Static quark free energies at finite temperature with two flavors of improved Wilson quarks

Tsukuba-Tokyo collaboration

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Motivations

- **Heavy quark free energy in hot matter**

  1. Channel dependence of "potential" (1c, 8c, $\bar{3}$c, 6c)
  2. Effective running coupling at $T \neq 0$
  3. Debye screening mass at $T \neq 0$
  4. Relation to p-QCD at high T

- **Full-QCD lattice simulation**

  We use improved Wilson fermion action.

  1. not many works
  2. Comparison with staggered fermion action

Free energy and Potential on a lattice

✓ Quark- antiquark "potential" (normalized free energy)

$$\frac{V(r,T)}{T} = -\ln \frac{\left\langle \mathrm{Tr}\,\Omega^{\dagger}(\vec{x})\,\mathrm{Tr}\,\Omega(\vec{0})\right\rangle}{\left\langle \mathrm{Tr}\,\Omega\right\rangle^{2}}$$

Polyakov loop

$$\mathrm{Tr}\,\Omega(\vec{x}) = \mathrm{Tr}\prod_{\tau=0}^{N_t-1} U_4(\tau,\vec{x})$$

$$\Longrightarrow \quad V(r,T)\big|_{r\to\infty} = 0: \text{ normalization}$$

$$\left\langle \mathrm{Tr}\,\Omega^{\dagger}(\vec{x})\,\mathrm{Tr}\,\Omega(\vec{0})\right\rangle \xrightarrow{|\vec{x}|\to\infty} \left\langle \mathrm{Tr}\,\Omega^{\dagger}(\vec{x})\right\rangle\left\langle \mathrm{Tr}\,\Omega(\vec{0})\right\rangle$$

✓ Separation to each channel after **Coulomb gauge fixing**

$$3_c \otimes \bar{3}_c = 1_c \oplus 8_c \quad \Longrightarrow \quad e^{-V_{\bar{Q}Q}(r,T)/T} = \frac{1}{9}e^{-V_1/T} + \frac{8}{9}e^{-V_8/T}$$

$$3_c \otimes 3_c = \bar{3}_c \oplus 6_c \quad \Longrightarrow \quad e^{-V_{QQ}(r,T)/T} = \frac{1}{3}e^{-V_{\bar{3}}/T} + \frac{2}{3}e^{-V_6/T}$$

Numerical simulations

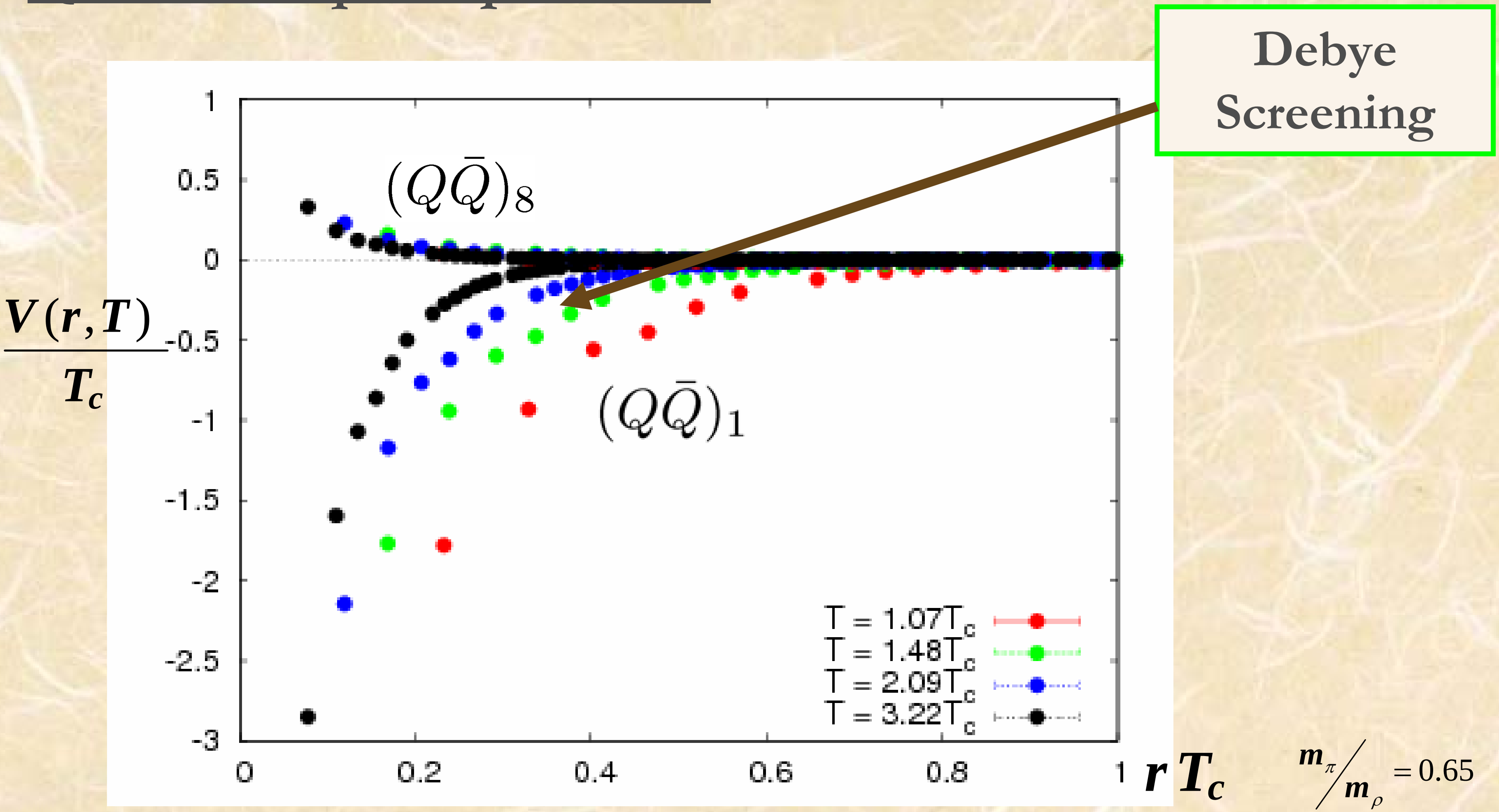
Parameters

- ✓ Lattice size: $N_s^3 \times N_t = 16^3 \times 4$, $T = \dfrac{1}{N_t a}$
- ✓ Gauge action: RG Iwasaki improved action
- ✓ Fermion action: Clover improved Wilson action (2-flavor)
- ✓ Quark mass & Temperature

  $m_\pi / m_\rho = 0.65: \ T = 1.0T_c \sim 3.2T_c$ (9 points)

  $m_\pi / m_\rho = 0.80: \ T = 1.0T_c \sim 2.5T_c$ (6 points)
- ✓ # of Configurations: 500 confs. (5000 traj.)
- ✓ Gauge fixing: Coulomb gauge

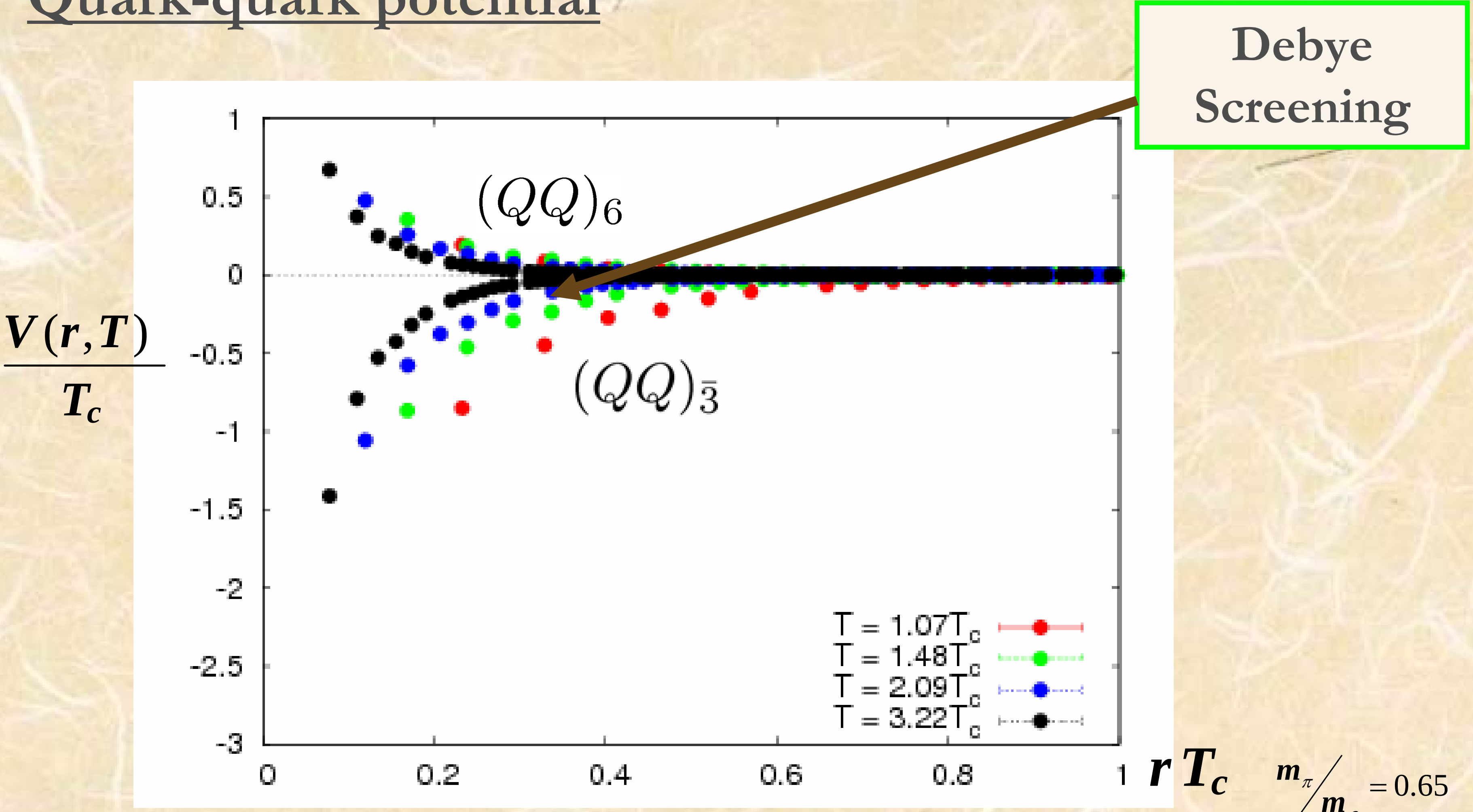
Quark-antiquark potential



1_c channel: attractive force

8_c channel: repulsive force

Quark-quark potential



$m_\pi / m_\rho = 0.65$

$\bar{3}_c$ channel: attractive force

6_c channel: repulsive force

Screening effect

Phenomenological potential

$$V(r,T) = C(M)\frac{\alpha(T)}{r}e^{-m_D(T)r}$$: screened Coulomb potential

$$C(M) = \left\langle \sum_{a=1}^{8} t_1^a \cdot t_2^a \right\rangle_M$$: Casimir factor

$$C(1) = -\frac{4}{3}, C(8) = \frac{1}{6}, C(\bar{3}) = -\frac{2}{3}, C(6) = \frac{1}{3}$$

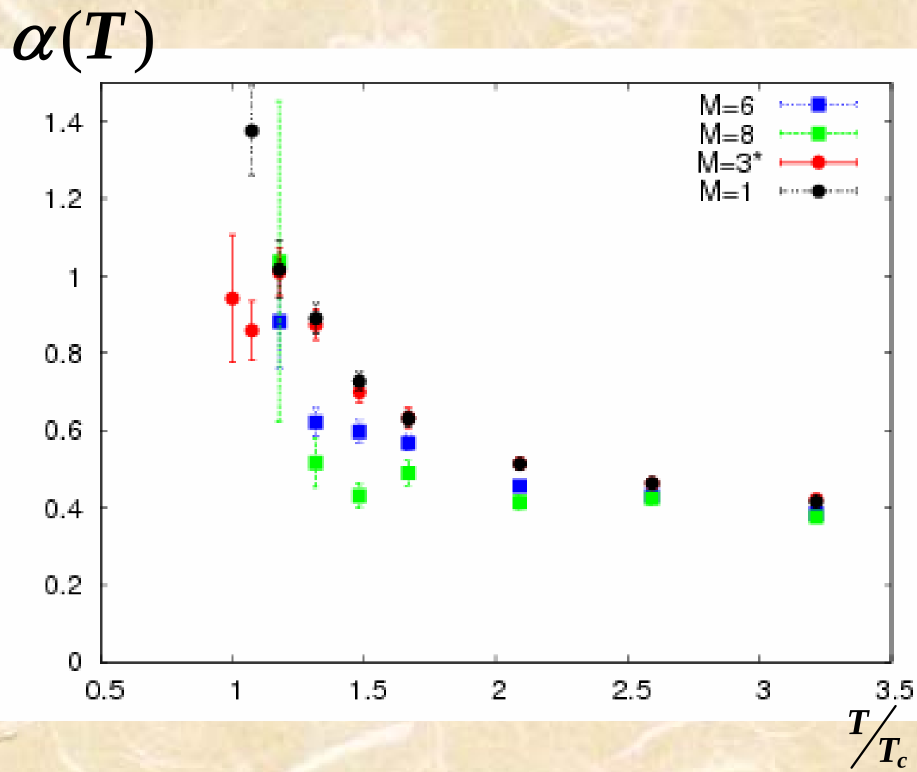
$\alpha(T)$: "effective" running coupling

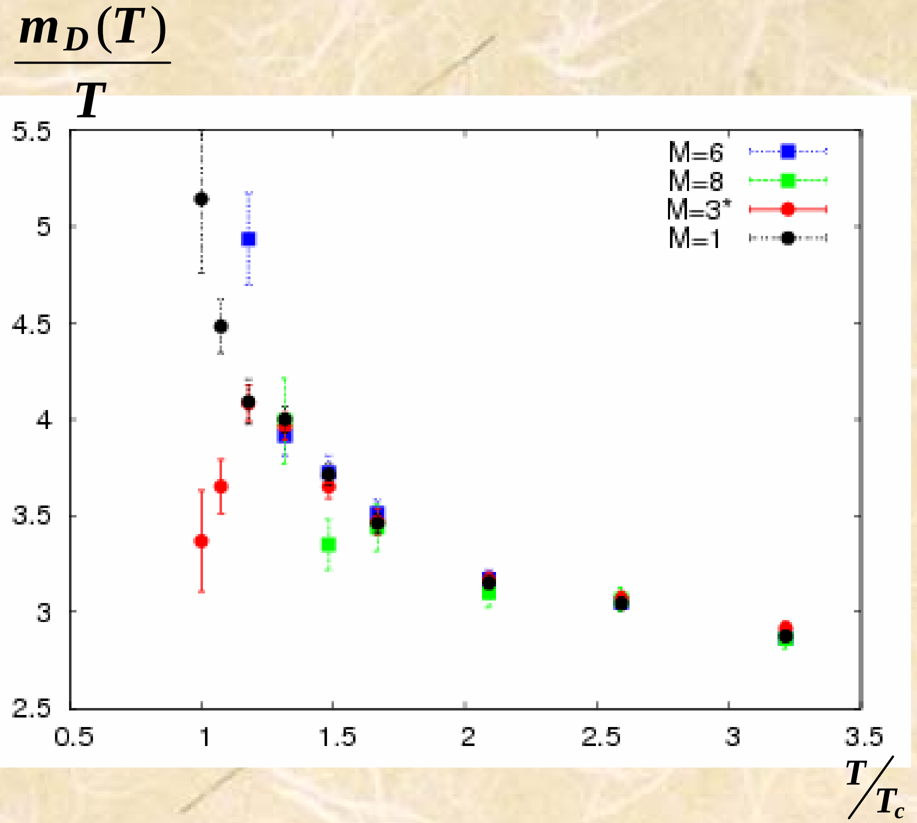
$m_D(T)$: Debye screening mass

⇩

Fitting the potentials of each channel with $\alpha(T)$ and $m_D(T)$ as free parameters.

Results of $\alpha(T)$ and $m_D(T)$.





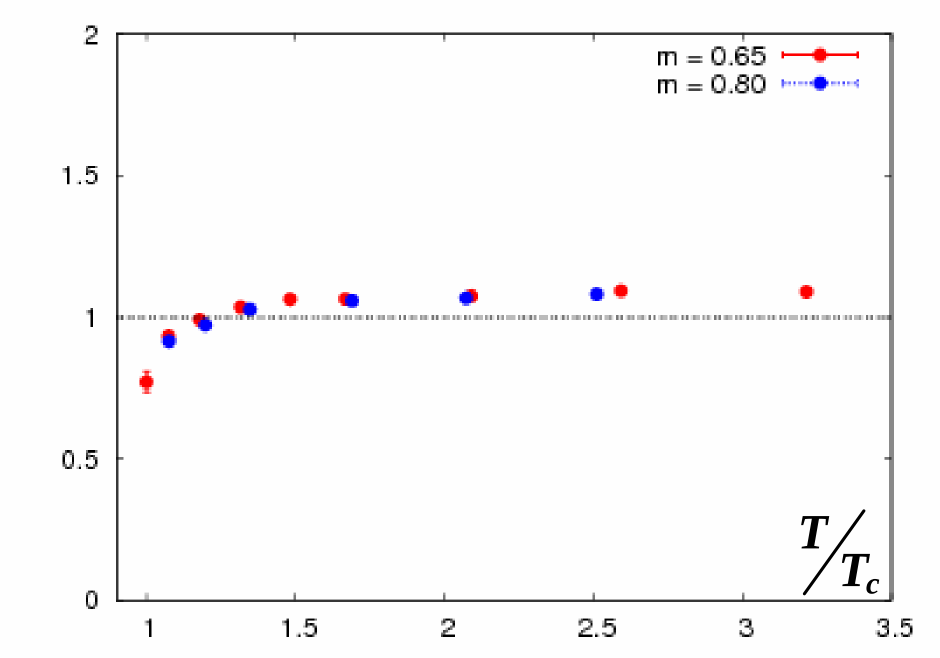
$$V(r,T) = C(M)\frac{\alpha(T)}{r}e^{-m_D(T)r}$$

For $T > 2.5\,T$c, potentials of each channel can be written by **the same parameters**: $\alpha(T)$ and $m_D(T)$.

Relation between $\alpha(T)$ and $m_D(T)$.

- Effective running coupling $g_{\mathrm{eff}}(T) \equiv \sqrt{4\pi\alpha(T)}$
- Screening mass $m_D(T) \overset{?}{=} \sqrt{1+\frac{N_f}{6}}\, g_{\mathrm{eff}}(T)T$

$$\frac{m_D(T)}{\sqrt{1+\frac{N_f}{6}}\, g_{\mathrm{eff}}(T)T}$$



for 1c potential

$m_D(T)$ is described by effective running coupling $g_{\mathrm{eff}}(T)$ with 10% accuracy.

$m_D(T)$ on a lattice vs. perturbative screening mass

- 2-loop running coupling

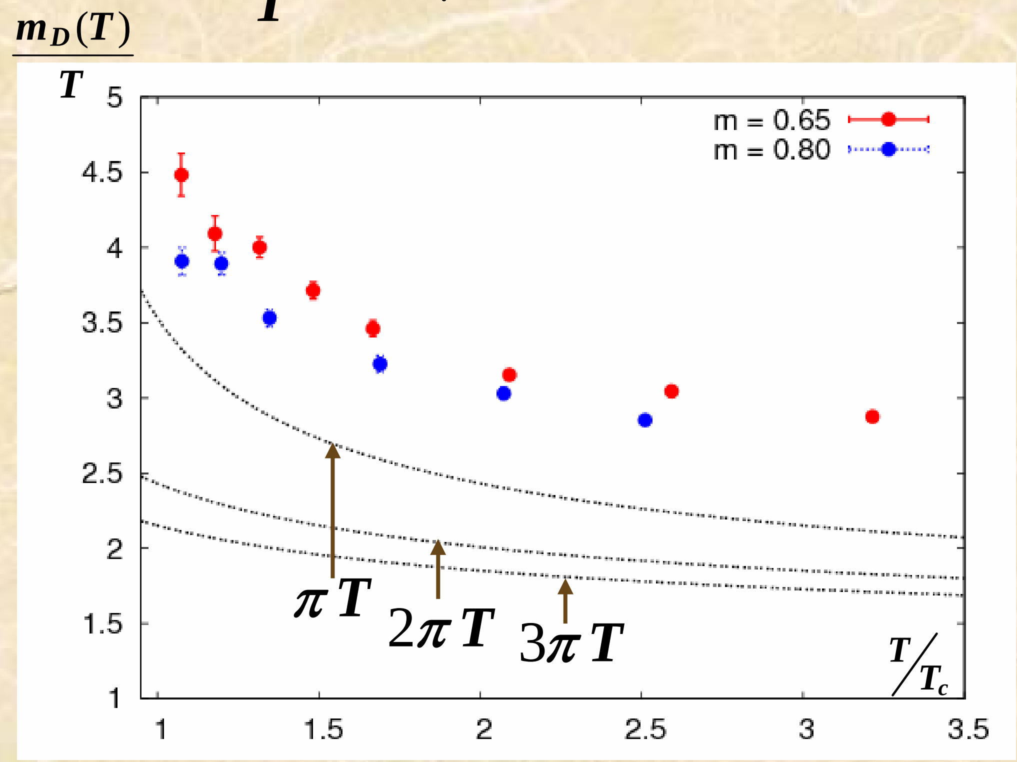
$$g^{-2}(T) = \beta_0 \ln\left(\frac{\mu}{\Lambda_{\overline{MS}}}\right)^2 + \frac{\beta_1}{\beta_0} \ln\ln\left(\frac{\mu}{\Lambda_{\overline{MS}}}\right)^2$$

$$\Lambda_{\overline{MS}}^{N_f=2} \approx 261\ \text{MeV}$$

$$\mu = \pi T,\ 2\pi T,\ 3\pi T$$

- Leading order perturbation

$$\frac{m_D(T)}{T} = \sqrt{1+\frac{N_f}{6}}\, g_{\text{2-loop}}(T)$$



Lattice screening mass is **not** reproduced by the LO-type screening mass.

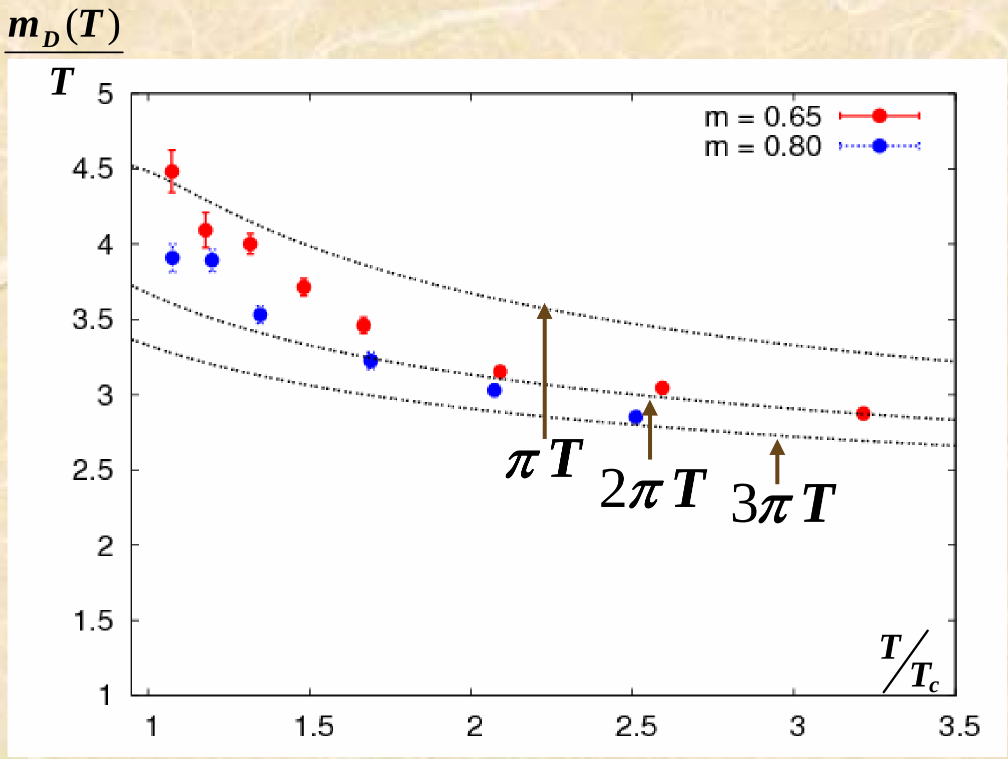
$m_D(T)$ on a lattice vs. perturbative screening mass

- Next-to-leading order perturbation

Rebhan, PRD 48 (1993) 48

$$\frac{m_D^{NLO}(T)}{T} = \sqrt{1+\frac{N_f}{6}}\, g(T)\left[1+g(T)\frac{3}{2\pi}\sqrt{\frac{1}{1+\frac{N_f}{6}}}\left(\ln\frac{2m_D}{m_{mag}}-\frac{1}{2}\right)+o(g^2)\right]$$

Magnetic screening mass: $m_{mag} = C_m\, g^2(T)\,T$



Fitting $m_D(T = 3.2T_c)$

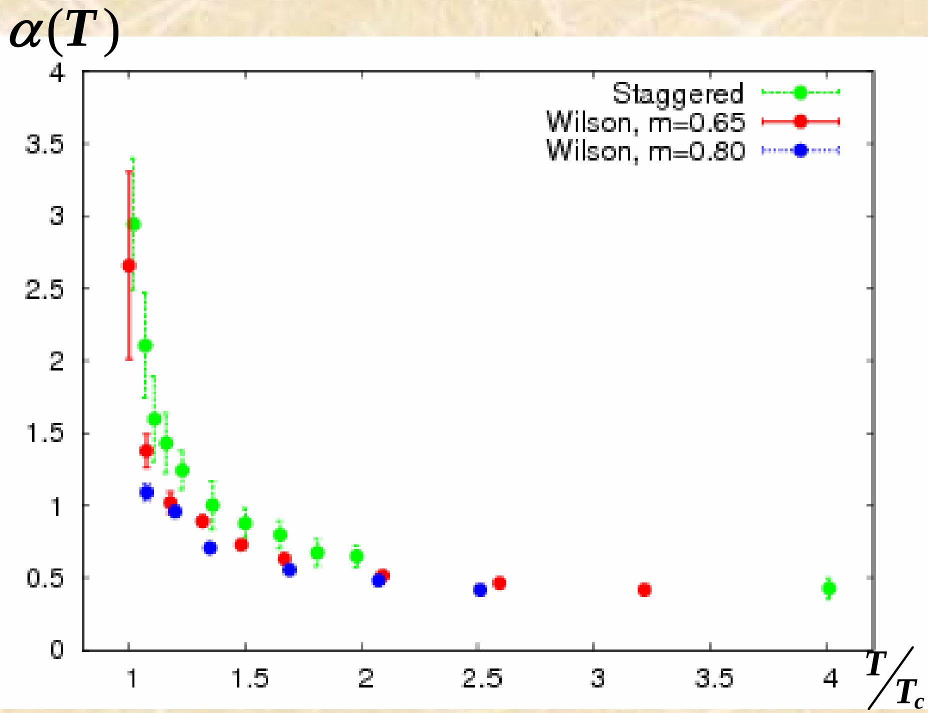
$\Rightarrow C_m = 0.37$

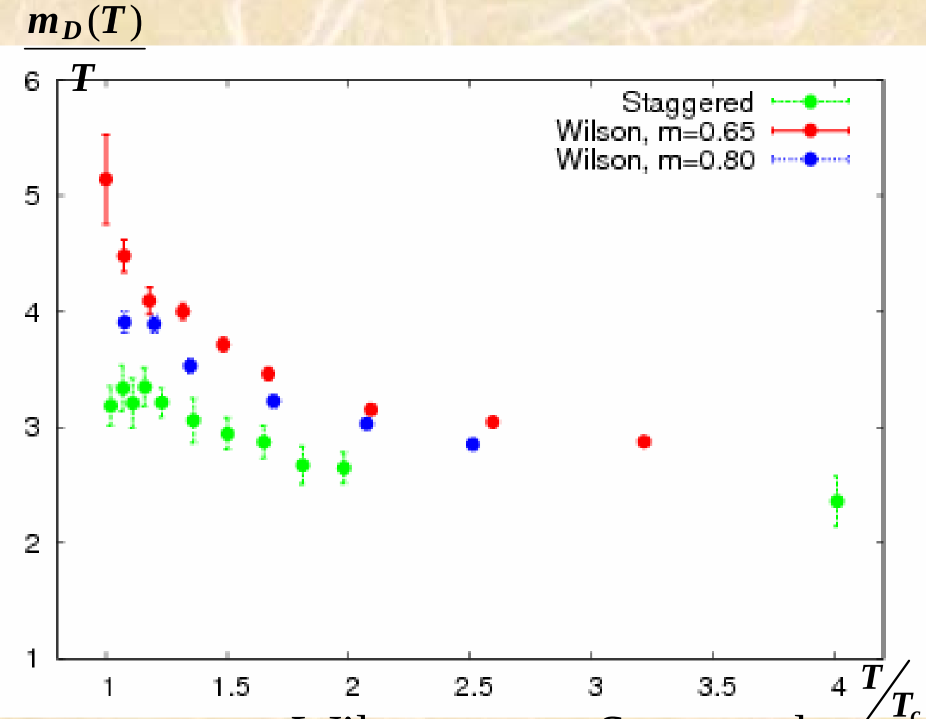
Lattice screening mass is well reproduced by the **NLO-type** screening mass at $T > 2T_c$.

Comparison with staggered fermion

- Improved staggered fermion with $N_s^3 \times N_t = 16^3 \times 4, \quad m_\pi / m_\rho \approx 0.70$

Kaczmarek and Zantow, PRD 71 (2005) 114510





$\alpha^{\text{Wilson}} \lesssim \alpha^{\text{Staggered}}$ $\qquad$ $m_D^{\text{Wilson}} \gtrsim m_D^{\text{Staggered}}$

Systematic error due to the difference of actions

may be 20% or more.

Summary

- Heavy quark "potential"

⇒ Lattice QCD simulation using 2-flavor Wilson fermion action

1c, $\bar{3}$c : attractive force   8c, 6c : repulsive force

- Screening effect ⇒ potentials are fitted by

$$V(r,T) = C(M)\frac{\alpha(T)}{r}e^{-m_D(T)r}$$

$T > 2.5T_c$ • $\alpha(T)$ and $m_D(T)$ are independent on channel.

$T \gtrsim T_c$ $m_D(T) \approx \sqrt{1+\frac{N_f}{6}}\, g_{\rm eff}(T)T$ $\left(g_{\rm eff}(T) \equiv \sqrt{4\pi\alpha(T)}\right)$

effective running coupling

$T > 2T_c$ $m_D(T) \approx \sqrt{1+\frac{N_f}{6}}\, g(T)T\left[1+C_{NLO}\, g(T)\right]$

2-loop running coupling

- Comparison with staggered fermion action

⇒ Systematic error is 20% or more