The decay constants f_{B^+} and f_{D^+} from three-flavor lattice QCD

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Abstract

We present new results for f_{B^+} and f_{D^+} from the MILC $2+1$ flavor $a = 0.09\,\mathrm{fm}$ “fine” lattice. We use clover heavy quarks in the Fermilab interpretation and improved staggered light quarks. Lattice results from partially quenched QCD fix the parameters of staggered chiral perturbation theory which is used in the extrapolation to the physical decay constants.

1. Introduction

The D meson decay constants, when compared to precise experimental results are a critical check of lattice methods need for f_B. In In Reference [1] we predicted $f_{D^+} = 201 \pm 3 \pm 17$ MeV in good agreement with the CLEO-c measurement $f_{D^+} = 223 \pm 17 \pm 3$ MeV revealed just days later [2]. This poster presents new preliminary results for B meson decay constants. Precise determinations of f_B, f_{B_s} and the ratio f_{B_s}/f_B are need to study the Standard Model picture of B-$\bar{B}$ and B_s-$\bar{B}_s$ mixing. Preliminary results for mixing matrix elements leading to $f_{B_{d,s}}\sqrt{B_{d,s}}$ are also being presented at this conference [3].

Our published D meson decay constants are based on results from the MILC $a = 0.12$ fm “coarse” lattice and a subset of the $a = 0.09$ fm “fine” lattice results presented here. We have since computed f_B on all three fine lattice ensembles, extended the D meson calculation to the third ensemble, and increased the number of valence quark mass values in the analysis from 5 to 10 for the $\beta = 7.11$ ensemble. We are also working to extend the number of configurations used in the decay constant analysis at $a = 0.12$ and we are repeating the calculation at $a = 0.15$ fm.

2. Simulation details

Lattice details are tabulated below. The mass of the two light sea quark flavors is m_l. The heavier single flavor quark has a mass m_h near the strange quark mass. Upsilon spectroscopy tells us the heavy quark potential scale $r_1 = 0.317(7)$ fm [4]. The last column lists the number of valence quarks masses, m_q, used in the extrapolations.

a [fm]	am_h	am_l	β	r_1/a	configs	# m_q
0.09	0.031	0.0031	7.08	3.71(1)	435	11
		0.0062	7.09	3.69(1)	557	8
		0.0124	7.11	3.71(1)	518	10

The decay constant f_{H_q} for a meson Hq is defined by

$$\left\langle 0 \mid A_\mu \mid H_q(p) \right\rangle = i f_{H_q} p_\mu \ . \tag{1}$$

The combination $\phi_{Hq} = f_{H_q}\sqrt{m_{H_q}}$ emerges from a combined fit to 2-pt functions

$$C_O(t) = \left\langle O^\dagger_{H_q}(t) \;\; O_{H_q}(0) \right\rangle \tag{2}$$

$$C_{A_4}(t) = \left\langle \; A_4(t) \;\; O_{H_q}(0) \right\rangle \ , \tag{3}$$

where O_{H_q} is a smeared or local operator.
The axial current renormalization is taken to be

$$Z^{Qq}_{A4} = \rho^{Qq}_{A4} \sqrt{Z^{QQ}_{V_4} Z^{qq}_{V_4}} \ . \tag{4}$$

The factors $Z^{ff}_{V_4}$ are computed nonperturbatively while the factor ρ^{Qq}_{A4} is known to one loop order and is close to unity.

3. NLO Staggered χPT

With staggered quarks the (squared) pseudoscalar meson masses are split

$$M^2_{ab,\xi} = (m_a + m_b)\mu + a^2\Delta_\xi \ , \tag{5}$$

where m_a, m_b are quark masses, the (sixteen) mesons are labeled by their taste representation $\xi = P, A, T, V, I$ and $\Delta_P = 0$.
At next to leading order in χPT the expression for the decay constants is

$$\phi_{Hq} = \Phi_H \left[1 + \Delta f_H(m_q, m_l, m_h) + p_H(m_q, m_l, m_h)\right] \ . \tag{6}$$

With staggered quarks

$$\Delta f_H = -\frac{1 + 3g^2_{H^*H\pi}}{2(4\pi f_\pi)^2}\left[\bar{h}_q + h^I_q + a^2\left(\delta'_A h^A_q + \delta'_V h^V_q\right)\right] \ . \tag{7}$$

Taste breaking effects arise at finite a from the meson mass splittings and the δ'_A and δ'_V terms [5]. Finite a effects dilute the logarithmic behavior, however, the QCD “chiral logarithm” is recovered in the continuum limit.
The analytic terms are

$$p = \frac{1}{2(4\pi f_\pi)^2}\left[p_1(m_l, m_h) + p_2(m_q)\right] \tag{8}$$

$$p_1 = f_1(\Lambda_\chi)\left[\frac{11}{9}\mu(2m_l + m_h) + a^2\left(\frac{3}{2}\bar{\Delta} + \frac{1}{3}\Delta_I\right)\right] \tag{9}$$

$$p_2 = f_2(\Lambda_\chi)\left[\frac{5}{3}\mu m_q + a^2\left(\frac{3}{2}\bar{\Delta} - \frac{2}{3}\Delta_I\right)\right] \ , \tag{10}$$

where $\bar{\Delta} = \frac{1}{16}\sum_\xi n_\xi \Delta_\xi$ and $n_\xi = 1$, 4, 6, 4, 1 for $\xi = P$, A, T, V, I. The $O(a^2)$ terms ensure that the Λ_χ dependence in f_1 and f_2 cancels that of Δf_q.
Equation 6 parameterizes our chiral extrapolations. We fit all ϕ_{Hq} at each lattice spacing to determine the parameters. Constraints (value and width) for μ, Δ_ξ, f_π, δ'_A and δ'_V come from χPT for lattice pions and kaons [6]. Coupling $g^2_{D^*D\pi}$ is likewise constrained by the CLEO measurement. From HQS we expect $g^2_{B^*B\pi} \approx g^2_{D^*D\pi}$. Remaining parameters Φ, f_1 and f_2 are determined in the fit.
To obtain physical results we set $\Delta_\xi = \delta'_{A,V} = 0$, $m_h \to m_s$ and $m_l \to (m_u + m_d)/2$. Then ϕ_d (ϕ_s) is found in the limit $m_q \to m_d$ (m_s).

4. D meson decay constants

In the figure below are separate D meson chiral extrapolations for the coarse and fine lattices. A total of 60 combinations of (m_q, m_l) are fit for the coarse lattice and 29 combinations for the fine lattice. For each fit, the subset of points along the $m_q = m_l$ direction is shown. The solid curve denotes the fit *including* a^2 effects while the broken curve shows the fit after the a^2 effects arising in Equations (5,7, 10) have been set to zero. Physical values of the decay constants are indicated by filled squares. The physical ϕ_{D_s} value is projected into the plane of the figure.

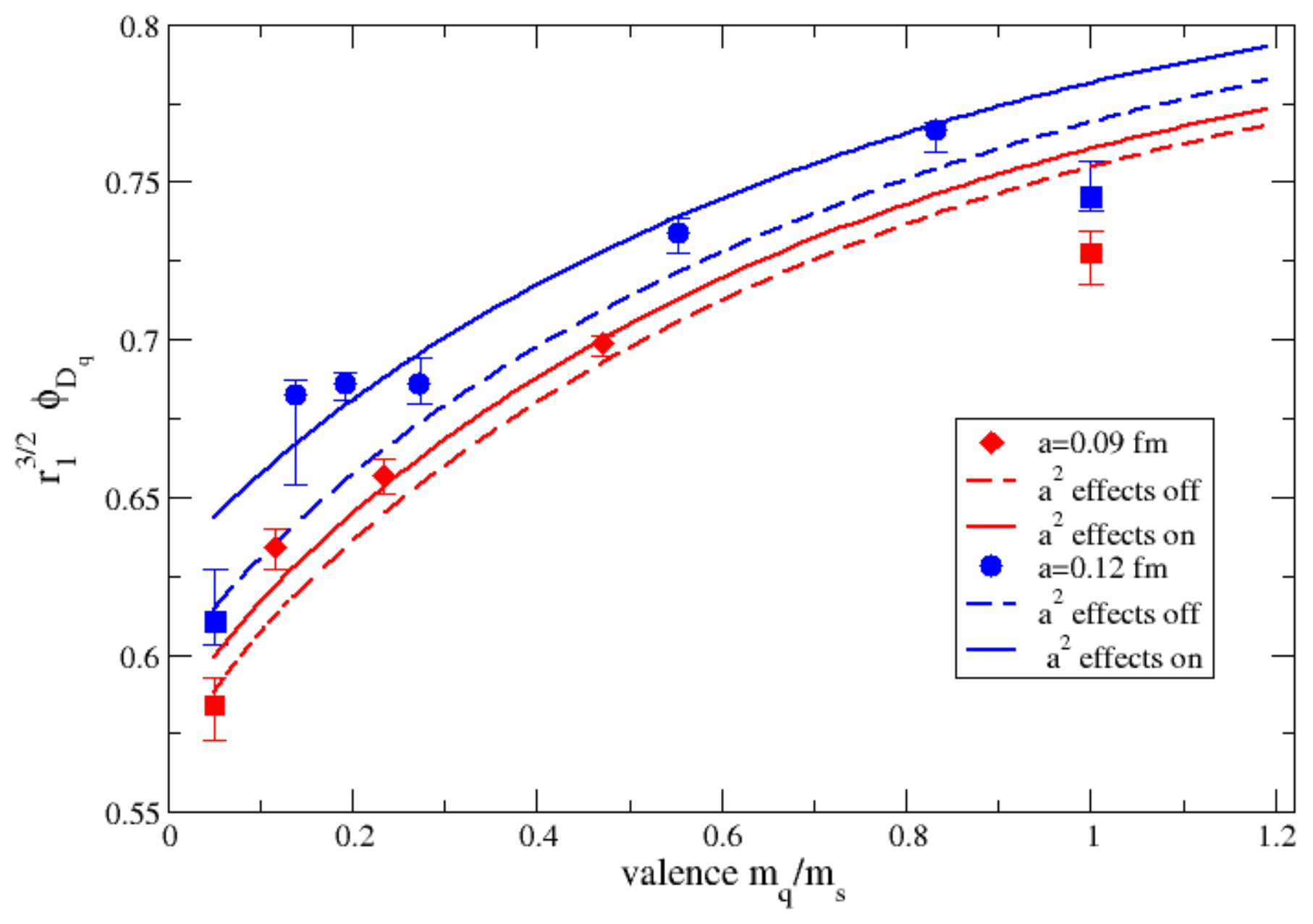

Figure 1: *Chiral extrapolation for f_{D^+} f_{D_s} at $a = 0.09$ fm (red) and at $a = 0.12$ fm (blue). Only statistical errors are shown.*

This result is consistent with our published result. We refrain from quoting new values for f_{D^+} and f_{D_s} until a complete analysis of systematic effects has been completed.
Many systematic effects, common to f_D and f_{D_s}, are expected to cancel in the ratio:

$$f_{D_s}/f_{D^+} = 1.21 \pm 0.01 \pm 0.04 \qquad . \tag{11}$$

The central value and statistical error are from the fine lattice. The ratio from the coarse lattice agrees within statistical errors. The systematic (second) uncertainty we take to be mainly due to the chiral extrapolation.

5. B meson decay constants

Below we show our PRELIMINARY result from the fine lattice.

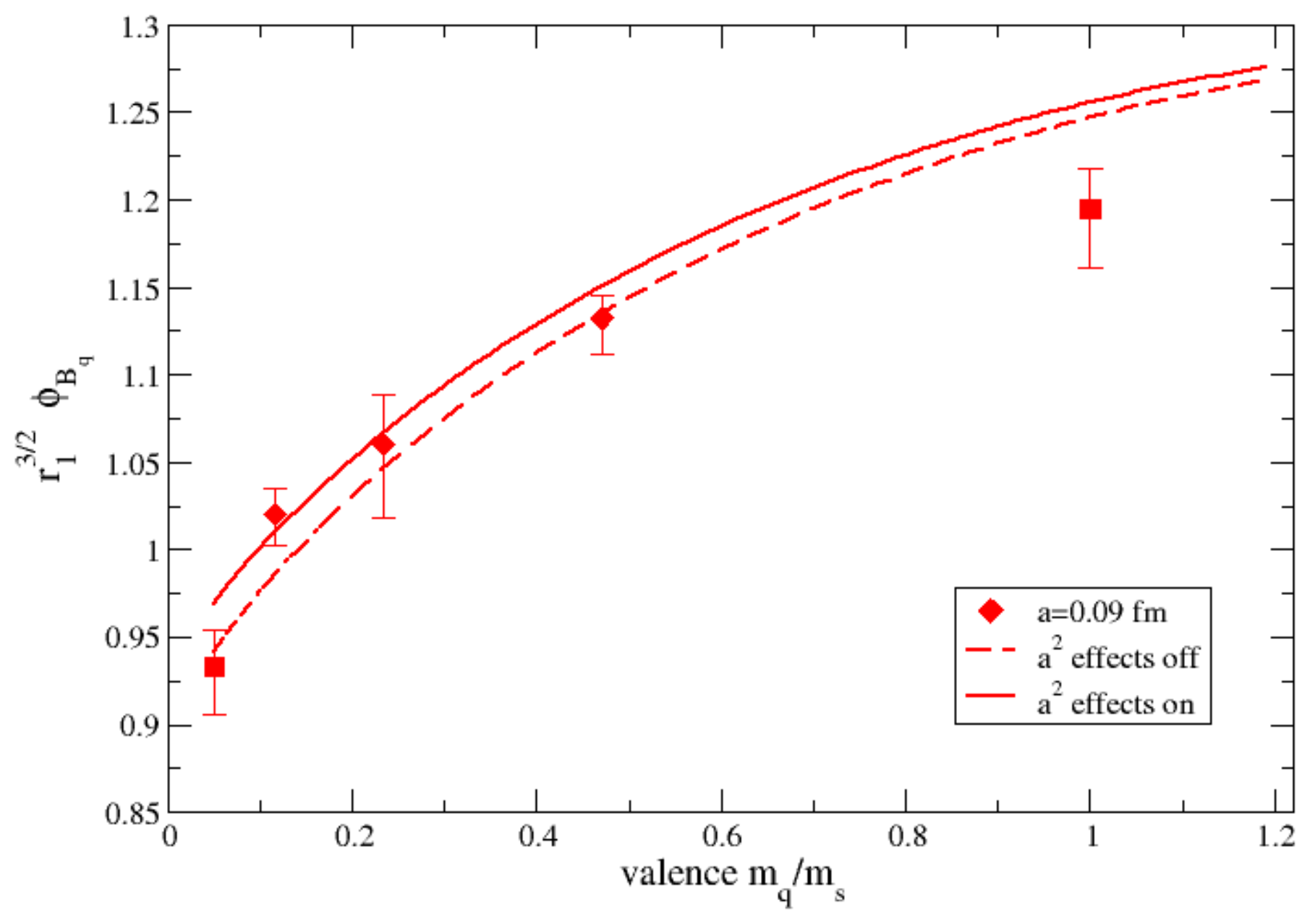

Figure 2: *Chiral extrapolation for f_B and f_{B_s} at $a = 0.09$ fm. Only statistical errors are shown.*

Our preliminary result is consistent with values $f_{B^+} = 199 \pm 6 \pm 35$ MeV and $f_{B_s} = 253 \pm 7 \pm 41$ MeV. Uncertainties are statistical and systematic respectively. The systematic error is predominantly due to the non-perturbative current renormalization which has not been completed.
Again, we take the ratio of decay constants:

$$f_{B_s}/f_{B^+} = 1.27 \pm 0.02 \pm 0.06 \tag{12}$$

and consider the chiral extrapolation the largest source of uncertainty.
In ratios of B to D decay constants we anticipate some cancellation of common statistical errors as well as common systematic effects since the bootstrap analysis preserves correlations.

$$f_{B^+}/f_{D^+} = 0.95 \pm 0.03 \pm 0.06 \tag{13}$$

$$f_{B_s}/f_{D_s} = 0.99 \pm 0.02 \pm 0.06 \tag{14}$$

$$R = (f_{B_s}/f_{B^+})/(f_{D_s}/f_{D^+}) = +1.04 \pm 0.01 \pm 0.02 \tag{15}$$

The systematic uncertainty for the first two ratios is dominated by a combination of effects due to tuning of the input bare m_c and m_b masses, heavy quark discretization effects and the chiral extrapolations. A more detailed analysis of systematic effects is underway. For the double ratio the effects above are reduced.
The deviation of the double ratio from one is a measure of $SU(3)$ flavor symmetry breaking among the decay constants and is expected to be small $R - 1 \approx -3.3\%$ [7].

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