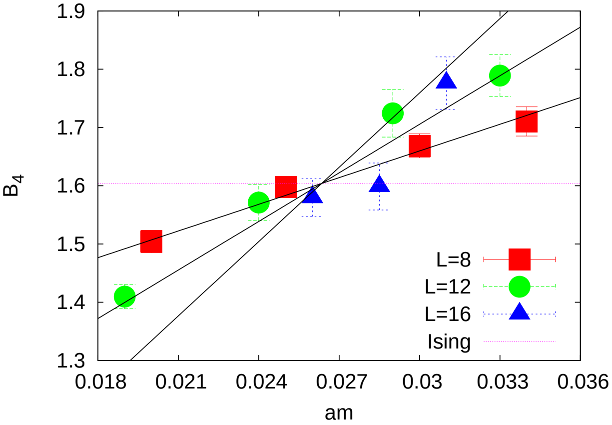


- New RHMC (Clark & Kennedy) exact and more efficient

$N_f = 3$: critical quark mass for 2nd order P.T. at $\mu = 0$ down by $\sim 25\%$!

Lattice renorm.? No: corresponding $\frac{m_\pi}{T_c}$ down by $\sim 10\%$

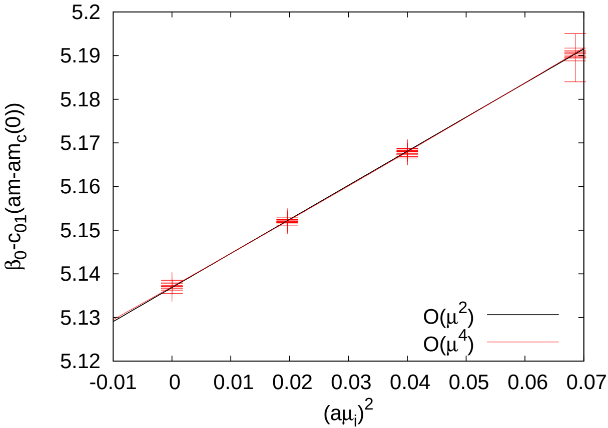
$N_f = 3$, $\mu = 0$: finite size scaling



$\xi \propto |m - m_c|^{-\nu} \;\; \longrightarrow \;\; B_4(m,L) = B_4^{Ising} + bL^{1/\nu}(m - m_c)$

Fit: $\nu = 0.67(13)$ versus $\nu^{Ising} \approx 0.63$

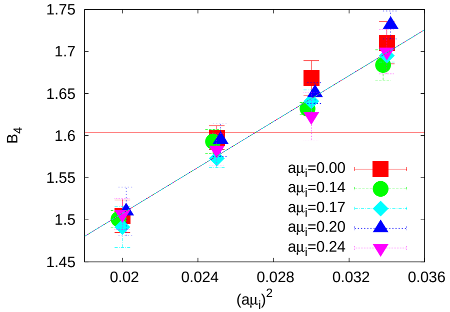
$N_f = 3$, $\mu \neq 0$: pseudo-critical temperature

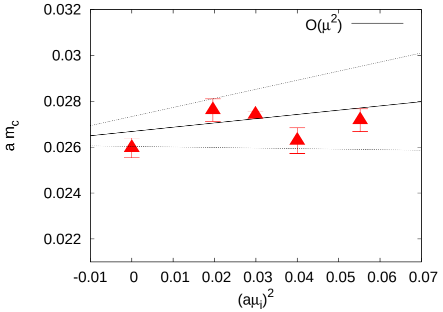


Correction $\sim$ 10% from R-algorithm:

$$\frac{T_c(\mu,m)}{T_c(0,m_0^c)} = 1 + 2.111(17)\left(\frac{m-m_0^c}{\pi T_c}\right) - 0.667(6)\left(\frac{\mu}{\pi T_c}\right)^2 + 0.23(9)\left(\frac{\mu}{\pi T_c}\right)^4 + \ldots$$

$N_f = 3$, $\mu \neq 0$: critical mass versus μ





No significant variation in $am_c \propto \frac{m_c}{T_c}$ versus $a\mu \propto \frac{\mu}{T}$

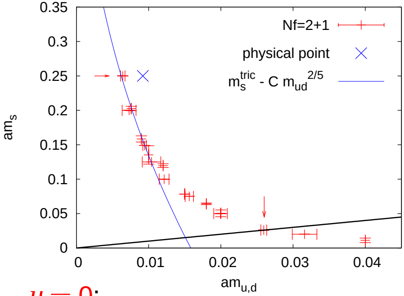
$$\Rightarrow \frac{d\ m_c}{d(\mu/T)^2} \approx \frac{d\ T_c}{d(\mu/T)^2}$$

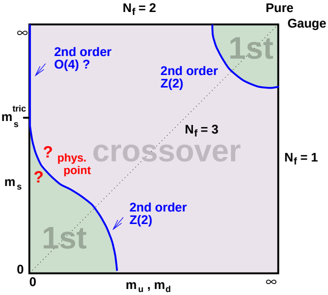
$$\frac{m_c(\mu)}{m_c(\mu=0)} = 1 - 0.7(4)\left(\frac{\mu}{\pi T}\right)^2 + \ldots$$

The first-order region shrinks when μ is turned on

Extend to $N_f = 2+1$

Line of second-order phase transitions in the quark mass plane $(m_{u,d}, m_s)$





$\mu = 0$:

- data consistent with tricritical point at $m_{u,d} = 0,\ m_s \sim 2.8T_c$
- physical point in crossover region

Fodor & Katz

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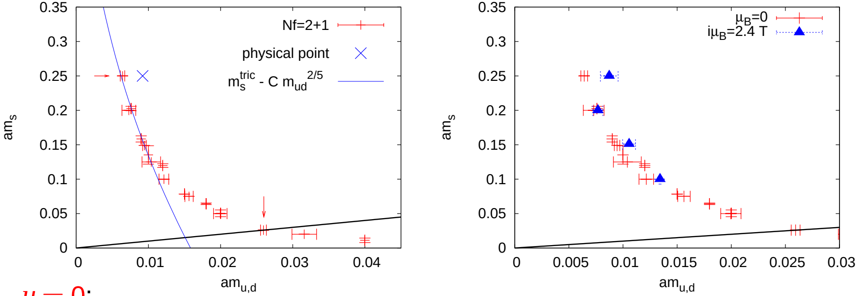


$\mu = 0$:

- data consistent with tricritical point at $m_{u,d} = 0,\ m_s \sim 2.8T_c$
- physical point in crossover region Fodor & Katz

$\mu \neq 0$ imaginary:

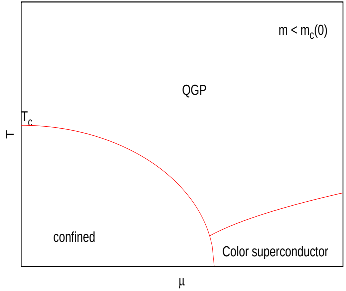
- am_c increases slightly $\Rightarrow$ decreases slightly with real μ

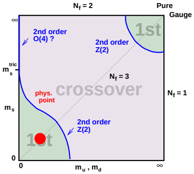
$$\Rightarrow \frac{d\ m_c}{d(\mu/T)^2} \lesssim \frac{d\ T_c}{d(\mu/T)^2}$$

The first-order region shrinks when μ is turned on

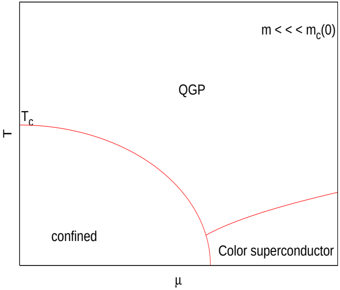
Resulting phase diagram (simplest possibility)

Standard scenario





Exotic scenario



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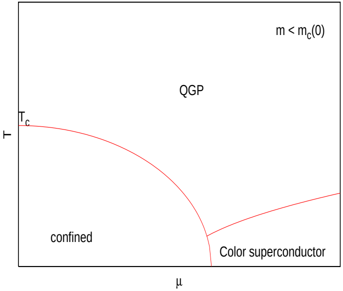

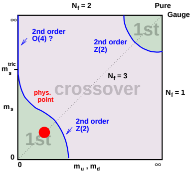

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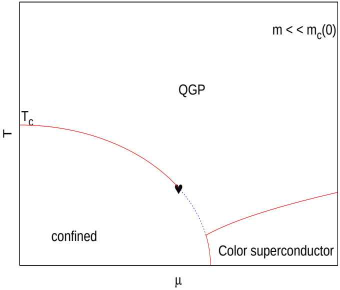

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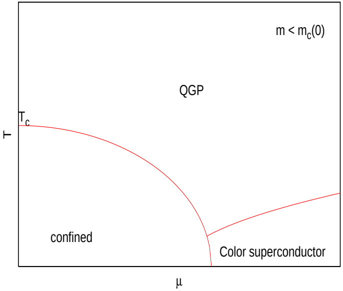

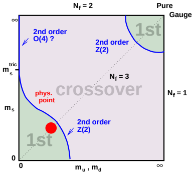

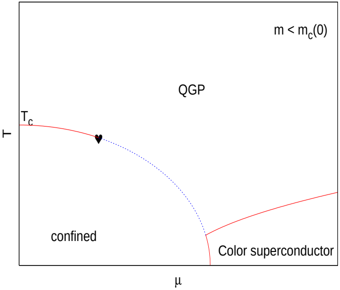

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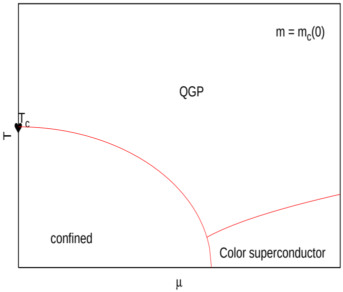

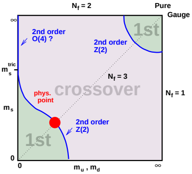

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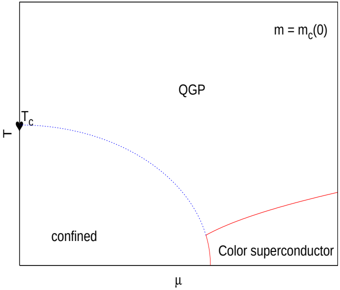

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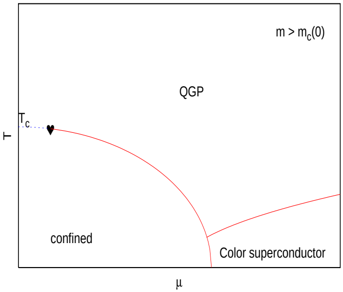

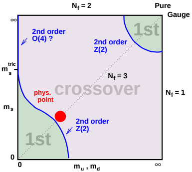

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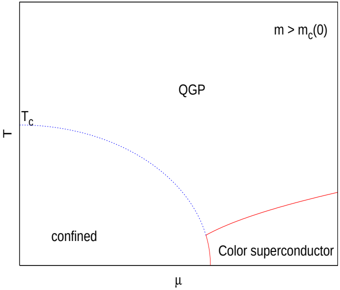

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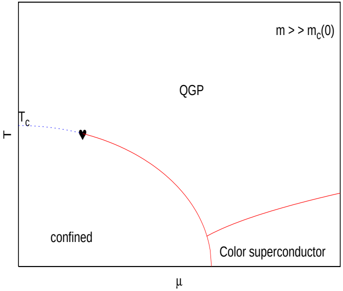

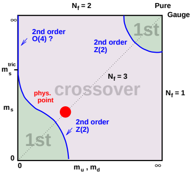

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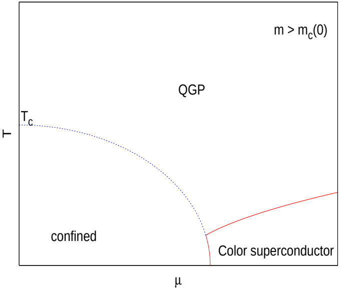

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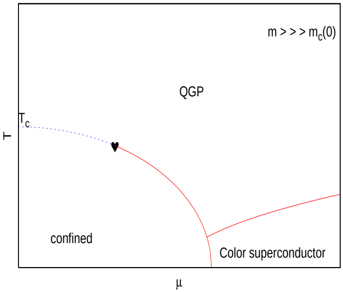

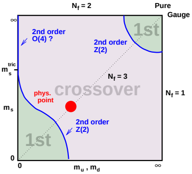

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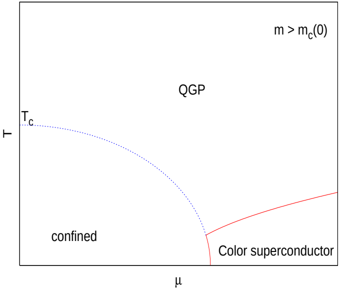
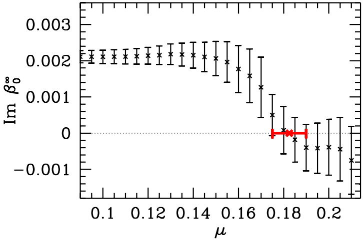


Contradiction with other lattice studies?

Caveat: at $N_f = 3$, $\mu = 0$ critical point, m_π varies by **factor** ~ 4
with Dirac discretization Bielefeld, MILC

- Fodor & Katz: $(T_E, \mu_E) = (162(2), 120(13))$ MeV ?

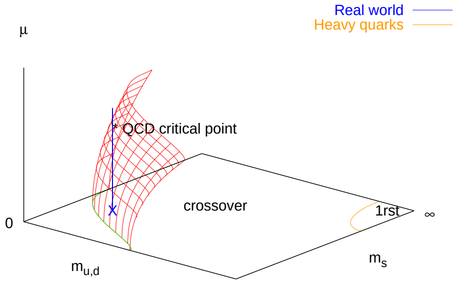


F&K keep $(m_q a)$ fixed, while $a(\beta_c)$ increases with μ
Lighter quarks at larger μ cause the phase transition
(dominant effect in our study)

- Gavai & Gupta: $\mu_E/T_E \lesssim 1$?
  cutoff effects + different theory $N_f = 2$

Conclusions

- Need finer lattice $N_t \geq 6$ for qualitative determination of phase diagram



- $\frac{m_c(\mu)}{m_c(\mu=0)} = 1 + \mathcal{O}(1)(\frac{\mu}{\pi T})^2 \rightarrow$ critical surface almost vertical

  - high sensitivity to quark masses (e.g. $m_u \neq m_d$)

  - $\mu_E / T_E \lesssim 1$ requires fine-tuning $\rightarrow$ unnatural

- Moderate computer resources with imaginary μ approach