

# Analysis of the finite temperature transition for 3 flavor QCD using p4-improved staggered fermions

Michael Cheng for the RBC-Bielefeld Collaboration

`michaelc@phys.columbia.edu`

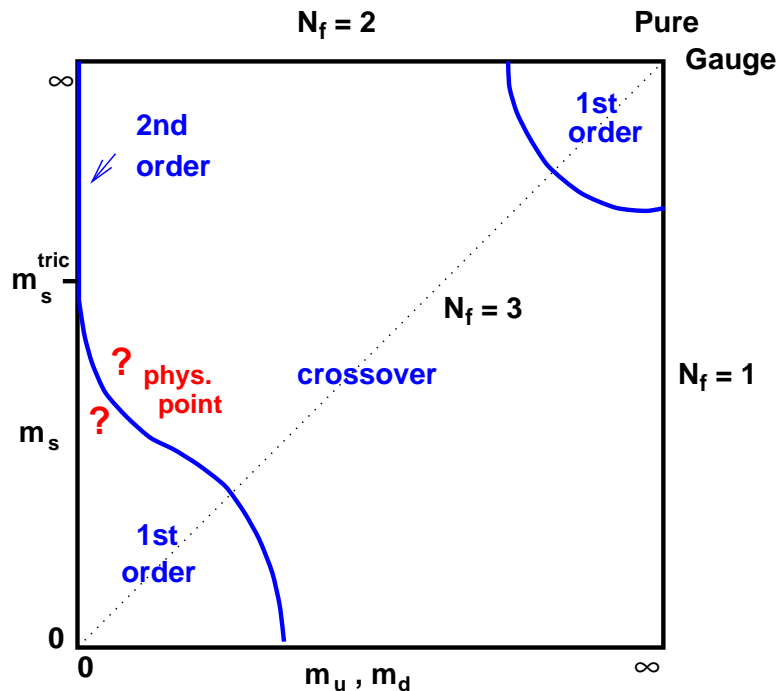
Columbia University

# Overview

We present a study of the phase transition of 3 flavor QCD for  $N_t = 4, 6$  with two different variants of p4-improved staggered fermions

- QCD thermodynamics
- p4 fermion action and link fattening
- Calculation details
- Locating  $\beta_c$  - observables and susceptibilities
- Zero temperature scale setting using  $r_0$
- “Physical” value for  $T_c$
- Scaling between  $N_t = 4$  and  $N_t = 6$

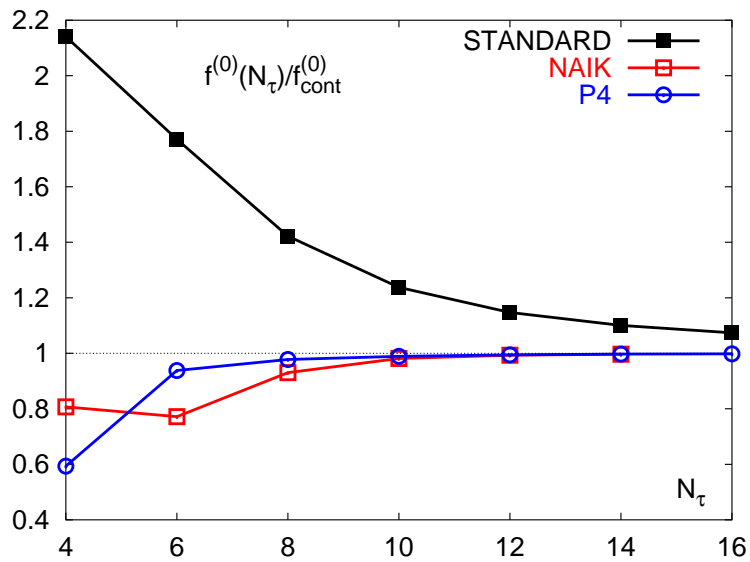
# QCD Thermodynamics



- Strongly interacting matter changes character immensely at high temperature
- Quarks become deconfined, forming a Quark-Gluon Plasma (QGP)
- Quantitative information about phase transition relevant for heavy ion collisions

- Study 3 flavor theory as preliminary to more realistic 2+1f calculation
- Study scaling of  $T_c$  between  $N_t = 4$  and  $N_t = 6$
- Gain information about location of the 3f critical point.

# P4 Action



- p4 action is variant of staggered fermions
- Improves rotational invariance of the quark propagator up to  $\mathcal{O}(p^4)$
- Better approach to continuum limit for thermodynamic observables in free field case.

● Fermion matrix is given by ( $c_1 = \frac{3}{4}$ ,  $c_{1,2} = \frac{1}{48}$  at tree-level):

$$M[U]_{ij} = \eta_i \left( c_1 \overleftrightarrow{j \leftarrow i j} + c_{1,2} \left[ \begin{array}{c} j \\ \uparrow \\ \downarrow \\ i \\ \downarrow \\ j \end{array} + \begin{array}{c} j \\ \leftarrow \\ \uparrow \\ i \\ \downarrow \\ j \end{array} + \begin{array}{c} j \\ \uparrow \\ \downarrow \\ i \\ \leftarrow \\ j \end{array} + \begin{array}{c} j \\ \leftarrow \\ \uparrow \\ i \\ \leftarrow \\ j \end{array} \right] \right)$$

# Link Fattening

- Staggered fermions exhibit flavor-symmetry breaking in meson spectrum
- Dynamics of transition dominated by light mesons  $\rightarrow$  reduce  $\mathcal{O}(a^2)$  errors by introducing link fattening.
- One link term can be modified like:

$$c_1 \longrightarrow + c_3 \Sigma \left[ \begin{array}{c} \text{---} \text{---} \text{---} \\ \uparrow \quad \downarrow \\ \text{---} \end{array} \right] + c_5 \Sigma \left[ \begin{array}{c} \text{---} \text{---} \text{---} \\ \nearrow \quad \searrow \\ \downarrow \end{array} \right] + c_7 \Sigma \left[ \begin{array}{c} \text{---} \text{---} \text{---} \\ \nearrow \quad \searrow \\ \downarrow \end{array} \right]$$

- **p4fat7**: Parameters set by cancelling tree-level flavor symmetry violations
- **p4fat3**: Matches old p4 action used for 3f  $N_t = 4$  calculation ( $\omega = 0.2$ )

Action	$c_1$	$c_3$	$c_5$	$c_7$	$c_{1,2}$
<b>p4fat7</b>	$\frac{1}{8} - \frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$	$\frac{1}{384}$	$\frac{1}{48}$
<b>p4fat3</b>	$\frac{3}{4} \frac{1}{1+6\omega}$	$\frac{3}{4} \frac{\omega}{1+6\omega}$	0	0	$\frac{1}{48}$

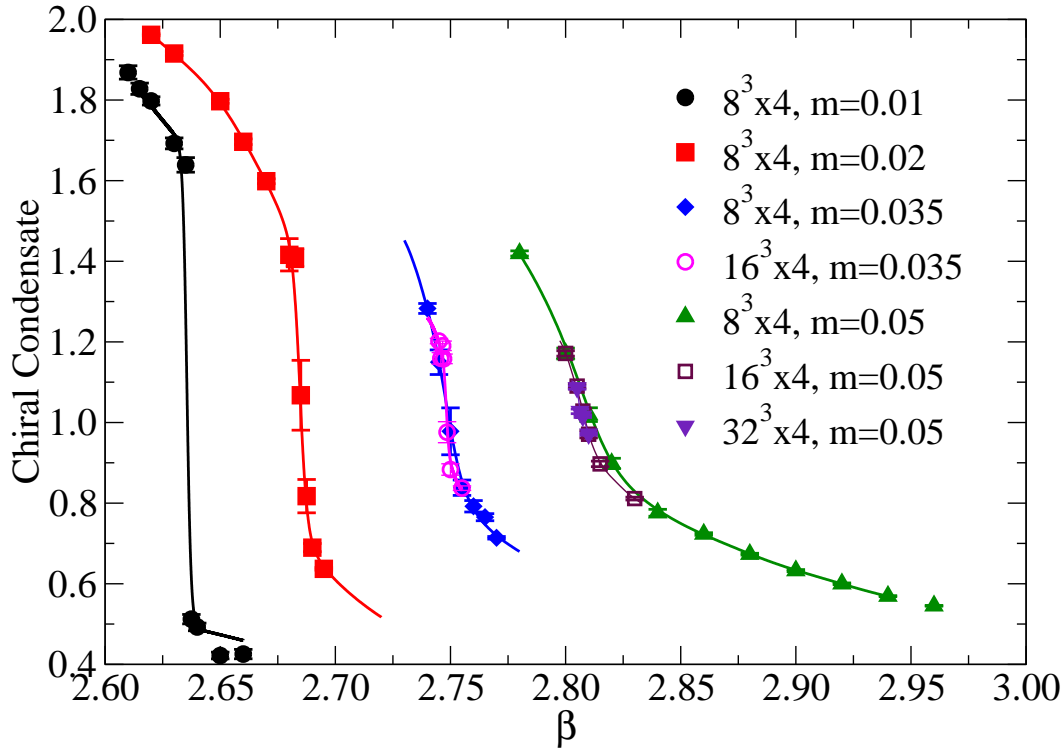
# Calculation Details

- Tree-level  $\mathcal{O}(a^2)$  improved Symanzik gauge action used - includes plaquette and planar rectangle terms
- Hybrid R algorithm used for evolutions,  $\delta t = 0.4m_q$
- Plaquette, rectangle, chiral condensate, and Polyakov loop measured on finite temperature lattices. Meson spectrum and Wilson Loop correlators measured on zero temperature lattices.

Action	$N_t$	Volumes	$m_q a$	# $\beta$
<b>p4fat7</b>	<b>4</b>	$8^3, 16^3, 32^3$	.01, .02, .035, .05, .1, .2	<b>72</b>
<b>p4fat7</b>	<b>6</b>	$16^3, 32^3$	.01, .02, .05, .2	<b>61</b>
<b>p4fat7</b>	<b>32</b>	$16^3$	.01, .02, .035, .05, .1, .2, .01,.02,.05	<b>9</b>
<b>p4fat3</b>	<b>4</b>	$8^3, 12^3, 16^3$	.005, .01, .025, .05, .1, .2	<b>BI</b>
<b>p4fat3</b>	<b>6</b>	$16^3$	.01, .02, .05, .1	<b>49</b>
<b>p4fat3</b>	<b>32</b>	$16^3$	.005, .01, .05, .01, .02	<b>5</b>

# Measurements

Chiral Condensate for p4fat7,  $N_t = 4$

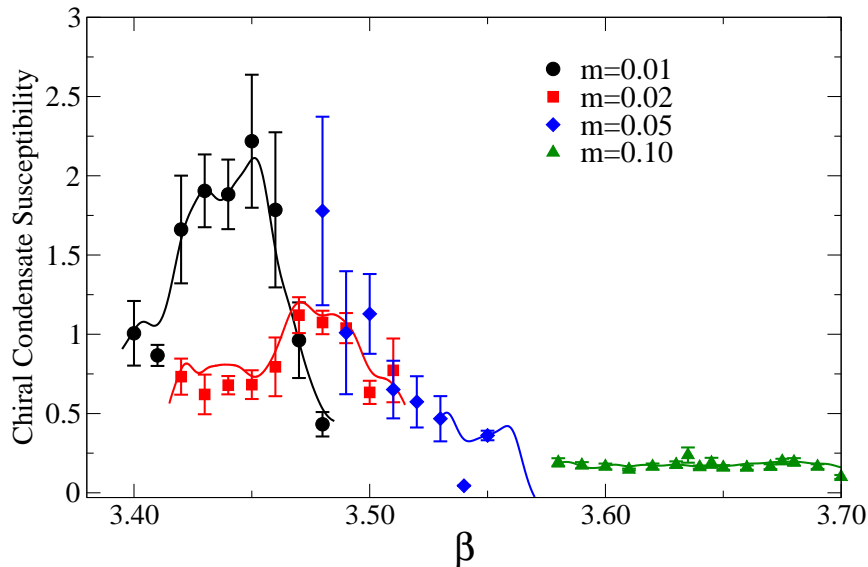


- $\bar{\psi}\psi$  (chiral order parameter) measured every 10 trajectories on finite temperature lattices with 10 random vectors
- $\bar{\psi}\psi$  for **p4fat7** shows good volume scaling

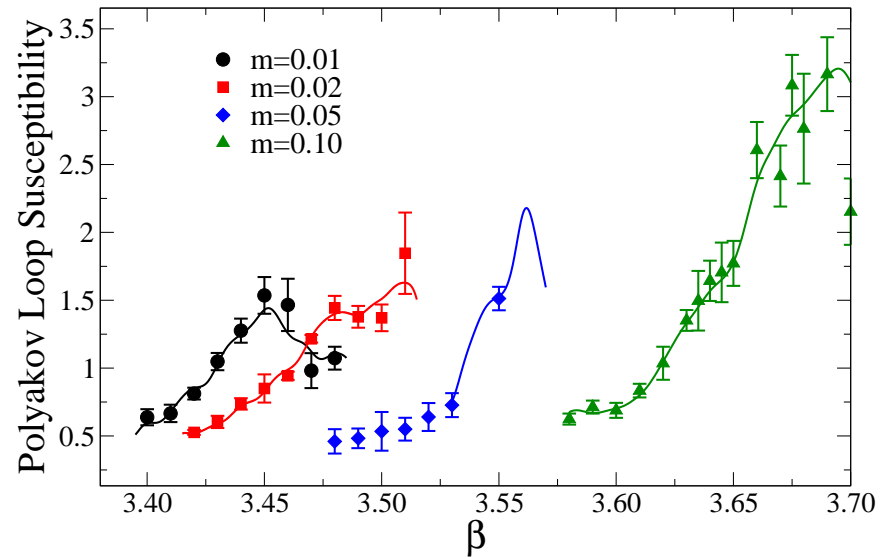
- **p4fat7** - Transition appears to be discontinuous (first order).

# Susceptibilities

Chiral Condensate Susceptibility for  $N_t = 6$



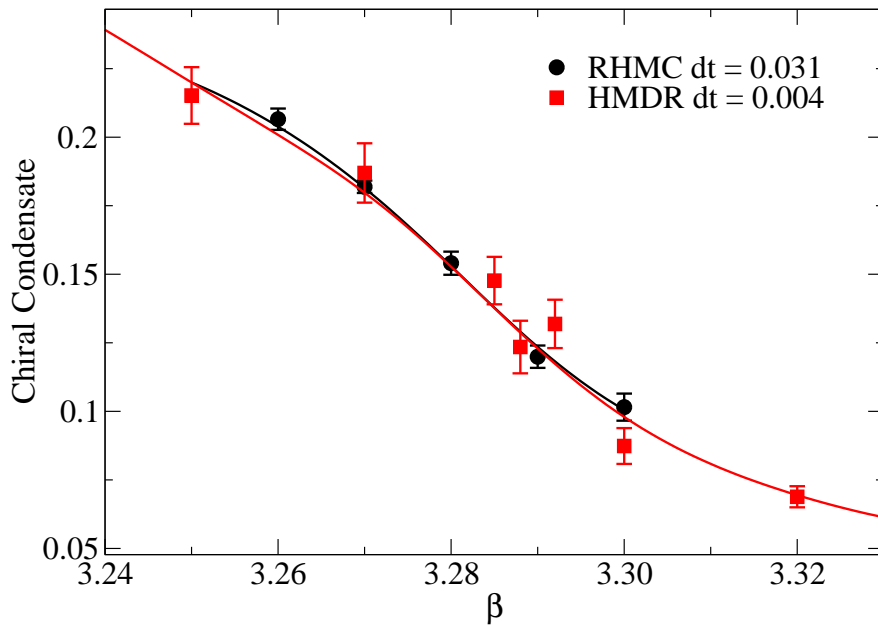
Polyakov Loop Susceptibility for  $N_t = 6$



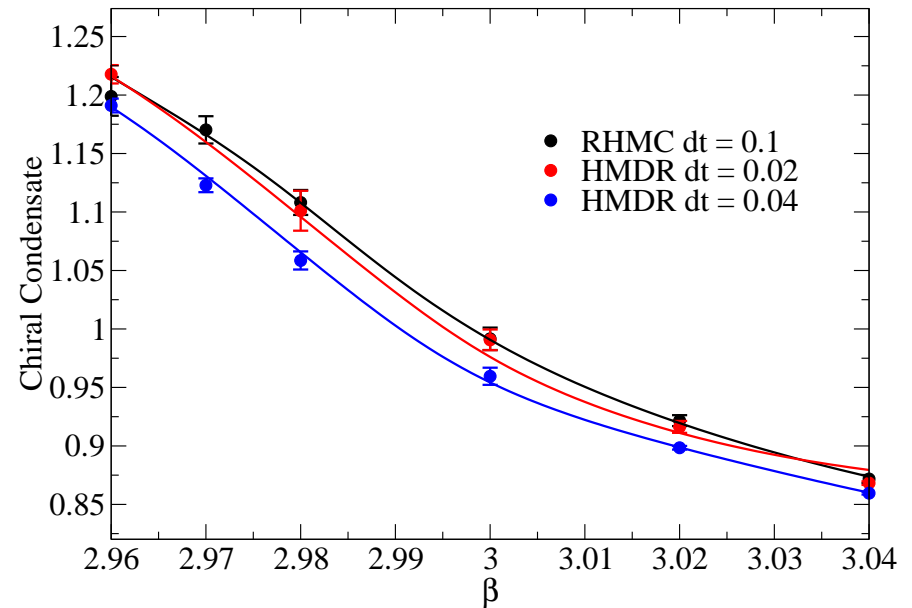
- Transition point determined by locating peak in susceptibilities:  $\chi_{\bar{\psi}\psi}$ ,  $\chi_L$ ,  $\chi_{plaq}$  with Ferrenberg-Swendsen reweighting
- **p4fat3** shows indications of smooth crossover
- **p4fat7** shows volume scaling indicative of a discontinuous 1st order transition up to rather large values of  $m_\pi$

# Comparison of HMDR with RHMC

RHMC vs. HMDR, p4fat3  $m_q=0.01, 8^3 \times 4$

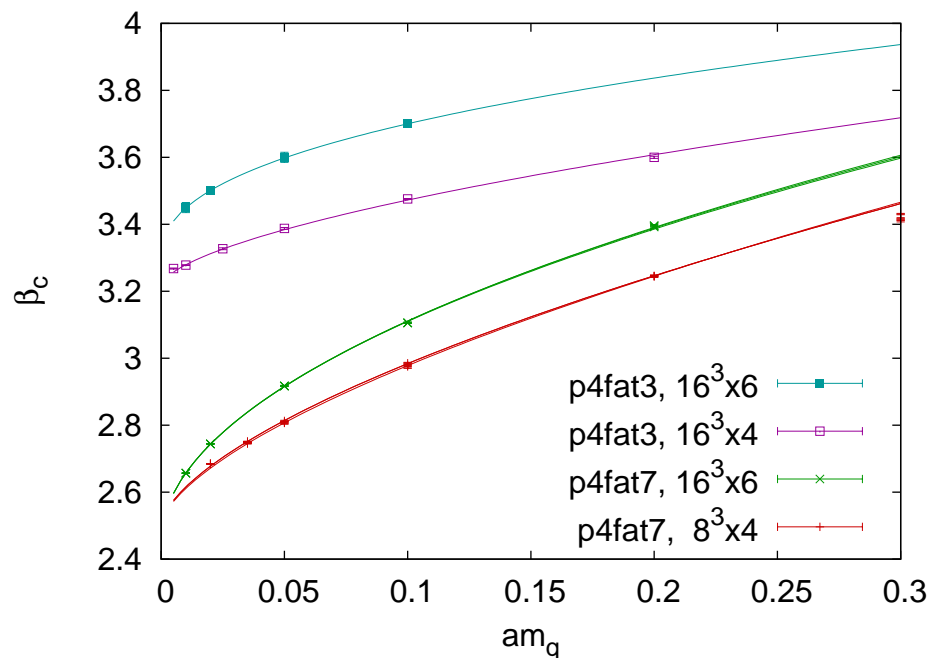


RHMC vs. HMDR, p4fat7,  $m_q=0.1, 8^3 \times 4$



- **p4fat3** on left shows good agreement, but only because HMDR uses  $8\times$  smaller step size ( $\delta t = 0.004$ ) compared to RHMC ( $\delta t = 0.031$ )
- **p4fat7** shows noticeable difference for HMDR ( $\delta t = 0.04, 0.02$ ) compared to RHMC ( $\delta t = 0.1$ )

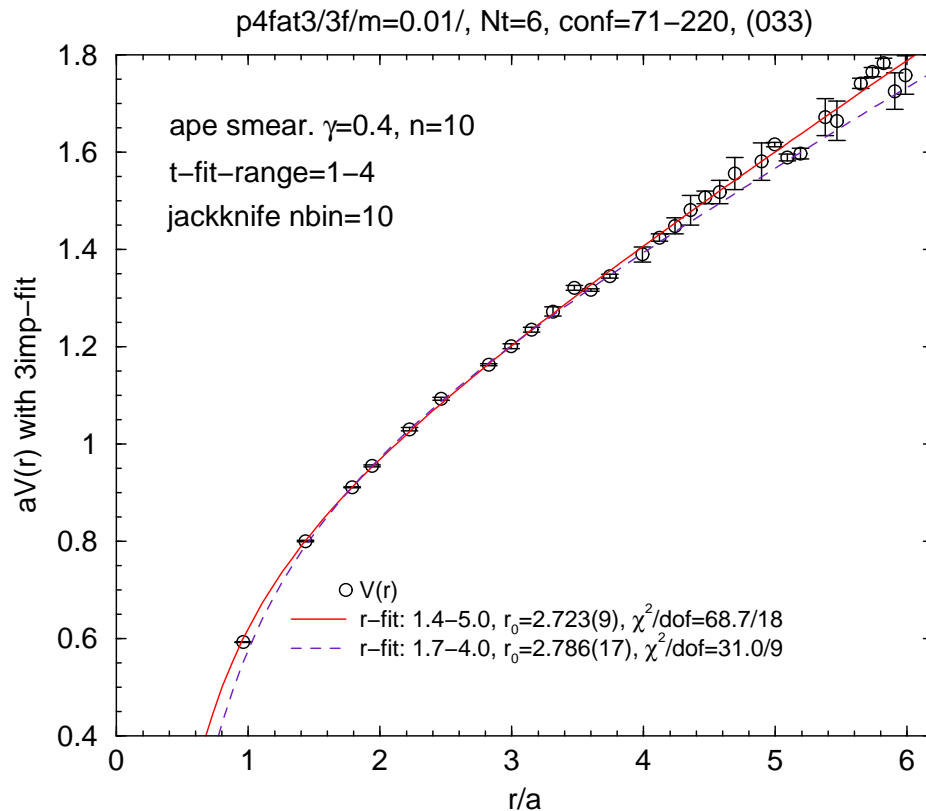
# $\beta_c$ vs. $m_q$



- For **p4fat7**, as  $m_q \rightarrow 0$ ,  $\beta_c(N_t = 4) \approx \beta_c(N_t = 6)$
- Appears to be first order even for large values of  $m_\pi$
- Indications of unphysical, first-order bulk phase transition for **p4fat7** interfering with finite temperature transition?

- $\beta_c$  for **p4fat3** indicates a smooth crossover transition down to relatively low values of  $m_\pi \rightarrow$  consistent with past studies.
- **p4fat3** scales as expected between  $N_t = 4$  and  $N_t = 6$

# Scale Setting - Heavy Quark Potential



Fit of  $V(r)$  for **p4fat3**,  $N_t = 6$

Lattice spacing  $a$  may be determined by short distance scale parameter  $r_0$ , known from lattice calculations ( $r_0 = 0.469(7)$  fm.).  $r_0$  defined by:

$$r^2 \frac{dV(r)}{dr} \Big|_{r=r_0} = 1.65$$

Heavy quark potential determined using Wilson loop correlators with APE smearing. 3-parameter fit to the ansatz:

$$V(r) = -\frac{\alpha}{r} + \sigma r + c$$

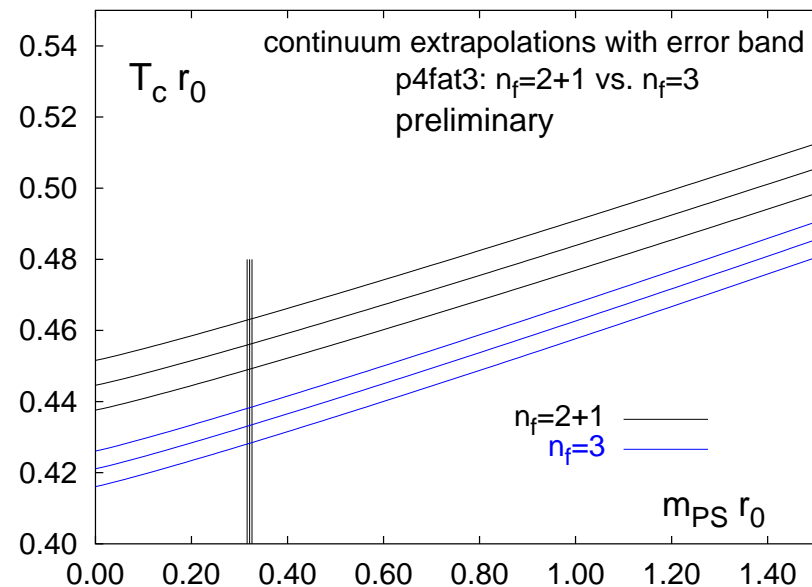
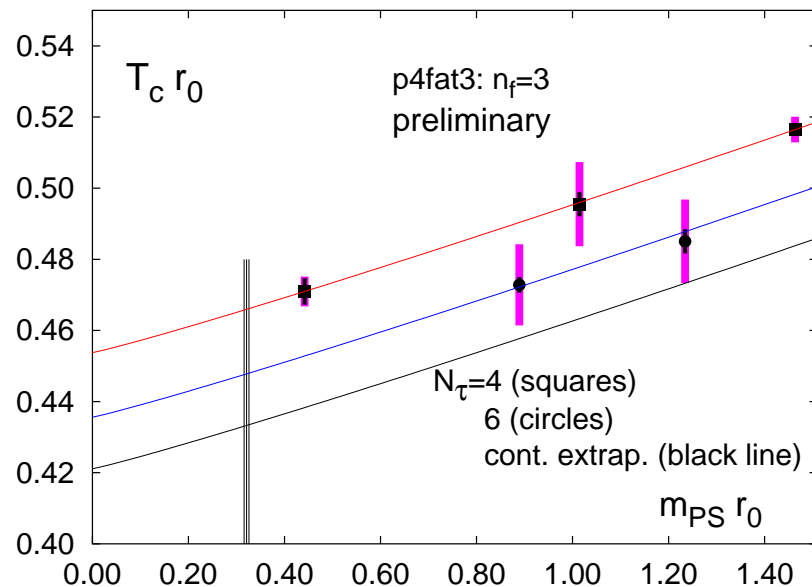
# Results for $T_c$

- Determination of the lattice spacing  $a$  via  $r_0$  allows us to express  $T_c$  in “physical” units via  $T = 1/N_t a$

Action	$N_t$	$m_\pi r_0$	$\beta_c$	$T_c r_0$
p4fat3	4	.4419	3.268(2)	.4719(42)
p4fat3	4	1.0144	3.327(10)	.4955(118)
p4fat3	4	1.4635	3.388(3)	.5165(32)
p4fat3	6	.8890	3.45(1)	.4728(114)
p4fat3	6	1.2344	3.48(1)	.4851(117)

- $\beta_c$  currently determined from  $\chi_{\bar{\psi}\psi}$
- $\Delta\beta_c = 0.01 \rightarrow 3\%$  error in scale determination.

# Results for $T_c$ (cont.)



- Simultaneous chiral, continuum fit to ansatz:  

$$T_c r_0(m_\pi r_0, N_t) = T_c r_0(0, 0) + A(m_\pi r_0)^d + B/N_t^2$$
- **p4fat3** yields a continuum extrapolated value  $T_c r_0 = 0.433(5)_{-1}^{+12}$  at the physical pion mass,  $m_\pi r_0 = 0.321(5)$
- $T_c r_0$  is  $\sim 5\%$  lower for 3f compared to 2+1f, in line with previous studies.

# Conclusions

- We have extended the existing 3f calculations of the transition temperature that use the p4 action by simulating with  $N_t = 6$ .
- More accurate determination of scale for  $N_t = 4, N_t = 6$ .
- Using **p4fat3**, we found  $T_c r_0 = 0.433(5)_{-1}^{+12}$  at the physical value of  $m_\pi$ . The “transition” seems to be a smooth crossover down to the lightest quark masses used, corresponding to  $m_\pi \approx 150 MeV$ .
- $T_c r_0$  for 3 flavors  $\sim 5\%$  less than for 2+1 flavors
- Suspect that **p4fat7** transition is distorted by unphysical bulk phase transition related to fat-link smearing. Currently under investigation.
- See also our 2+1f calculation (presented by Takashi Umeda), which improves upon this calculation by using the RHMC algorithm, as well as nearly physical values for the quark masses