$B \to K\ell^+\ell^-$ from Lattice QCD

Rajendra D. Jain

Department of Physics, University of Illinois at Urbana-Champaign
for the Fermilab/MILC collaboration

Abstract

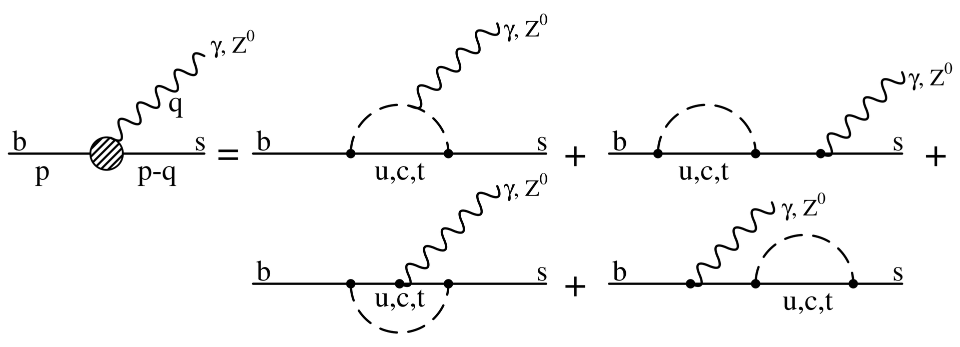
We present preliminary results of a first lattice calculation of the form factors relevant to the rare decay $B \to K\ell^+\ell^-$. Hadronic matrix elements are calculated on MILC lattices with 2+1 flavors of sea quark. We use the ASQTAD action for light quarks and the Fermilab action for the b quark.

1. Introduction

As flavor-changing neutral current processes, rare decays provide for precision tests of the Standard Model. The rare semi-leptonic decay $B \to K\ell^+\ell^-$ in particular is interesting as it tests the SM in the b-sector and because an experimental measurement of its rate should soon be within the reach of the current generation of B factories.

2. Continuum Theory

The leading-order contribution to the $B \to K\ell^+\ell^-$ rate comes from the electroweak penguin diagrams shown below:



Carrying out an operator product expansion yields effective hamiltonians for $b \to \gamma s$ and $b \to Zs$ transitions given by

$$\mathcal{H}^{\textit{eff.}}_{b\gamma^\mu s} = -i \sum_{i=u,c,t} V^*_{ib} V_{is} \frac{G_F}{\sqrt{2}} \frac{e}{8\pi^2} \Big\{ F_\gamma(x_i)[\bar{b}_L(q^2\gamma^\mu - q_\mu \not{q}) s_R] + F'_\gamma(x_i)[\bar{b}_L(i m_b \sigma^{\mu\nu} q_\nu)] s_R \Big\}$$

and

$$\mathcal{H}^{\textit{eff.}}_{bZ^\mu s} = -i \sum_{i=u,c,t} V^*_{ib} V_{is} \frac{G_F}{\sqrt{2}} \frac{e}{2\pi^2} M_Z^2 \tan(\Theta_W) F_Z(x_i)[\bar{b}_L \gamma^\mu s_L]$$

where the $F(x_i)$'s are known functions of $x_i \equiv m_i^2/m_W^2$. One thus finds that the hadronic matrix elements needed for $B \to K\ell^+\ell^-$ are $\langle B \,|\, \gamma^\mu \,|\, K \rangle$ and $\langle B \,|\, \sigma^{\mu\nu} q_\nu \,|\, K \rangle$. These have the standard parameterizations

$$\langle B(p) \,|\, \bar{b}\sigma^{\mu\nu} q_\nu s \,|\, K(k) \rangle = i \frac{f_T}{m_B + m_K} \Big\{ (p+k)^\mu q^2 - q^\mu (m_B^2 - m_K^2) \Big\}$$

and $$\langle B(p) \,|\, \bar{b}\gamma^\mu s \,|\, K(k) \rangle = f_+ (p^\mu + k^\mu - \frac{m_B^2 - m_K^2}{q^2} q^\mu) + f_0 \frac{m_B^2 - m_K^2}{q^2} q^\mu .$$

Rather than extracting f_+ and f_0 directly, we here quote results for $f_\parallel$ and $f_\perp$, which are related to f_+ and f_0 by

$$f_0 = \frac{\sqrt{2 m_B}}{m_B^2 - m_K^2} [(m_B - E) f_\parallel + (E^2 - m_K^2) f_\perp] \quad \text{and}$$

$$f_+ = \frac{1}{\sqrt{2 m_B}} [f_\parallel + (m_B - E) f_\perp],$$

where E is the energy of the K.

3. Lattices and Actions

The results we present employ an ensemble of 460 MILC lattices with 2+1 flavors of dynamical sea quarks. The light valence and sea quarks are simulated using the ASQTAD action[1], while for the b we use the Wilson-like Fermilab action[2]. It can been shown that, within this formalism, the $\mathcal{O}(a)$ discretiztion errors in two-quark current operators can be removed by a simple rotation of the heavy-quark field. The fermilab action is $\mathcal{O}(a)$ improved, while the ASQTAD and gauge field actions we have chosen have errors starting at $\mathcal{O}(\alpha_s a^2, a^4)$. Overall, we thus expect our results to be accurate up to $\mathcal{O}(a^2)$. Results presented here are for $a = 1/1.596$ GeV with $am_\ell = 0.02$, $am_s = 0.03$ and $\kappa_b = 0.086$.

4. Extracting Form Factors

The two- and three-point functions relevant to the decay $B \to K\ell^+\ell^-$ are (working in the rest frame of the B)

$$C_{3\Gamma}(\mathbf{p}; t_x, t_y) \equiv \sum_{\mathbf{xy}} e^{i\mathbf{p}\cdot\mathbf{x}} \left\langle \Phi_K^\dagger(0)\; \bar{s}(x)\Gamma b(x)\; \Phi_B(y) \right\rangle ,$$

$$C_{2B}(\mathbf{0}, y_t) \equiv \sum_{\mathbf{y}} \left\langle \Phi_B^\dagger(0) \Phi_B(y) \right\rangle ,$$

$$\text{and } C_2(\mathbf{p}; y_t)_K \equiv \sum_{\mathbf{y}} e^{i\mathbf{p}\cdot\mathbf{y}} \left\langle \Phi_K^\dagger(0) \Phi_K(y) \right\rangle$$

where Γ is one of either γ^μ or $\sigma^{\mu\nu}$ and the Φ's are interpolating fields. Throughout our analysis T, the sink time for the B in $C_{3\Gamma}$ is fixed at 16. We have found that the desired matrix element $\langle K(\mathbf{p}) \,|\, s\Gamma b \,|\, B(\mathbf{0}) \rangle$ can be extracted by analyzing the double ratio

$$R_\Gamma(\mathbf{p}; t, T) \equiv \frac{C_{3\Gamma}(\mathbf{p}; t, T) \cdot C_{3\Gamma}(\mathbf{p}; T-t, T)}{C_2(\mathbf{0}; T)_B \cdot C_2(\mathbf{p}; T)} .$$

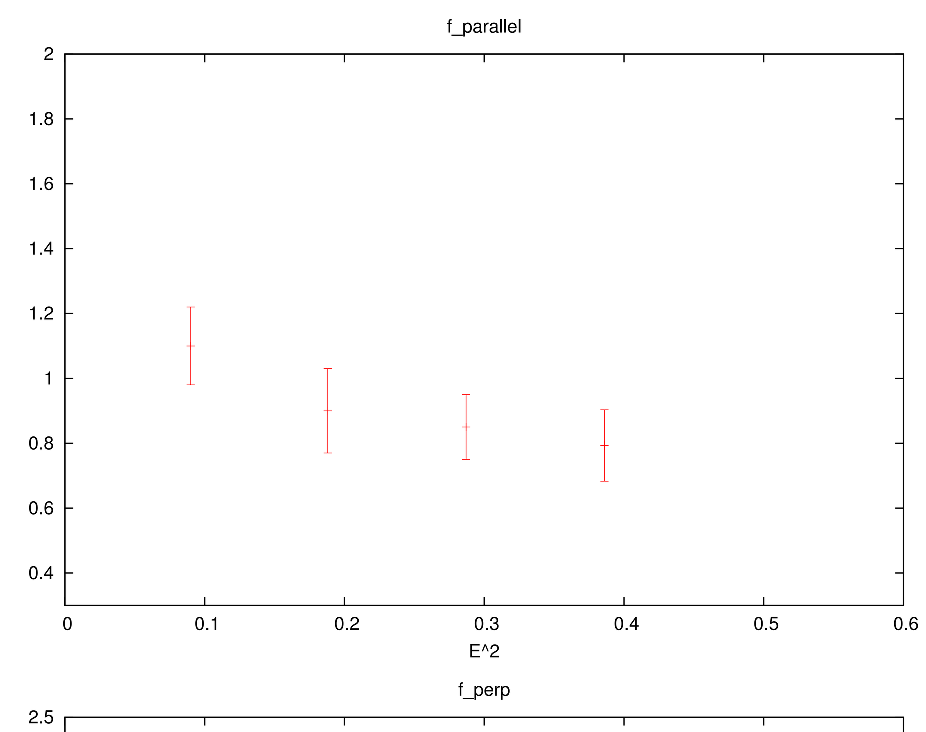
The use of staggered valence quarks and the subsequent appearence of doublers causes contributions to B-meson correlators whose signs oscillate in Euclidean time[4]. In fitting R values obtained from lattice data, our fits are currently limited by truncating the sum over states to include only the ground states and a single B oscillator. Expanding our expression for R then yields the functional form

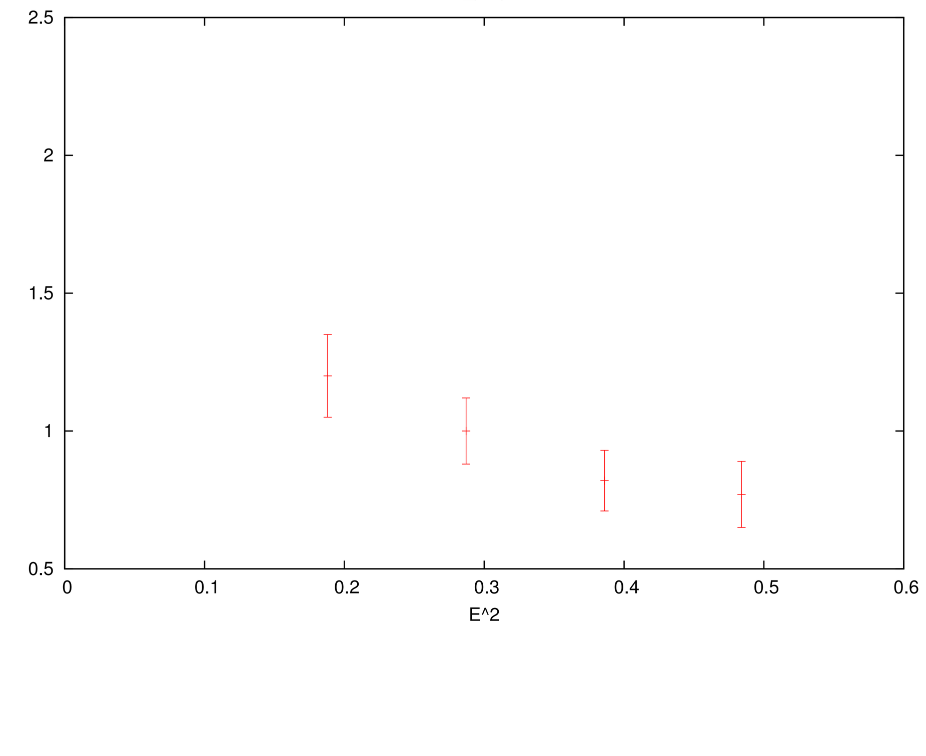
$$R_\Gamma(\mathbf{p}; t, T) \approx A + (-1)^t B \cdot \cosh(dE(\frac{T}{2} - t) + C e^{-dEt}$$

in terms of unknown parameters A,B,C, and dE. The parameter A is then related to the matrix element in which we are interested by $\langle K(\mathbf{p}) \,|\, s\Gamma b \,|\, B(\mathbf{0}) \rangle = \rho_\Gamma Z_V^{hh} Z_V^{ll} \sqrt{A}$. The Z_V's are non-perturbative renormalization constants which have been calculated and $\rho = 1 + \mathcal{O}(\alpha_s)$ is a perturbative renormalization which is known for the vector current matrix elements but which remains to be calculated for the tensor current.

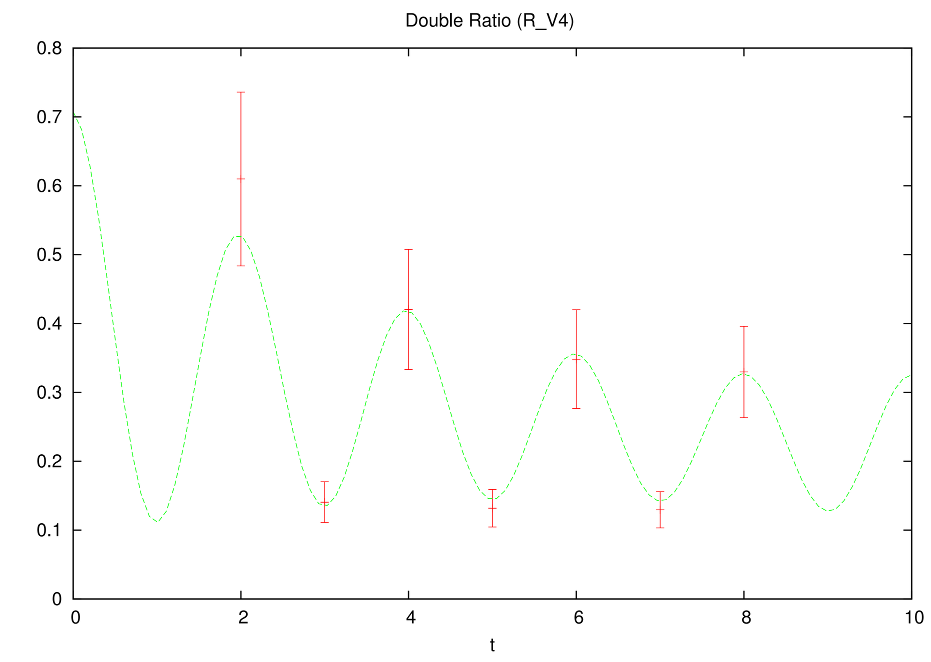
5. Results

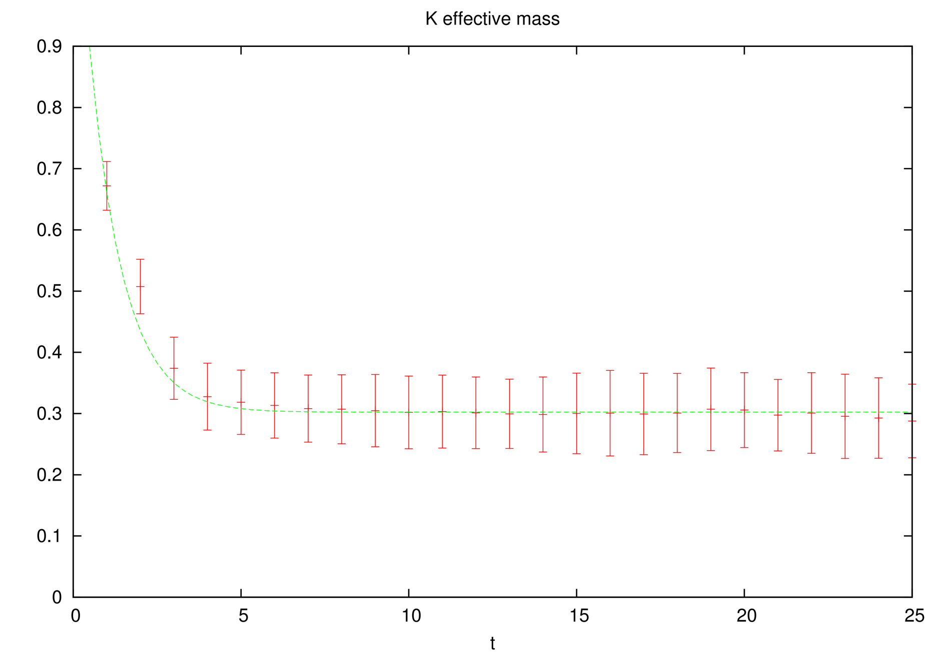
We have obtained results for $f_\parallel$ and $f_\perp$ as shown below. They are plotted in lattice units as a function of the square of the kaon energy, E. Results for f_T are still underway.





These results have been extracted by fitting to the double ratio, R, as discussed above. An example of such a fit is shown below for $\Gamma = V^4$ and $\mathbf{p}_K = (0,0,0)$. The line represents a fit to the data in the range $3 < t < 8$. Also shown is an effective mass plot extracted from the K two-point correlator with the fit range being $2 < t < 14$. Fittng to the two-points was essential for extracting the B and K masses and energies. χ^2 for all fits shown were in the range $0.6 - 1.2$.





6. Conclusion and Outlook

Though this work is still in its beginning stages, the above demonstrates a framework for making a lattice prediction of the form factors leading to $B \to K\ell^+\ell^-$. The first step in improving our analyis will be to study the systematic errors introduced by fitting to the double ratio of section 4. We plan to study the robustness of this procedure, as well as to explore alternate methods such as directly fitting to our two- and three- point functions. Once the errors introduced by fitting are understood, this analysis will need to be repeated on other MILC ensembles so that we can study the lattices spacing and light-quark mass dependence of our results. Additionally ρ_σ, the perturbative correction to tensor current matrix elements, will need to be computed. Finally, we plan to examine methods for improving our statistical errors, such as including multiple sources times, smearing, or by varying T.

References

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[3] M. Wingate and J. Shigemitsu, [arXiv:hep-lat/0211014].