A Preliminary Calculation of the B_S Mass and Width Difference Hadronic Matrix Elements

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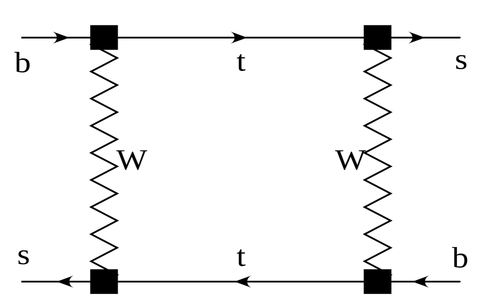
ABSTRACT

We present a preliminary calculation of the hadronic matrix elements relevant to the B_s mass and width difference. The calculation is done on MILC lattices with 2+1 sea quarks. We use the Asqtad action for the light valence quarks and the Fermilab action for the b quark.



1 Introduction

1.1 The mass and width difference in the B_s meson system

The B_s meson and its antiparticle are able to mix via the diagram above. It is the mass and width difference between the mass eigenstates that are of interest in the determination of CKM matrix elements and CP violation . The $B_s - \bar{B}_s$ meson system's mass difference, Δm_s, is dependent on CKM matrix elements and is parametrized as

$$\Delta m_s = \frac{G_F^2}{6\pi^2} m_W^2 \eta_B S_0(m_t, m_W) |V_{ts} V_{tb}^*|^2 \langle \bar{B}_s | Q | B_s \rangle. \quad (1)$$

The hadronic matrix element is conventionally parametrized as

$$\langle \bar{B}_s | Q | B_s \rangle = \langle \bar{B}_s | \bar{b}\gamma_\mu (1-\gamma_5) s \bar{b} \gamma^\mu (1-\gamma_5) s | B_s \rangle = \frac{8}{3} m_{B_s}^2 f_{B_s}^2 B_{B_s}. \quad (2)$$

f_{B_s} is the decay constant of the B_s meson, and B_{B_s} is the bag parameter.
The difference in decay rates of the eigenstates in the neutral B meson system is another measurable quantity which is sensitive to CP violation. The width difference

$$\Delta\Gamma_{B_s} = \frac{G_F^2 m_b^2}{6\pi^2 m_{B_s}} |V_{cb}^* V_{cs}|^2 [F(\frac{m_c^2}{m_b^2}) \langle \bar{B}_s | Q | B_s \rangle + F_S(\frac{m_c^2}{m_b^2}) \langle \bar{B}_s | Q_S | B_s \rangle][1 + O(\frac{\Lambda_{QCD}}{m_b})] \quad (3)$$

is determined by two hadronic matrix elements at the leading order in the heavy quark expansion. Q is the familiar operator from the mass difference, and Q_S is parametrized in a similar way

$$\langle \bar{B}_s | Q_S | B_s \rangle = \langle \bar{B}_s | \bar{s}(1-\gamma_5) b \bar{s}(1-\gamma_5) b | B_s \rangle = -\frac{5}{3} \frac{m_{B_s}^2}{(m_b + m_s)^2} m_{B_s}^2 f_{B_s}^2 B_S$$

$$= -\frac{5}{3} m_{B_s}^2 f_{B_s}^2 B_S'. \quad (4)$$

1.2 Lattice, Actions, and Operators

We performed the following calculations on the MILC coarse lattices ($a = 0.12 fm$) with 2+1 sea quarks on 592 configurations. The sea quarks are simulated using the Asqtad improved action, where errors are introduced at $O(a^4, a^2\alpha)$.
The valence s quark is also simulated using the Asqtad action, and the heavy b quark is handled using the Fermilab action, with errors starting at $O(a\alpha_s, a^2)$.
For tree level $O(a)$ improvement of the operator we found that a rotation of the b quark via [1] is all that is necessary. In order to include higher order effects additional six dimensional operators will have to be included. These matrix elements are already calculated as described below.

a^{-1}	Sea masses	Light mass (m_l)	Heavy mass (m_h)
1.596 + 0.025 GeV	$u/d = 0.01$ $s = 0.05$	s=0.0415	κ_b=0.086

2 The Open Propagator

The calculation of the general 4-quark matrix element

$$\langle \bar{H}_q(x) | \bar{q}\Gamma_1 h \bar{q} \Gamma_2 h(0) | H_q(y) \rangle \quad (5)$$

can be organized in such a way that only two propagator inversions are necessary in order to calculate all 24 possible matrix elements. After performing the wick contractions and fourier transforming (5) we obtain

$$\langle \bar{H}_q(x) | O_q^{\Delta h=2} | H_q(y) \rangle = \mathbf{Tr}[\Gamma_1 E_{hq}(t_x)] \mathbf{Tr}[\Gamma_2 E_{hq}(t_y)]$$

$$+ \mathbf{Tr}[\Gamma_1 E_{hq}(t_y)] \mathbf{Tr}[\Gamma_2 E_{hq}(t_x)] + \mathbf{Tr}[\Gamma_1 E_{hq}(t_x) \Gamma_2 E_{hq}(t_y)] + \mathbf{Tr}[\Gamma_1 E_{hq}(t_y) \Gamma_2 E_{hq}(t_x)] \quad (6)$$

where the traces are over color and spin indices. The "open propagator"

$$E_{hq,ij}^{ab}(t_x) = \gamma_5^{ac} G_{h,ki}^{*dc}(t_x, 0) G_{q,kj}^{db}(t_x, 0) \quad (7)$$

is all that is needed to construct the matrix element. This object is very small in size and can easily be saved and later used to construct all 2 and 3 point functions necessary for our calculation. [4].

3 Data and Fitting

3.1 Data

Fig. 1

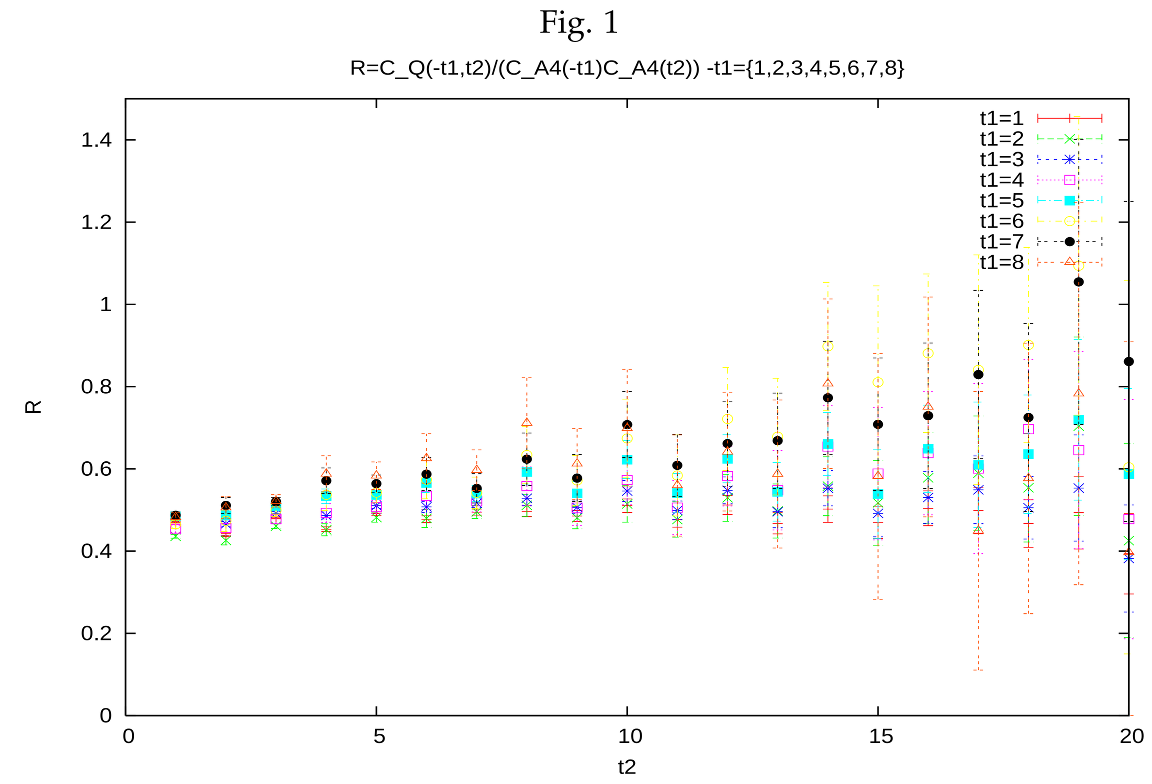


Fig 1. is the ratio of the 3 point function and 2 point functions

$$R(-t_1, t_2) = \frac{3}{8} \frac{C_Q(-t_1, t_2)}{C_{A4}(-t_1) C_{A4}(t_2)} \quad (8)$$

where

$$C_{A4}(t) = \sum_{\vec{x}} \langle \bar{b}(\vec{x}, t) \gamma_5 s(\vec{x}, t) \bar{s}(0) \gamma_0 \gamma_5 b(0) \rangle \quad (9)$$

and

$$C_Q(t_1, t_2) = \sum_{\vec{x}, \vec{y}} \langle \bar{b}(\vec{x}, t_1) \gamma_5 s(\vec{x}, t_1) [Q(0)] \bar{b}(\vec{y}, t_2) \gamma_5 s(\vec{y}, t_2) \rangle \quad (10)$$

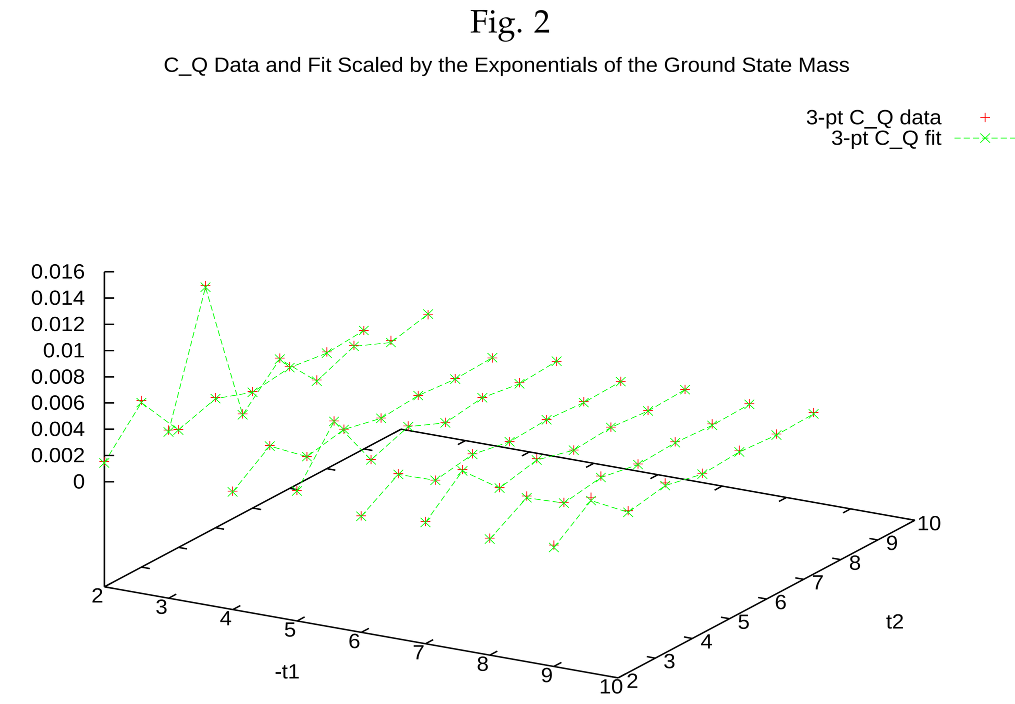
The correlation functions have naive valence quarks which contain doublers that cause higher energy 0^+ states to contribute. These states oscillate in euclidean time and must be taken into account in the fits [2]. In the limit $-t_1, t_2 \to \infty$ R becomes the bag parameter

$$B_{B_s} = \frac{3}{8} \frac{\langle \bar{B}_s | \bar{b}\gamma_\mu (1-\gamma_5) s \bar{b} \gamma^\mu (1-\gamma_5) s | B_s \rangle}{m_{B_s}^2 |\langle \bar{b}\gamma_0 \gamma_5 s | B_s \rangle|^2}. \quad (11)$$

The oscillating states make the identification of a plateau and fitting directly to the ratio difficult. We are examining other ratios to determine the possibility of fitting to these, but are currently fitting to C_Q directly.
The analysis of B_S' is analogous.

3.2 Correlation function fitting

Fig. 2



In order to extract the parameters of interest we performed constrained fits [3] simultaneously to 3 correlation functions: the 2 point functions

$$C_Z(t) \to_{t\to\infty} \frac{1}{2m_{B_s}} |\langle \bar{s}\gamma_5 b | B \rangle|^2 e^{-m_{B_s} t}, \quad (12)$$

$$C_{A_4}(t) \to_{t\to\infty} \frac{1}{2m_{B_s}} \langle \bar{s}\gamma_5 b | B \rangle \langle B | \bar{b}\gamma_0\gamma_5 s \rangle e^{-m_{B_s} t} \quad (13)$$

and 3 point function

$$C_Q(-t_1, t_2) \to_{t_1, t_2 \to\infty} \frac{1}{(2m_{B_s})^2} |\langle \bar{s}\gamma_5 b | B \rangle|^2 \langle \bar{B} | Q | B \rangle e^{-m_{B_s} t_1} e^{-m_{B_s} t_2}. \quad (14)$$

C_Z allows the overlap parameters in C_Q to be removed and the matrix element isolated. C_{A_4} is used to determine f_{B_s} and can be used to isolate B_{B_s}. It should be noted that the parameter most directly of phenomenological interest, $f_{B_s}\sqrt{B_{B_s}}$, can be extracted by combining just C_Z and C_Q.
In addition to the ground state other excited states contribute, in particular the opposite parity oscillating states arising from the naive valence quark. For our best fits we included the first 4 states (2 regular and 2 oscillating), with a $\chi^2 \approx 0.8$, with t_1 and t_2 taken over $t_{min} = 2, t_{max} = 9$. These fits are shown in Fig. 2 and Fig. 3 for Q and Fig. 4 for Q_S.
We looked at a variety of time slices and ranges along with including additional states in the model function. The fit results were typically consistent within 25% of the error bars of the best fit. We calculated Q_S in an identical way.

Fig. 3

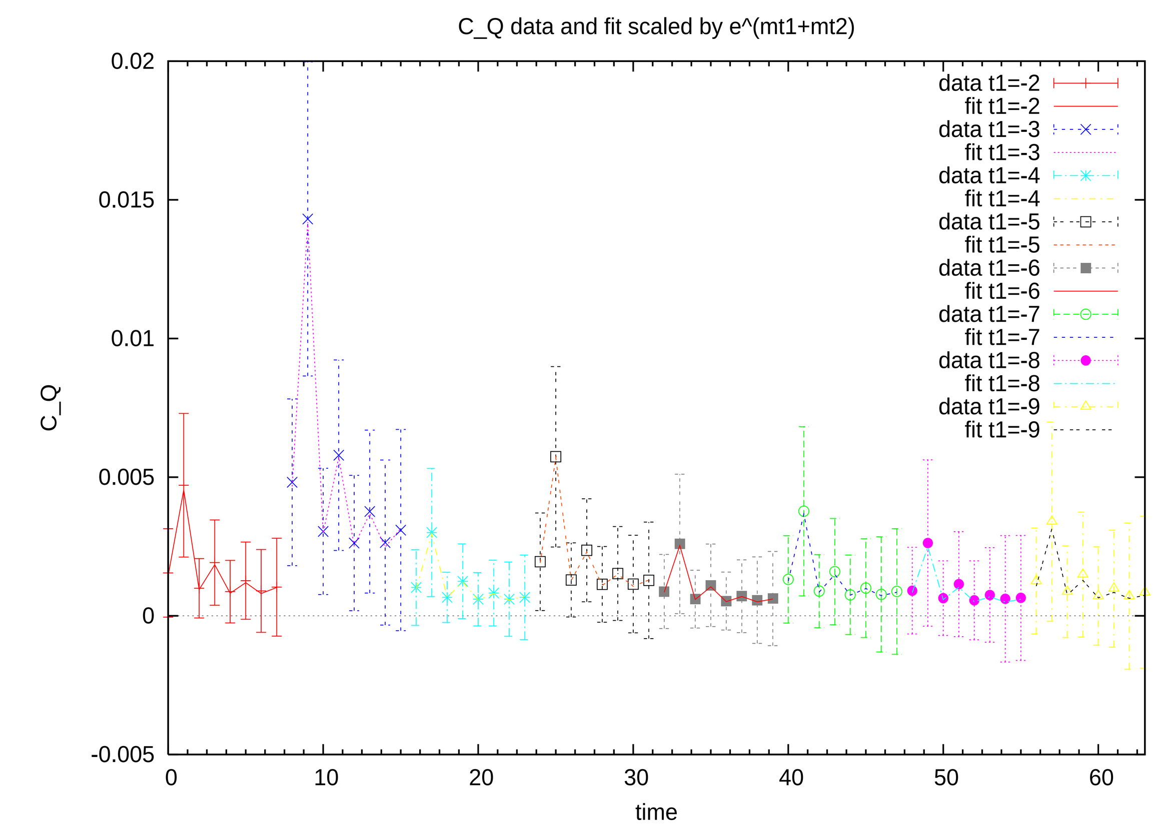
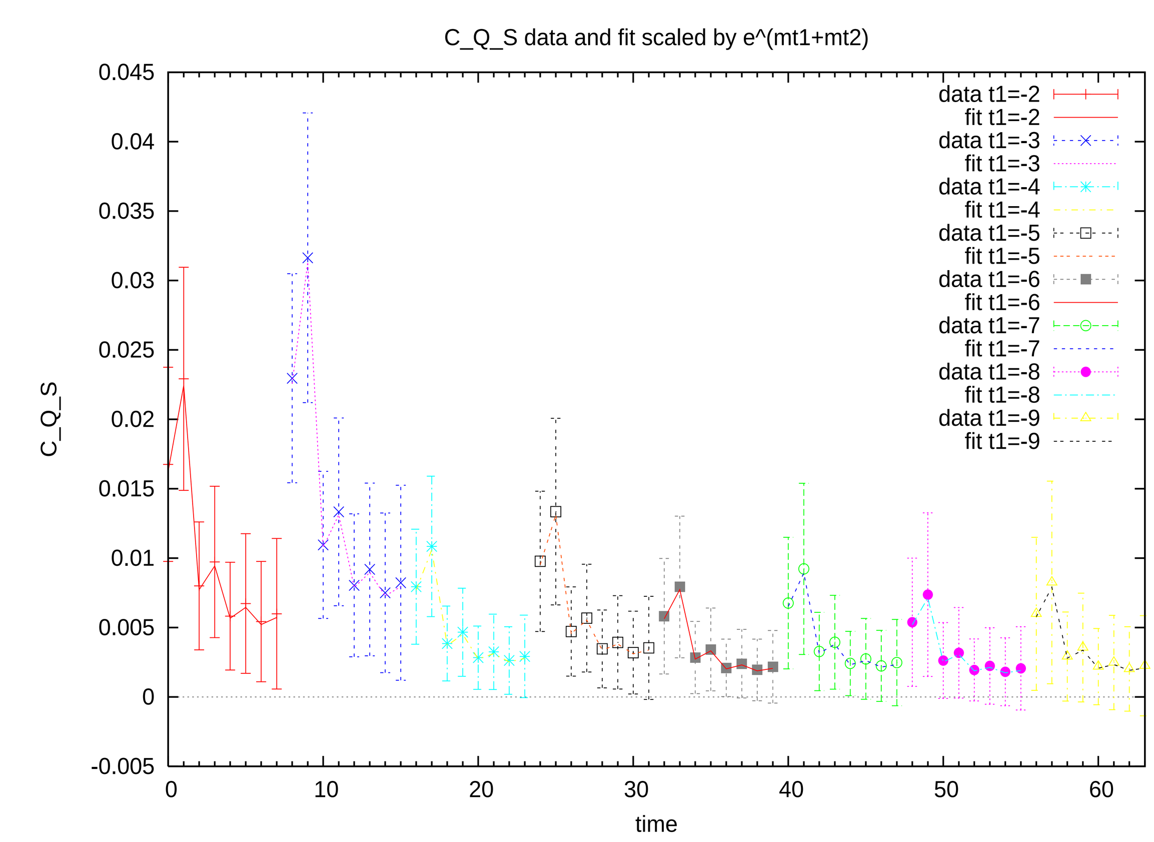


Fig. 4



Each separate set of points and lines in the above 2 figures represents the data (points) and fit (line) at a particular time source slice, i.e. $-t_1$ fixed and t_2 varied.

3.3 Results

The errors reported are χ^2 errors from the fitting and systematic errors due to uncertainty in the lattice spacing. The results do not include the renormalization coefficients. Our results are

Units	B_{B_s}	$f_{B_s}\sqrt{B_{B_s}}$	B_S'	$f_{B_s}\sqrt{B_S'}$
a	0.61 +/- 0.07	0.161 +/- 0.007	2.44 +/- 0.21	0.333 +/- 0.009
GeV	0.61 +/- 0.07	0.255 +/- 0.012	2.44 +/- 0.21	0.527 +/- 0.018

4 Summary and Outlook

- The statistical uncertainties of this calculation are straightforward to reduce. Specifically, we plan to repeat the calculation on the same ensemble, but with different time sources. In addition, we want to examine smearing's effects on our results. Smearing should reduce the contamination of excited states in the correlation functions.
- At this point the calculation is done only at tree level with $O(a)$ improvement. We are planning to include the perturbative matching at one loop order at which point additional operators must be included. Including these operators will be straight forward once their coefficients have been calculated. The NLO operators in the $1/m_b$ expansion contribute significantly to $\Delta\Gamma_s$, and will also have to be calculated [6].
- We are planning to repeat this calculation on the available MILC ensembles for various sea quark masses and lattice spacings in order to observe its mass and lattice spacing dependence. We also plan to study the B_d system and to extract ξ.

References

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