WHY (STAGGERED FERMIONS)$^{1/4}$

FAIL

AT FINITE DENSITY

M. Golterman, Y. Shamir, and B. Svetitsky
SF State U. and Tel Aviv U.

In a nutshell: (hep-lat/0602026)

- $\Delta[U] \equiv \mathrm{Det}\,[D(U) + m] = \prod \eta_i$. The η_i are complex. When $\mu \neq 0$ the Det is complex.
- Reweighting uses $\Delta[U]^{1/4}$. What is its phase?
  - e.g., take $\eta^{1/4}$ in cut plane $\Rightarrow$ $\Delta^{1/4}$ jumps when any η_i crosses cut.
- Cf. “ideal prescription” – no jumps. Requires very good taste symmetry $=$ very near continuum limit
- Far from continuum limit, taste symmetry is broken $\Rightarrow$ the jumps return $\Rightarrow$ enormous systematic errors

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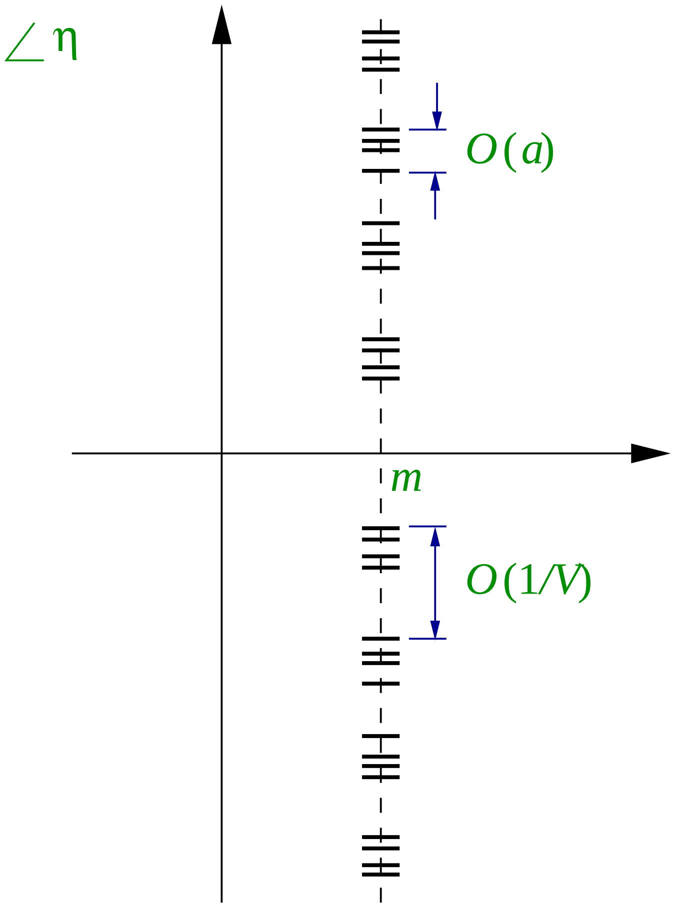
- "Far from . . ." $=$ Foreseeable future!

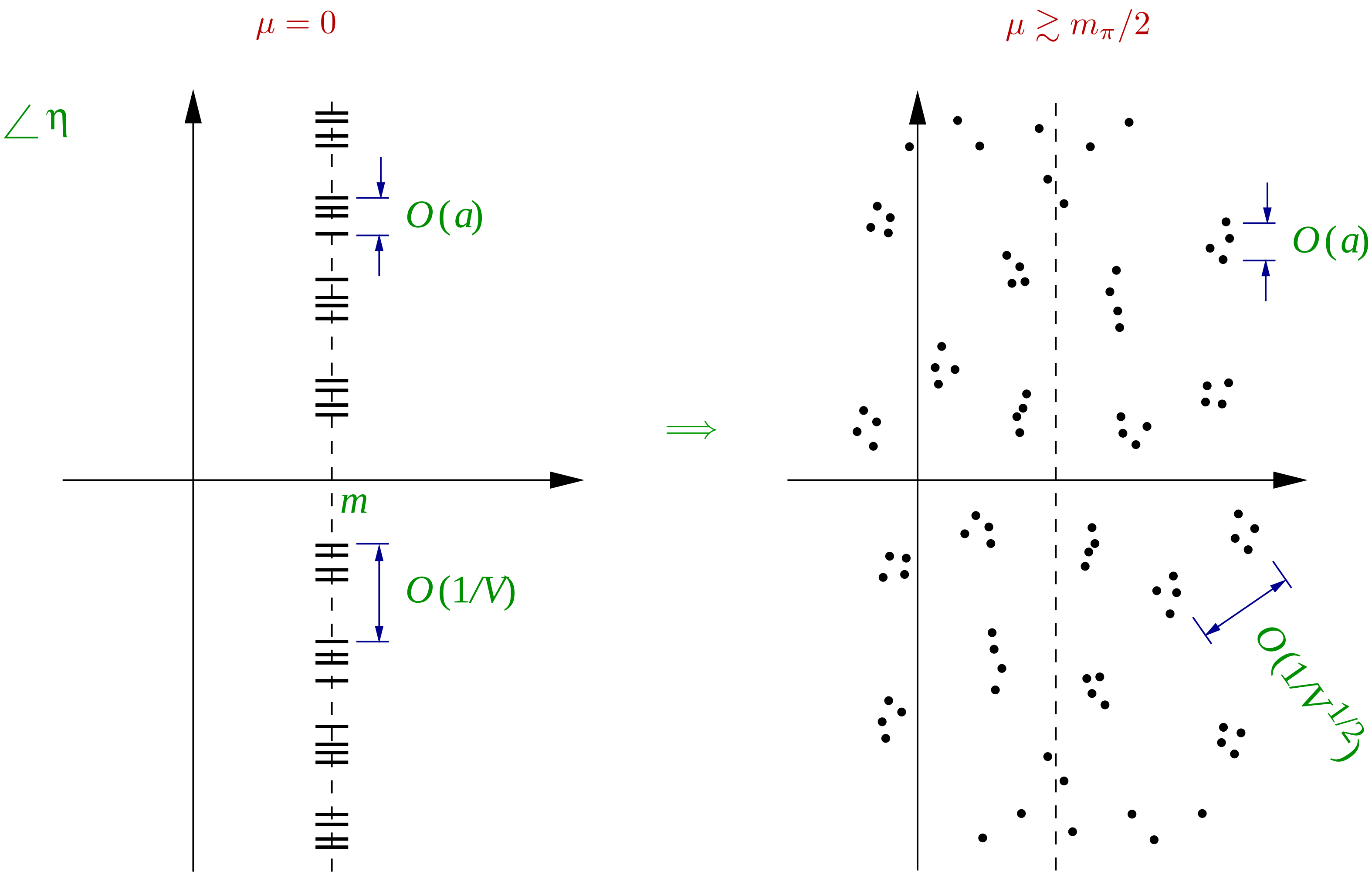
Eigenvalues of staggered $D(U) + m$

$\mu = 0$

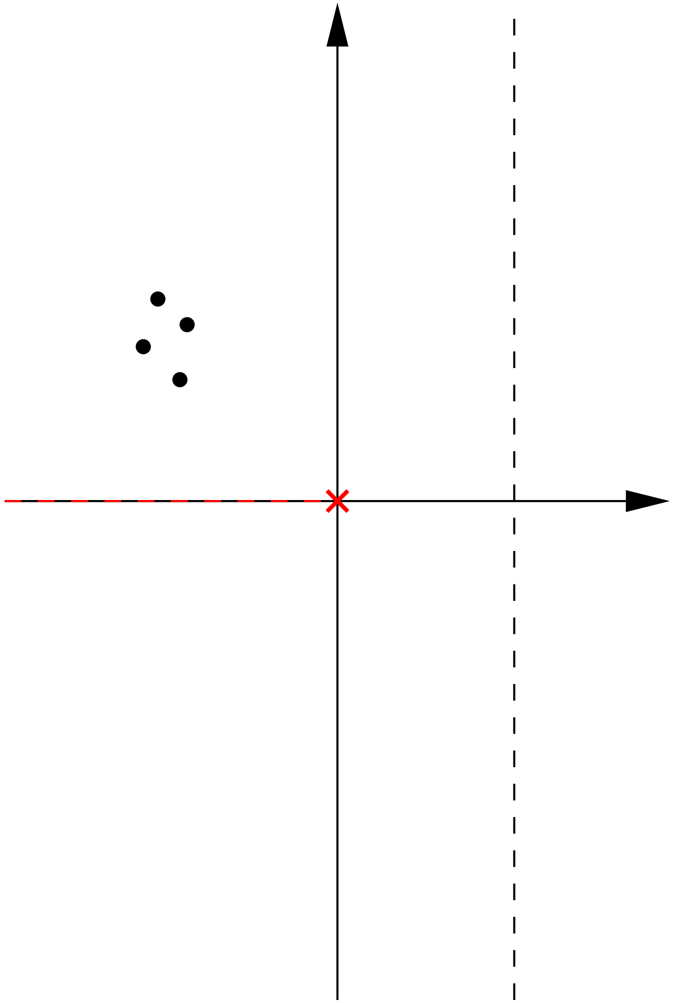


Eigenvalues of staggered $D(U) + m$

(Akemann et al. hep-th/0411030)



Fourth root of $\Delta[U] = \mathrm{Det}\,[D(U) + m]$



One quartet at a time:

$$\left(\prod_1^4 \eta_i\right)^{1/4} = \prod_1^4 \eta_i^{1/4} \quad ?$$

(“product of roots”)

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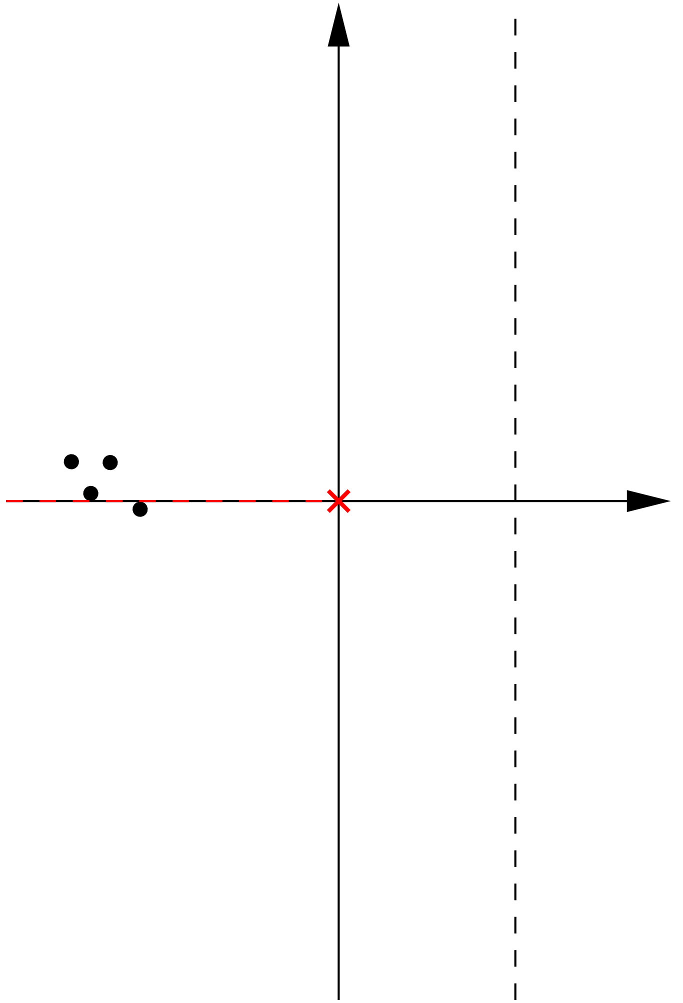

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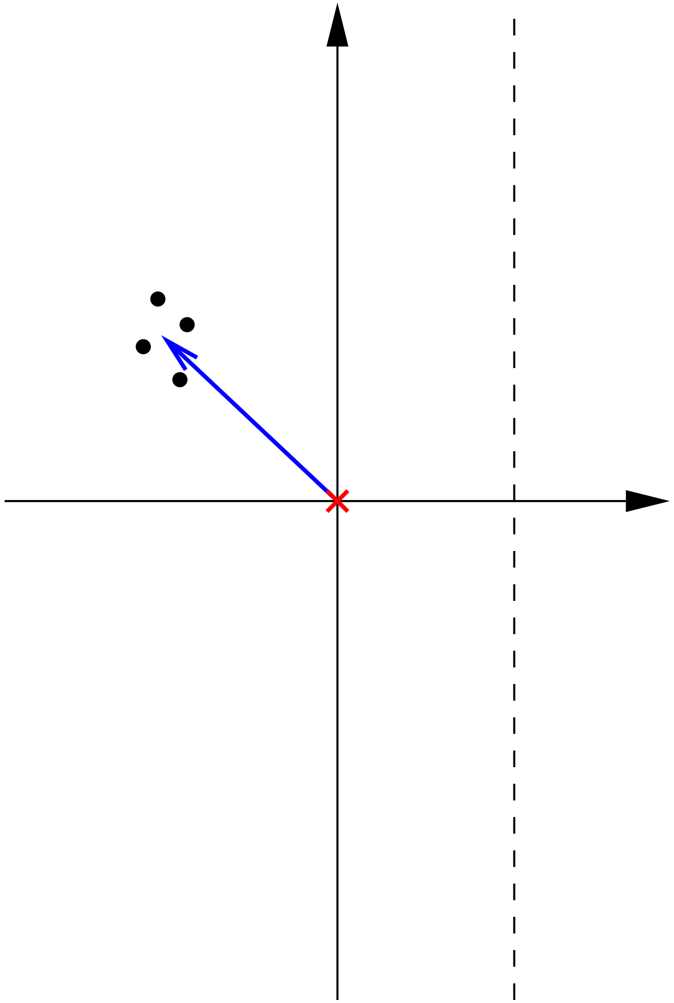
("product of roots")

Change $U \longrightarrow$ eigenvalues move.

$\Rightarrow$ jumps in phase by $\pi/2$ whenever any η_i crosses cut.

Fourth root of $\Delta[U] = \mathrm{Det}\,[D(U) + m]$



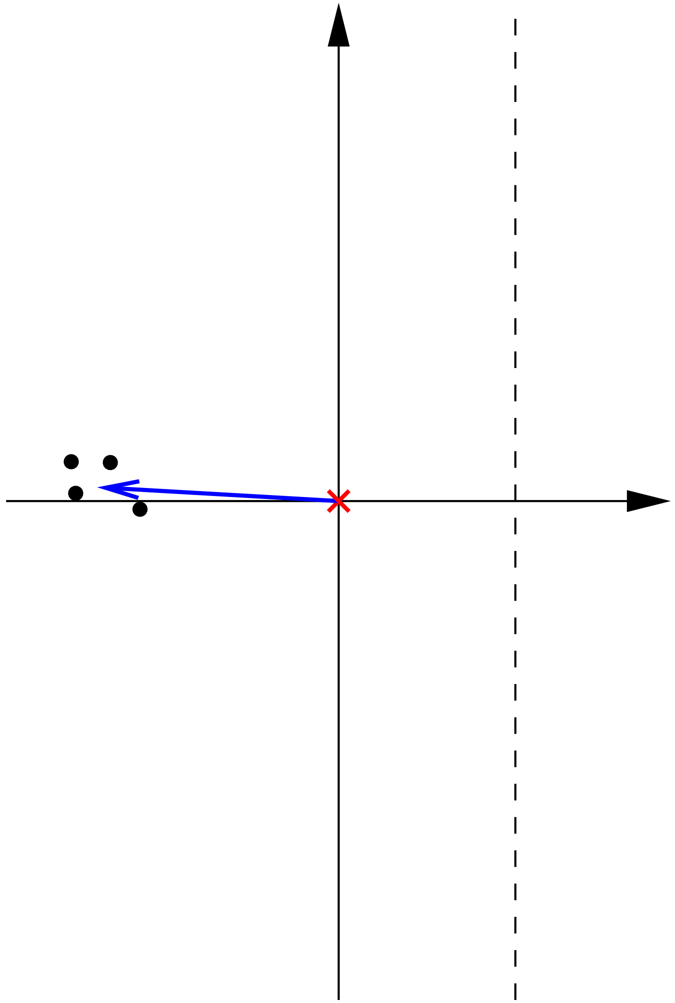
Solution ("ideal prescription"):

Choose phase of

$$\left(\prod_1^4 \eta_i\right)^{1/4}$$

to point to the center of the quartet!

Fourth root of $\Delta[U] = \mathrm{Det}\,[D(U) + m]$



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$\Rightarrow$ NO jumps in phase when any η_i crosses axis.

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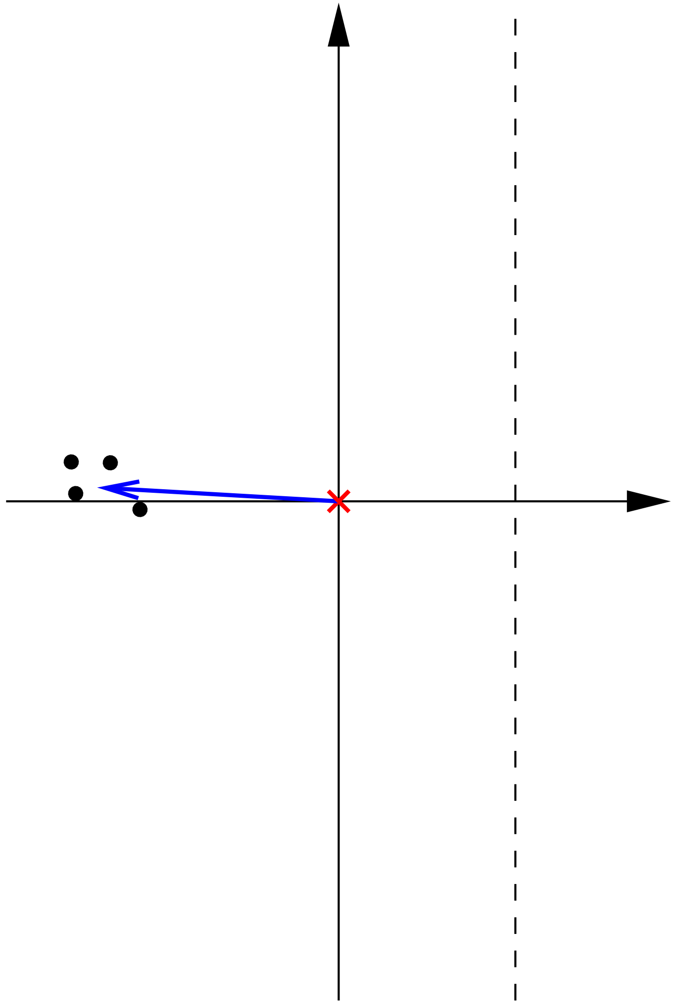

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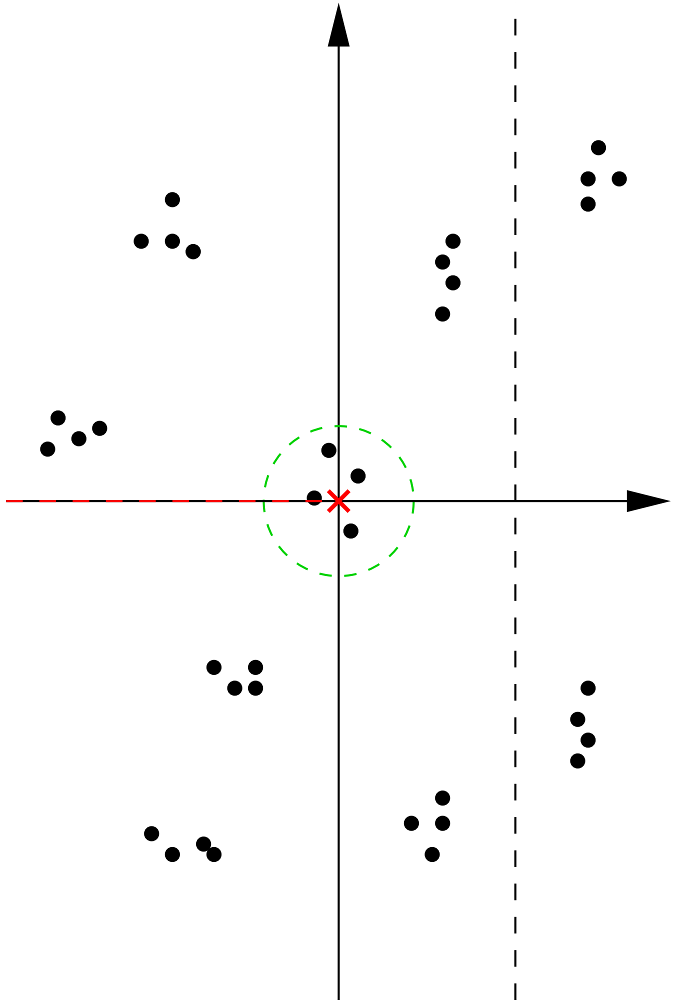
$\Rightarrow$ NO jumps in phase when any η_i crosses axis.

Smooth replacement of four tastes by one quark per flavor

Problematic configurations:



NO clear definition of phase of root!

$\Rightarrow$ Such configurations should be dropped.

Probability of prob. config:

$O(a^2 V \Lambda^6)$	quenched
$O[(a\sqrt{V}\Lambda^3)^3]$	reweighted

$=$ systematic error of algorithm

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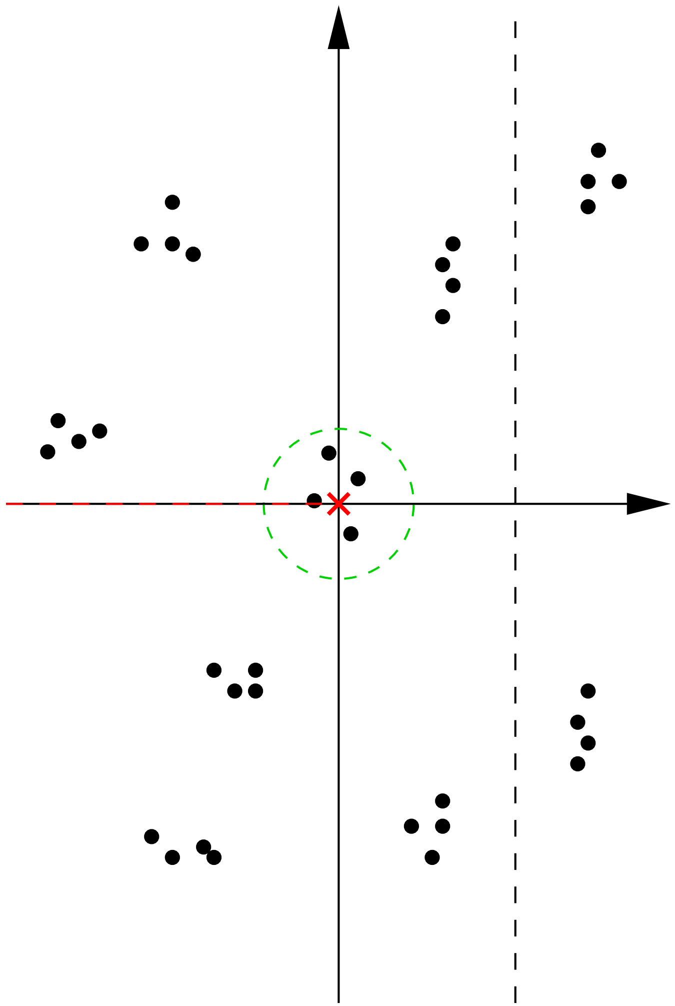

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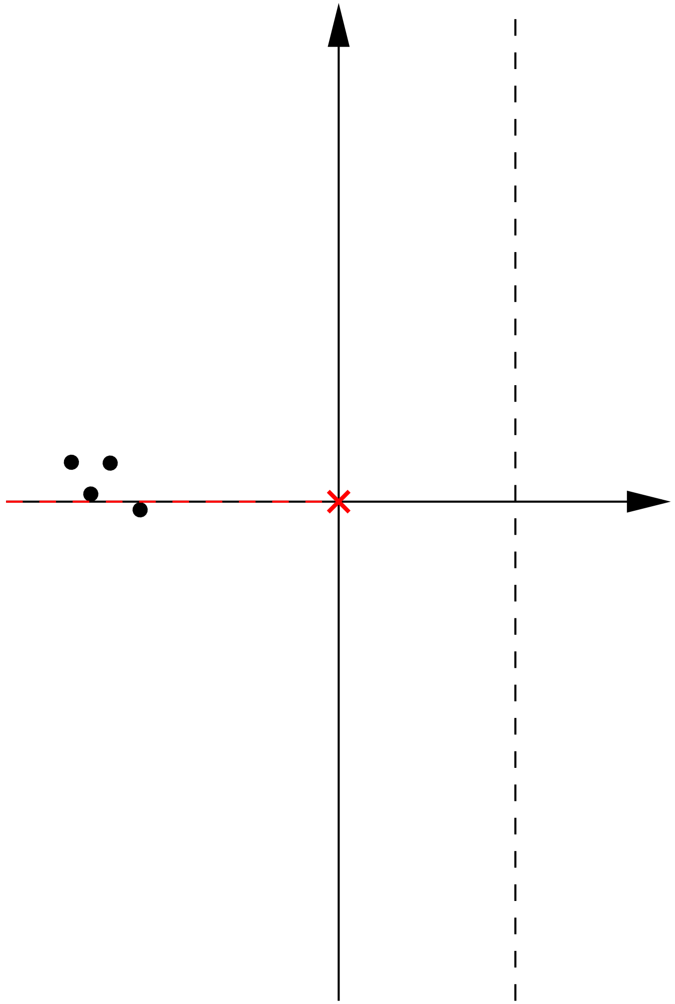
Note:

Volume required is fixed by physics (e.g., $m_\pi L \gtrsim 3$).
We must take $a \to 0$ before $V \to \infty$.

- similar to requirement $a \to 0$ before $m \to 0$ (`Bernard hep-lat/0412030`)

So much for the "ideal prescription." **How bad can things get?**



"Product of roots":

$$\left(\prod_1^4 \eta_i\right)^{1/4} = \prod_1^4 \eta_i^{1/4}$$

We should drop configs where quartets enter "exclusion zone" of width $\sim a$ along the cut.

Systematic error $= O(aV\Lambda^5)$

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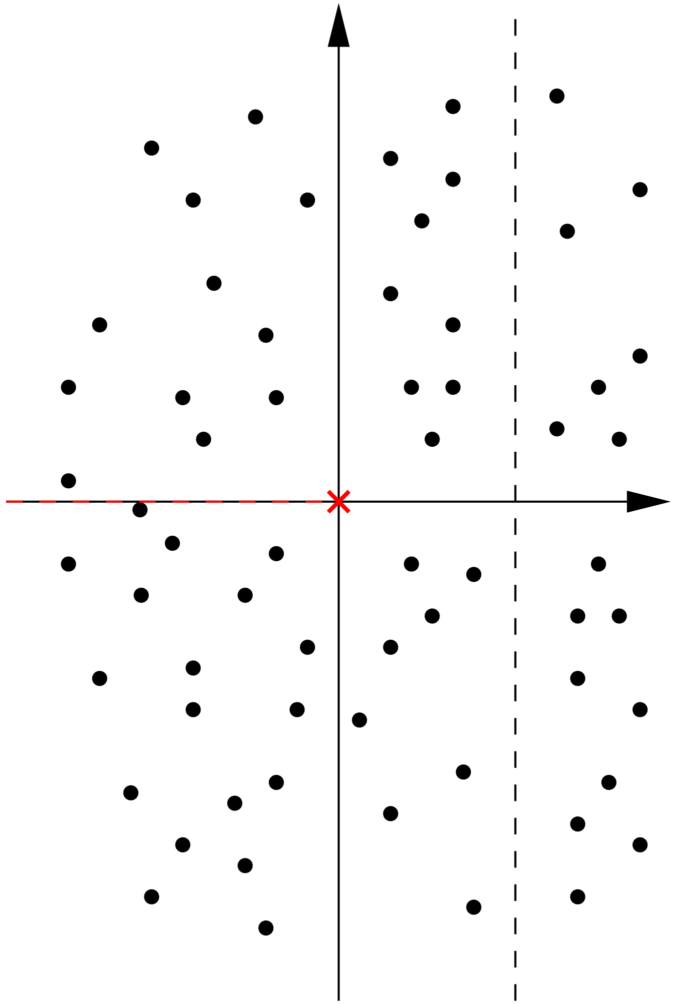

Far from continuum — NO quartets:

Phase flips by $\pi/2$ at random

Systematic error $=$???

Where are we today?

($\mu > m_\pi/2 \Longrightarrow \mu_B > 210 \text{ MeV}$)

Require: $a\sqrt{V}\Lambda^3 \ll 1$ for well-separated quartets. E.g. for $a\sqrt{V}\Lambda^3 = 1/5$,

$$L \gtrsim 3m_\pi^{-1},\ T \sim T_c \quad \Longrightarrow \quad a^{-1} > 4 \text{ GeV}$$

Cf. large calculation: $T \simeq \Lambda$, $Ta = 1/N_t = 1/4$ (so $a^{-1} \sim 800 \text{ MeV}$), $V\Lambda^4 \simeq 27$ (Fodor & Katz hep-lat/0402006)

$a\sqrt{V}\Lambda^3 \simeq 1.3 \quad \Longrightarrow$ No quartets. Also: $\prod \xi_i$ vs. $\prod \eta_i \quad \Longrightarrow$ No quartets.

Estimate systematic errors as if there were quartets:

“Ideal prescription” $\left(a\sqrt{V}\Lambda^3\right)^3 \simeq 220\%$

“Product of roots” $aV\Lambda^5 \simeq 600\%$

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Summary:

- $\Delta[U] \equiv \mathrm{Det}\,[D(U)+m] = \prod \eta_i$. The η_i are complex. When $\mu \neq 0$ the Det is complex.
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