Matrix elements and diquark correlations in quenched QCD with overlap fermions

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Simulation details

Quenched QCD with Wilson gauge action, 6-hit Metropolis algorithm (acceptance ≈ 0.5), 100 configurations separated by 10,000 upgrades.

Overlap quark propagators calculated for a single point source, all 12 color-spin combinations, after gauge fixing to the Landau gauge.

$18^3 \times 64$ lattice: $\beta = 6$, $a^{-1} = 2.17(9)\mathrm{GeV}$, $\rho = 1.4$,
$am_q = 0.03, 0.04, 0.06, 0.08, 0.1, 0.25, 0.5, 0.75$

$14^3 \times 48$ lattice: $\beta = 5.85$, $a^{-1} = 1.49(4)\mathrm{GeV}$, $\rho = 1.6$,
$am_q = 0.03, 0.04, 0.053, 0.08, 0.106, 0.132, 0.33, 0.66, 0.99$

Zolotarev approx. with 12 poles (used for first 55 $18^3 \times 64$ configurations) then Chebyshev approx. (degree $100 \sim 500$), after Ritz projection of the lowest 12 ($18^3 \times 64$)   40 ($14^3 \times 48$) eigenvectors of H^2.

Convergence criteria: $|x^{-1/2} - \sum T_n(x)| < 10^{-8}$, $|D^\dagger D\psi - \chi|^2 < 10^{-7}$.

F90 shared memory code, run on 16 and 32 proc. IBM-p690 at BU and NCSA.

$\Delta S = 2$ operators

$$
\begin{aligned}
O_1 &= [\bar{s}^a \gamma_\mu (1-\gamma_5) d^a][\bar{s}^b \gamma^\mu (1-\gamma_5) d^b] \\
O_2 &= [\bar{s}^a (1-\gamma_5) d^a][\bar{s}^b (1-\gamma_5) d^b] \\
O_3 &= [\bar{s}^a (1-\gamma_5) d^b][\bar{s}^b (1-\gamma_5) d^a] \\
O_4 &= [\bar{s}^a (1-\gamma_5) d^a][\bar{s}^b (1+\gamma_5) d^b] \\
O_5 &= [\bar{s}^a (1-\gamma_5) d^b][\bar{s}^b (1+\gamma_5) d^a]
\end{aligned}
$$

$O_2 - O_5$, plus the parity transformed $\tilde{O}_1 - \tilde{O}_3$ appear in extensions of the standard model. Their matrix elements are parametrized by

$B_1(\mu) = B_K(\mu) \equiv \frac{\langle \bar{K}^0 | O_1(\mu) | K^0 \rangle}{\frac{16}{3} M_K^2 F_K^2}, \quad B_i(\mu) \equiv -\frac{\langle \bar{K}^0 | O_i(\mu) | K^0 \rangle}{N_i \left(\frac{\sqrt{2} F_K M_K^2}{m_s(\mu) + m_d(\mu)} \right)^2}$ with

$N_i = \{5/3, -1/3, -2, -2/3\}, i = 2 - 5$ Note: $B_{7,8}^{3/2} = B_{5,4}$

Bare matrix elements

We calculate

$$\mathcal{B}^1_{PP}(x_0,y_0) = \frac{\sum_{\vec{x},\vec{y}}\langle P(x)O_1(0)P(y)\rangle}{\frac{8}{3}\sum_{\vec{x},\vec{y}}\langle P(x)\bar{A}_0(0)\rangle\langle\bar{A}_0(0)P(y)\rangle} \xrightarrow{a\ll x_0\ll T/2\ll y_0\ll T} B_1$$

$$\mathcal{B}^i_{PP}(x_0,y_0) = \frac{\sum_{\vec{x},\vec{y}}\langle P(x)O_i(0)P(y)\rangle}{N_i\sum_{\vec{x},\vec{y}}\langle P(x)\bar{P}(0)\rangle\langle\bar{P}(0)P(y)\rangle} \xrightarrow{a\ll x_0\ll T/2\ll y_0\ll T} B_i$$

and fit to a constant in the symmetric time intervals given by: $12 \leq x_0/a \leq 19$ and $45 \leq y_0/a \leq 52$ for $i{=}1,\cdots 5$, at $\beta = 6.0$; $10 \leq x_0/a \leq 12$ and $36 \leq y_0/a \leq 38$ for $i{=}1$ and $10 \leq x_0/a \leq 14$ and $34 \leq y_0/a \leq 38$ for $i{=}2,\cdots 5$, at $\beta = 5.85$

Non-perturbative renormalization

We work in the Landau gauge and calculate forward two-quark and four-quark Green functions with momentum p, forming the ratios

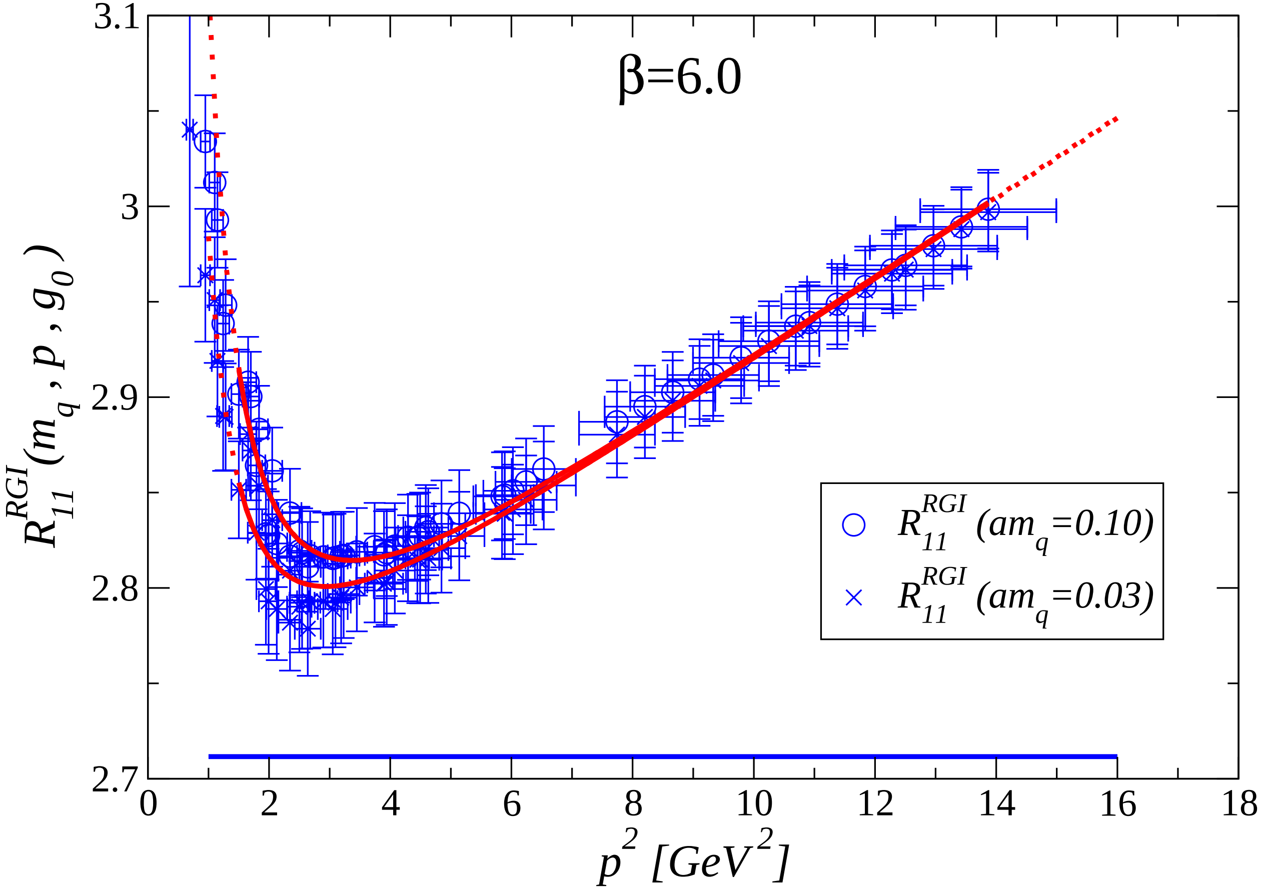
$$\mathcal{R}_{ij}^{\mathrm{RI}}(m_q, p, g_0) \equiv Z_A^2 \frac{\mathrm{Tr}\{\Lambda_V(m_q, p, g_0)\mathcal{P}_V\}^2}{\mathrm{Tr}\{\Lambda_{O_j}(m_q, p, g_0)\mathcal{P}_{O_i}\}}$$

with $i = j = 1$ for O_1, $i, j \in \{2, 3\}$ to for the mixing pair O_2 and O_3 and $i, j \in \{4, 5\}$ for the mixing pair O_4, O_5. $\mathcal{P}_O$ are normalized projectors onto the spin-color structure of the tree-level operators O.

We fit to

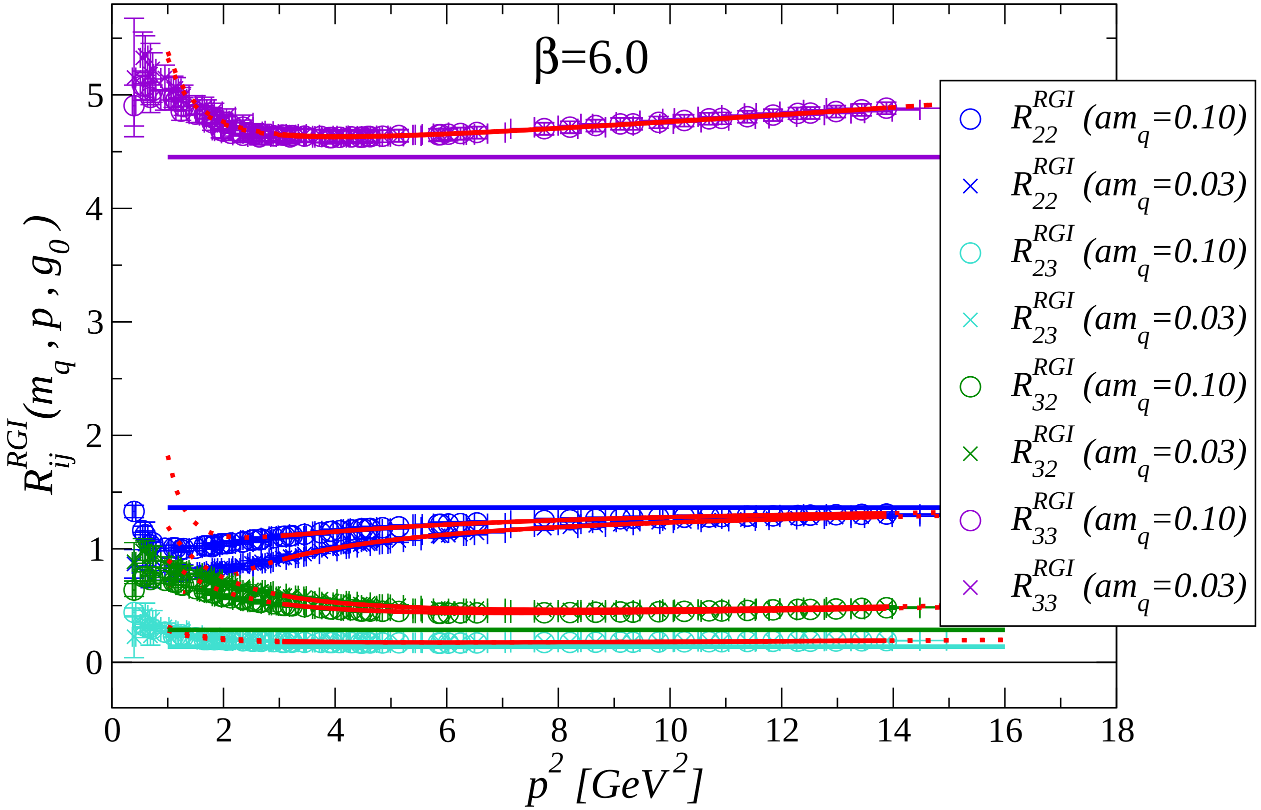
$$\mathcal{R}_{ij}^{\mathrm{RI}}(m_q, p, g_0) =$$
$$\cdots + \frac{A_{ij}^{(2)}(m_q, g_0, p^2)}{p^2} + U_{ik}^{\mathrm{RI}}(p^2)\, Z_{kj}^{\mathrm{RGI}}(g_0) + B_{ij}^{(2)}(g_0, p^2)\,(ap)^2 + \cdots \ ,$$

Renormalization, cont'd



Plot of the renormalization ratio $\mathcal{R}_{11}^{\mathrm{RGI}}(m_q, p, g_0)$ for the standard model operator O_1 as a function of four-momentum squared, p^2, for the most massive ($am_q = 0.10$, circles) and lightest ($am_q = 0.03$, crosses) quarks.

Renormalization, cont'd



Renormalization results for the operator pair $\{O_2,\ O_3\}$.

Results

Final results at the kaon mass in the RI/MOM scheme at $2\mathrm{GeV}$. The first error is statistical and the second is systematic. Also shown for comparison are the results of Donini et al., obtained using quenched, tree-level, $O(a)$-improved Wilson fermions.

i	B_i	
	This work	Donini et al.
1	$0.563(47)(30)$	$0.69(21)$
2	$0.865(72)(35)$	$0.70(9)$
3	$1.41(10)(13)$	$1.1(1)$
4	$0.938(48)(36)$	$1.1(1)$
5	$0.616(51)(59)$	$0.77(11)$

Results, cont'd

Results, at $\beta = 6.0$ and 5.85, for the ratios $D^{3/2}_{7,8} = \langle \pi^+ | Q^{3/2}_{7,8} | K^+ \rangle / F^2$ interpolated to the kaon point and best estimate for $\langle (\pi\pi)_{I=2} | Q_{7,8} | K^0 \rangle$ in the chiral (written as a $K^+ \to \pi^+$ matrix element). All results are given in the RI/MOM scheme at $2\mathrm{GeV}$ and units of GeV.

β	$D_7^{3/2}(M_K^2)$	$D_8^{3/2}(M_K^2)$	$[\frac{\langle \pi^+ \vert Q_7^{3/2} \vert K^+ \rangle}{F}]_\chi$	$[\frac{\langle \pi^+ \vert Q_8^{3/2} \vert K^+ \rangle}{F}]_\chi$
6.0	$1.63(14)(1)$	$7.7(11)(0)$	$0.163(14)(1)(21)$	$0.77(11)(0)(10)$
5.85	$1.38(34)(21)$	$6.6(9)(3)$	$0.138(34)(21)(18)$	$0.66(9)(3)(9)$

(see hep-lat/0605016)

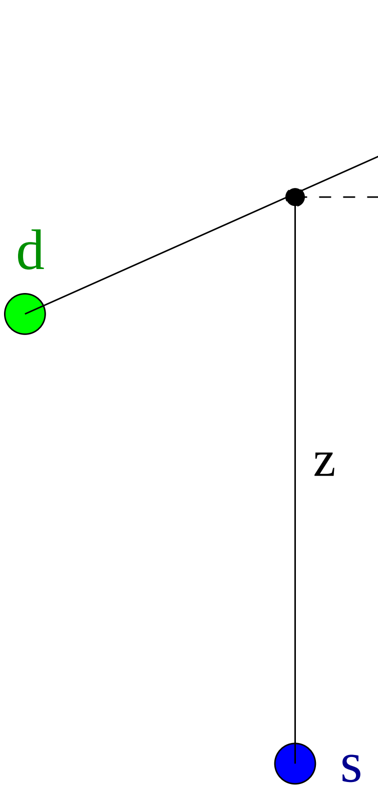
Baryon wave-functions and diquark correlations

Calculate

$< (\bar{u}\bar{d}\bar{s})(t)(uds)(0) >$

in the Landau or Coulomb gauge,

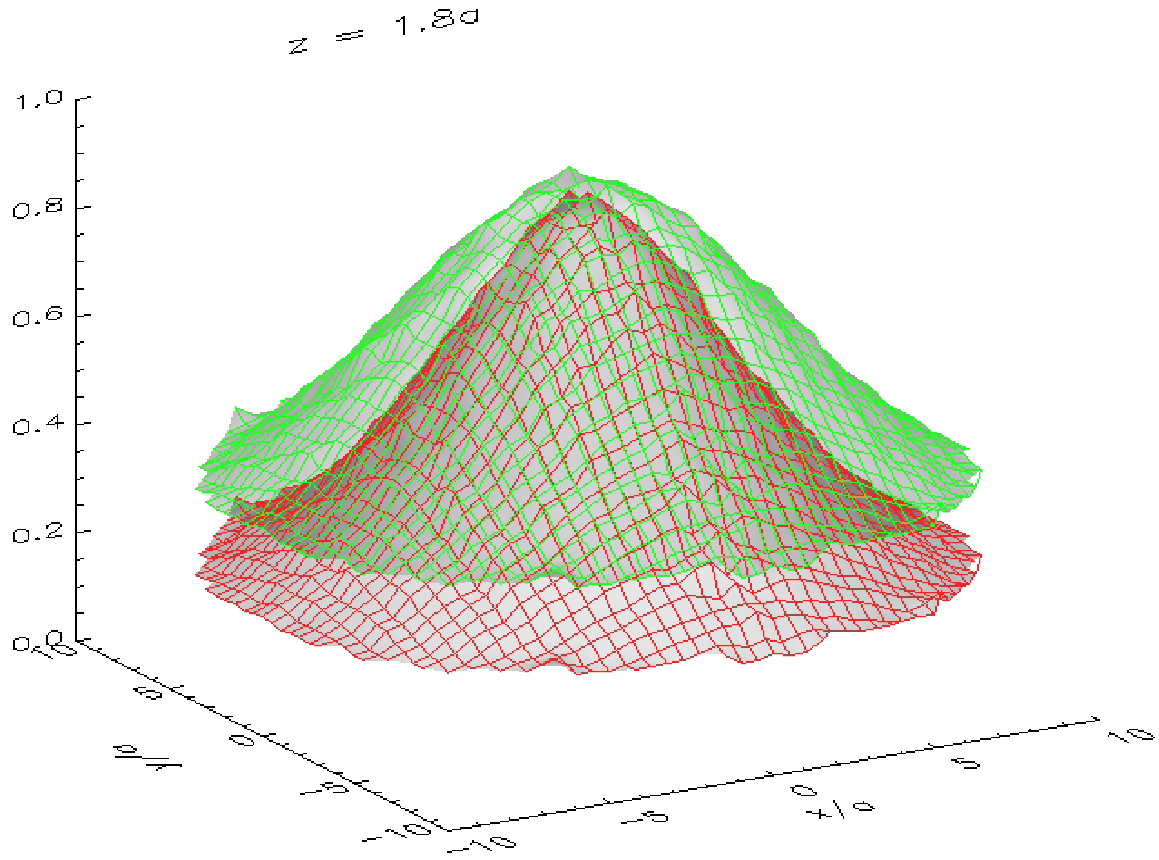
and plot vs. x, y at fixed z



(see also Hecht et al., hep-lat/9208005)

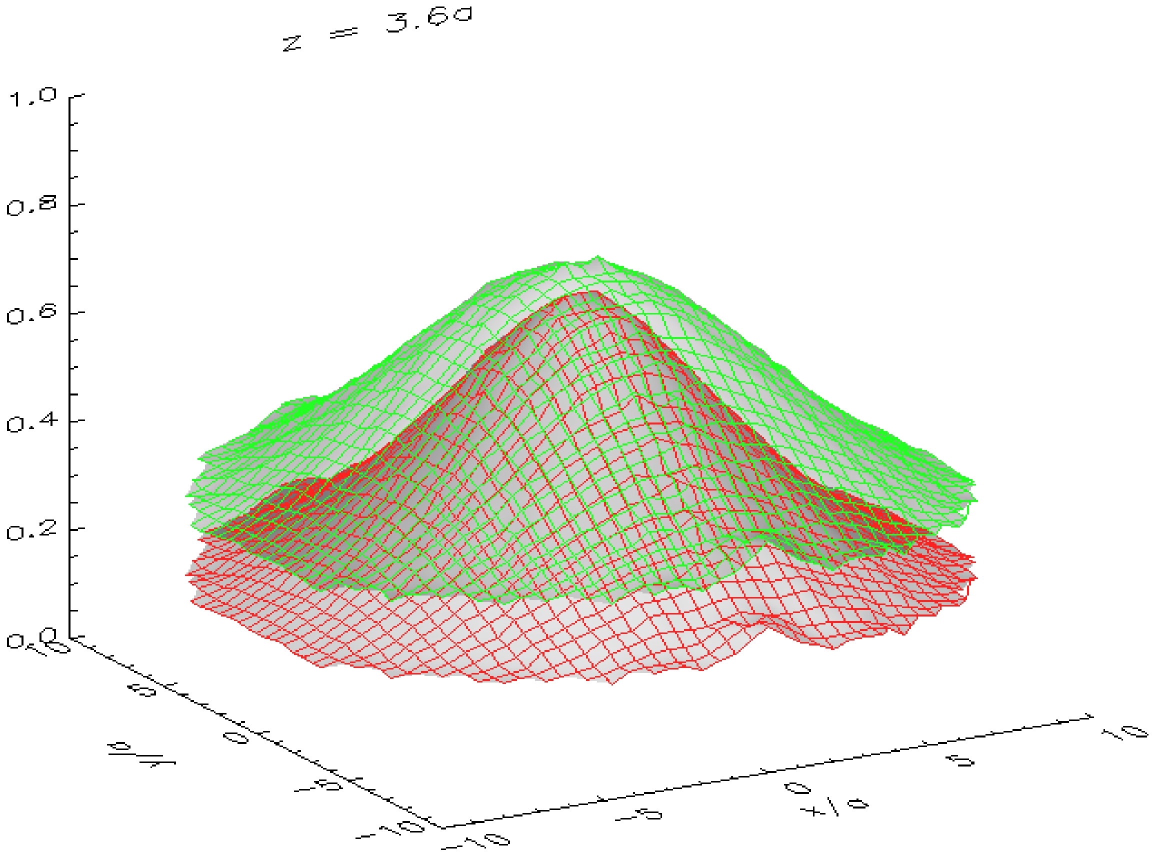
Baryon wave-functions

Red is for u, d in an isospin 0 state (Λ-like), green for u, d in an isospin 1 state (Σ-like) $(z = 1.8a)$



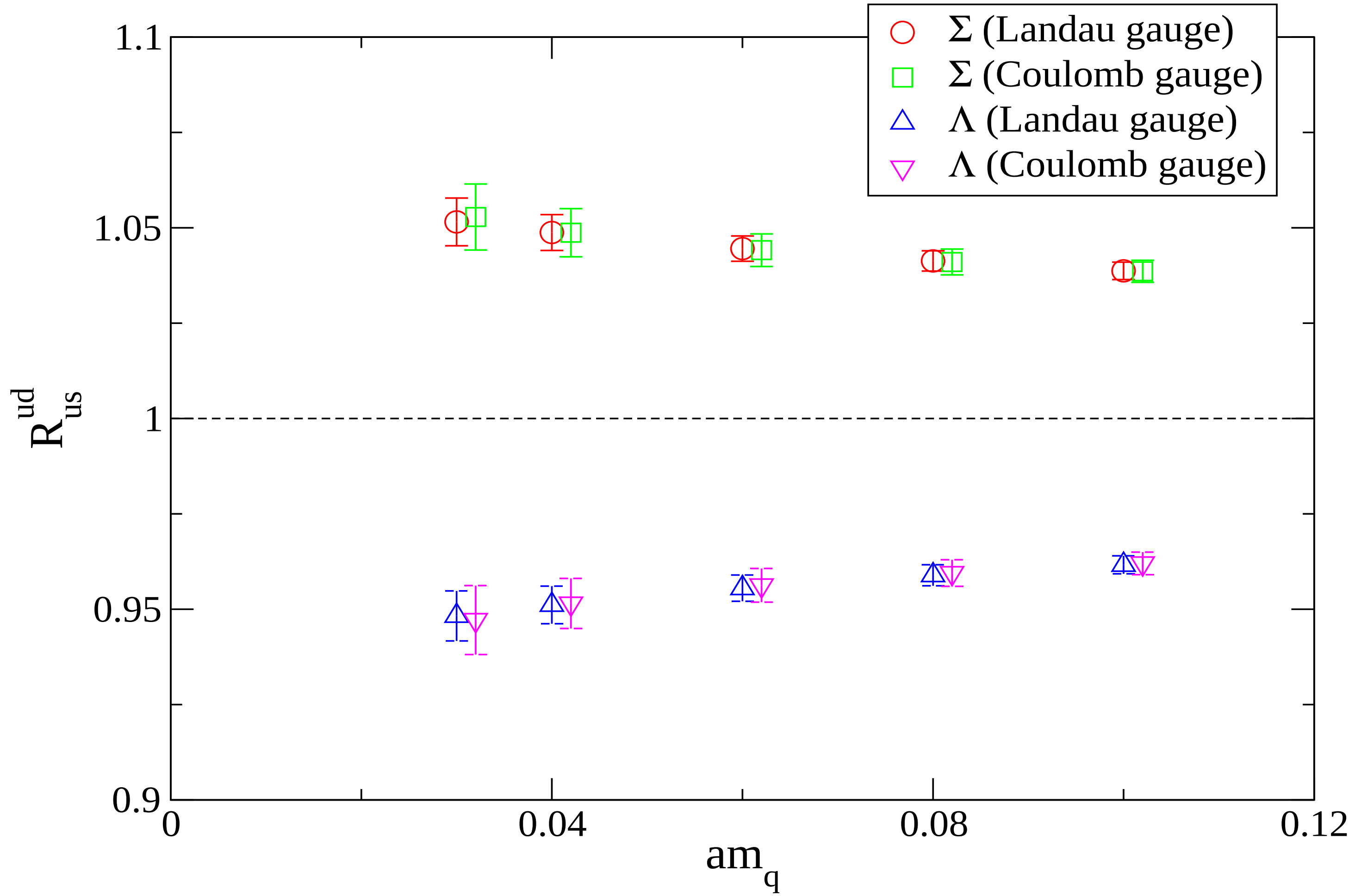
Baryon wave-functions

Red is for u, d in an isospin 0 state (Λ-like), green for u, d in an isospin 1 state (Σ-like) $(z = 3.6a)$



Diquark correlations

Ratio of mean u-d over mean u-s separation in the Λ-like and Σ-like states



Conclusions

The overlap discretization, at least insofar as valence quarks are concerned, can be used in large scale simulations, and, because of its very good symmetry properties, represents a choice method for QCD numerical calculations.

Our results provide validation of the method, some novel matrix element values, and evidence for strong diquark correlation in a flavor $\bar{3}$, spin-singlet state.