The B_s Meson Mixing in Full QCD

(HPQCD Collaboration)

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Major Achievement at the Tevatron in 2006

Two-sided bound on ΔM_s from DØ quickly followed by a precise measurement from CDF,

$$\Delta M_s = 17.31^{+0.33}_{-0.18} \pm 0.07 \ \ ps^{-1}$$

$$|B_s(H)\rangle = p|B_s\rangle + q|\overline{B}_s\rangle \qquad \Delta M_s = M_H - M_L$$

$$|B_s(L)\rangle = p|B_s\rangle - q|\overline{B}_s\rangle \qquad \Delta\Gamma_s = \Gamma_L - \Gamma_H$$

Unofficial world average (R.v.Kooten, FP&CP, April 2006)

$$\Delta\Gamma_s = 0.097^{+0.041}_{-0.042} \ ps^{-1} \rightarrow \left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} \approx 0.15 \pm 0.06$$

Theoretical predictions require Lattice input.

The HPQCD collaboration has studied hadronic matrix elements relevant for ΔM_s and $(\Delta\Gamma/\Gamma)_{B_s}$ using,

- MILC $N_f = 2 + 1$ configurations
- NRQCD b-quarks
- AsqTad light quarks

As in our studies of B leptonic and semileptonic decays, all action parameters fixed via light and heavy-heavy simulations prior to embarking on B physics.

a^{-1} ..... Υ 2S-1S splitting
m_b ...... Υ
m_s ...... Kaon

Relevant Four Fermion Operators

$$
\begin{aligned}
OL &\equiv [\bar{b}^i\, q^i]_{V-A}[\bar{b}^j\, q^j]_{V-A} \\
OS &\equiv [\bar{b}^i\, q^i]_{S-P}[\bar{b}^j\, q^j]_{S-P} \\
Q3 &\equiv [\bar{b}^i\, q^j]_{S-P}[\bar{b}^j\, q^i]_{S-P} \\
OLj1 &\equiv \frac{1}{2M}\left\{[\vec{\nabla}\bar{b}^i\cdot\vec{\gamma}\, q^i]_{V-A}[\bar{b}^j\, q^j]_{V-A} \;+\; [\bar{b}^i\, q^i]_{V-A}[\vec{\nabla}\bar{b}^j\cdot\vec{\gamma}\, q^j]_{V-A}\right\} \\
OSj1 &\equiv \frac{1}{2M}\left\{[\vec{\nabla}\bar{b}^i\cdot\vec{\gamma}\, q^i]_{S-P}[\bar{b}^j\, q^j]_{S-P} \;+\; [\bar{b}^i\, q^i]_{S-P}[\vec{\nabla}\bar{b}^j\cdot\vec{\gamma}\, q^j]_{S-P}\right\} \\
Q3j1 &\equiv \frac{1}{2M}\left\{[\vec{\nabla}\bar{b}^i\cdot\vec{\gamma}\, q^j]_{S-P}[\bar{b}^j\, q^i]_{S-P} \;+\; [\bar{b}^i\, q^j]_{S-P}[\vec{\nabla}\bar{b}^j\cdot\vec{\gamma}\, q^i]_{S-P}\right\}
\end{aligned}
$$

"i,j" are color indices.

OL which mixes with OS at $\mathcal{O}(\alpha_s)$ is the operator relevant for the mass difference ΔM_s.

For $\Delta\Gamma_s$ one needs either OS & OL or $Q3$ & OL. Recently Lenz & Nierste have argued that the latter pair leads to smaller theoretical uncertainties.$[\langle Q3\rangle = -\langle OS\rangle - \frac{1}{2}\langle OL\rangle + \mathcal{O}(1/M)]$

Matching

$$\frac{1}{2M_{B_s}a^3}a^6\,\langle OL\rangle^{\overline{\text{MS}}} = [1+\alpha_s\cdot\rho_{LL}]\langle OL\rangle_{eff}+\alpha_s\cdot\rho_{LS}\langle OS\rangle_{eff}$$
$$+\ \left[\langle OLj1\rangle_{eff}-\alpha_s\left(\zeta_{10}^{LL}\langle OL\rangle_{eff}+\zeta_{10}^{LS}\langle OS\rangle_{eff}\right)\right]$$

This quantity is defined to be

$$\frac{a^3}{2M_{B_s}}\,\langle OL\rangle^{\overline{\text{MS}}}\equiv\frac{a^3}{2M_{B_s}}\,\left[\tfrac{8}{3}f_{B_s}^2B_{B_s}M_{B_s}^2\right]=\tfrac{4}{3}a^3M_{B_s}\left(f_{B_s}^2B_{B_s}\right)$$

Similarly,

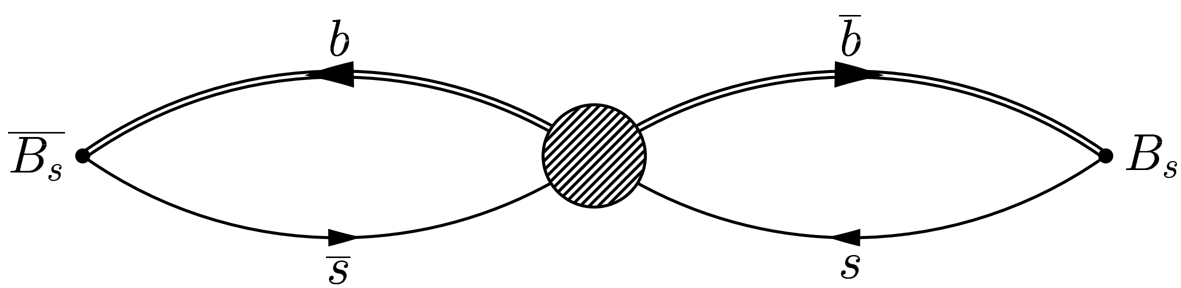
$$\frac{a^3}{2M_{B_s}}\,\langle OS\rangle^{\overline{\text{MS}}}=-\frac{5}{6}a^3M_{B_s}\left(f_{B_s}^2\frac{B_S}{R^2}\right)$$

$$\frac{a^3}{2M_{B_s}}\,\langle Q3\rangle^{\overline{\text{MS}}}=\ \frac{1}{6}a^3M_{B_s}\left(f_{B_s}^2\frac{\tilde{B}_S}{R^2}\right)$$

where $\frac{1}{R}=\frac{M_{B_s}}{(\overline{m}_b+\overline{m}_s)}$.

Four Fermion Operator Matrix Elements

$$C^{(4f)}(t_1, t_2) = \sum_{\vec{x}_1, \vec{x}_2} \langle 0 | \Phi_{\overline{B}_s}(\vec{x}_1, t_1) \, [\widehat{Q}]_{(0)} \, \Phi^\dagger_{B_s}(\vec{x}_2, -t_2) | 0 \rangle$$



We work with $1 \leq t_1, t_2 \leq 16$

Fitting

$$C^{(4f)}(t_1, t_2) = \sum_{j,k=0}^{N_{exp}-1} A_{jk} \, (-1)^{j \cdot t_1} \, (-1)^{k \cdot t_2} \, e^{-E_B^{(j)}(t_1-1)} \, e^{-E_B^{(k)}(t_2-1)}$$

$$C^B(t) = \sum_{\vec{x}} \langle 0 | \Phi_{B_s}(\vec{x}, t) \, \Phi^\dagger_{B_s}(0) | 0 \rangle = \sum_{j=0}^{N_{exp}-1} \xi_j \, (-1)^{j \cdot t} \, e^{-E_B^{(j)}(t-1)}$$

Fit directly to $C^{(4f)}(t_1, t_2)$ rather than take ratios.
Use Bayesian methods and let $N_{exp} \leq 7 - 9$.
Use close to the entire range $2 \leq t_1, t_2 \leq 16$.
Simultaneous fits with the B correlator.

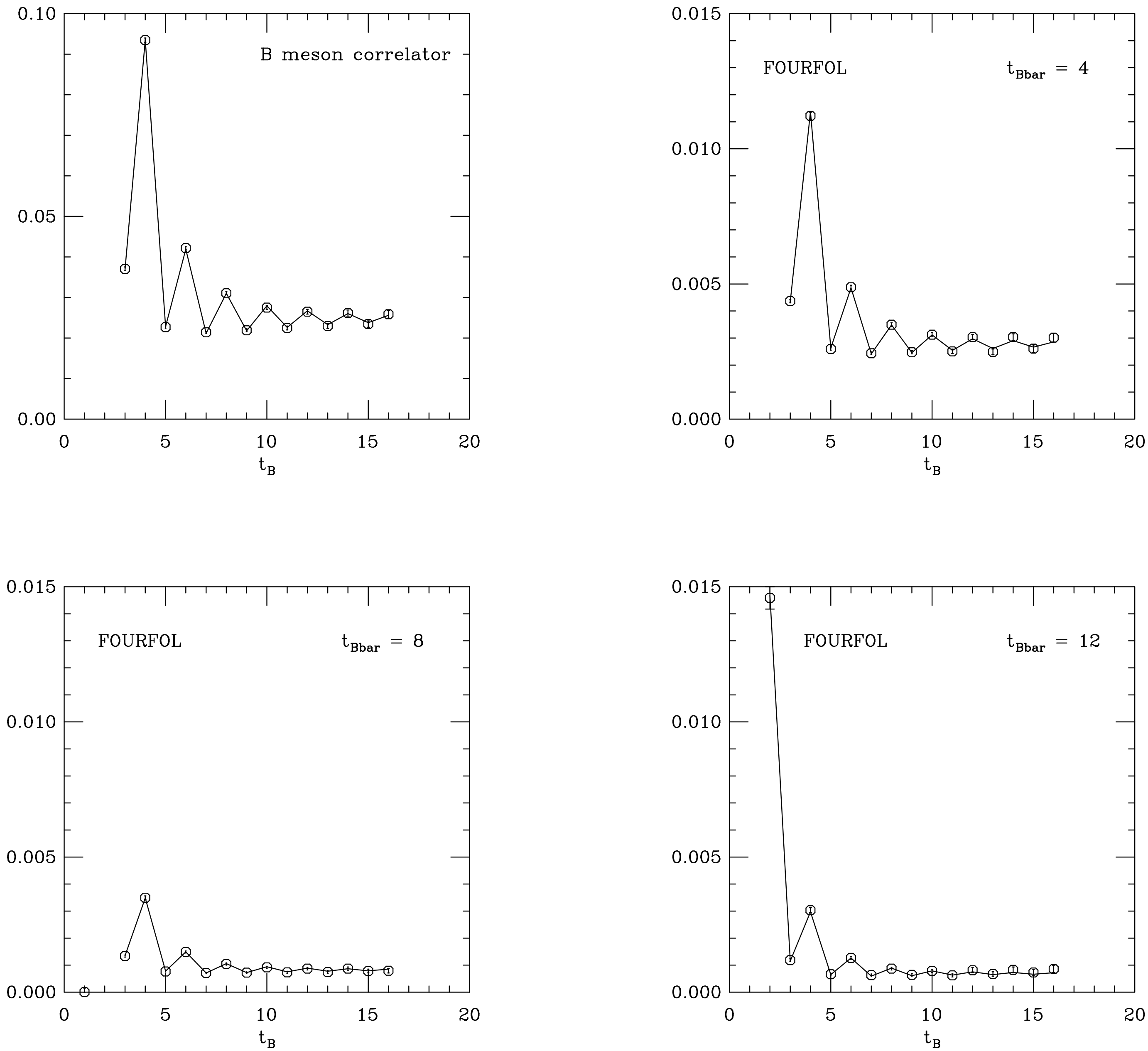
$$\frac{A_{00}}{\xi_0} = \langle \overline{B}_s | \hat{Q} | B_s \rangle_{eff} \equiv \langle \hat{Q} \rangle_{eff}$$

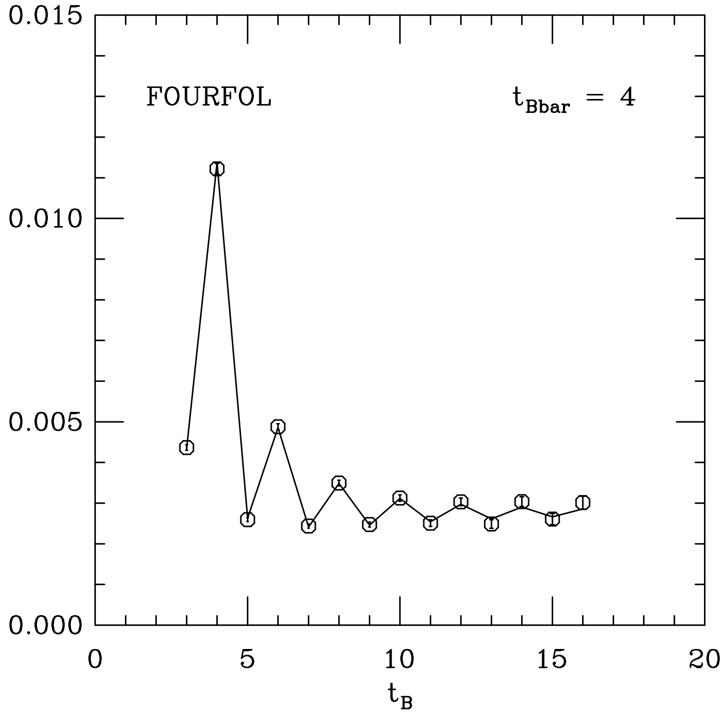
Fitting (cont'd)

Fits were more challenging than in our previous experience with B leptonic and semileptonic decay matrix elements. Hence the central values of fit parameters were fixed by using very narrow prior widths for some of the priors, i.e. essentially fits were carried out with $E_B^{(0)}$ and $E_B^{(1)}$ fixed to their known values from pure B correlator fits. "statistical + fitting errors " were then inflated to take into account what happens when prior widths are relaxed.

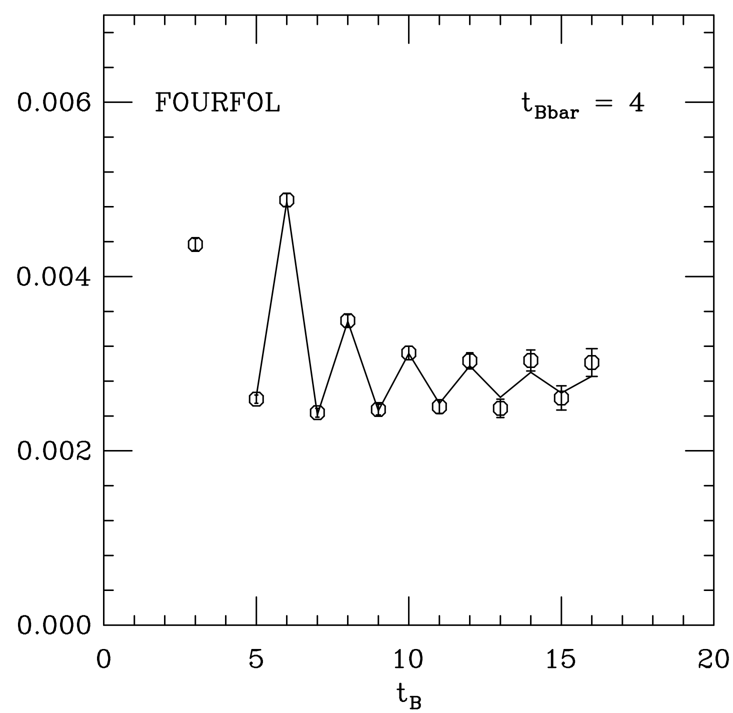
Following plots show

$$C^B(t)\, e^{E_B^{(0)}\cdot t} \quad and \quad C^{(4f)}(t_B, t_{Bbar})\, e^{E_B^{(0)}\cdot(t_B + t_{Bbar})}$$

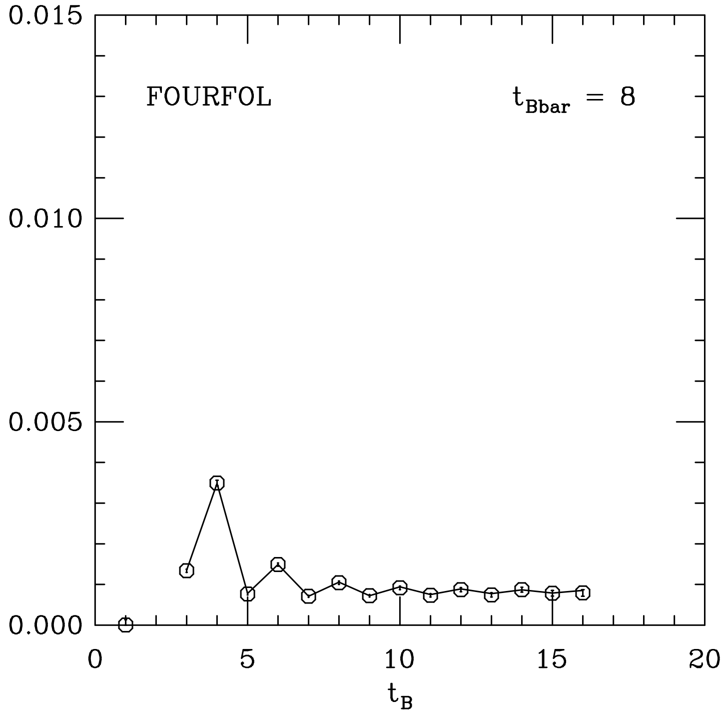

B meson correlator
FOURFOL
t_{Bbar} = 4
FOURFOL
t_{Bbar} = 8
FOURFOL
t_{Bbar} = 12
t_B
0.00
0.05
0.10
0.000
0.005
0.010
0.015
0
5
10
15
20



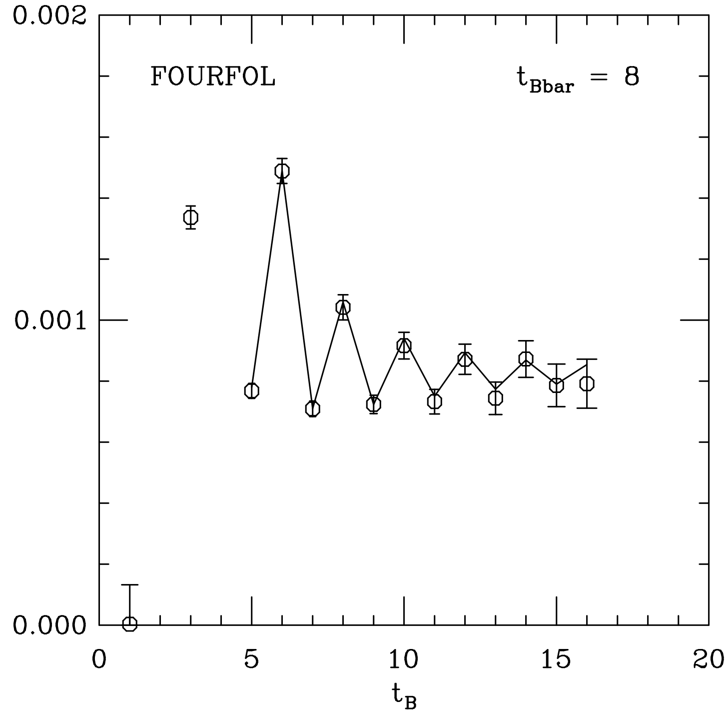
FOURFOL
t_{Bbar} = 4
0.015
0.010
0.005
0.000
0
5
10
15
20
t_B



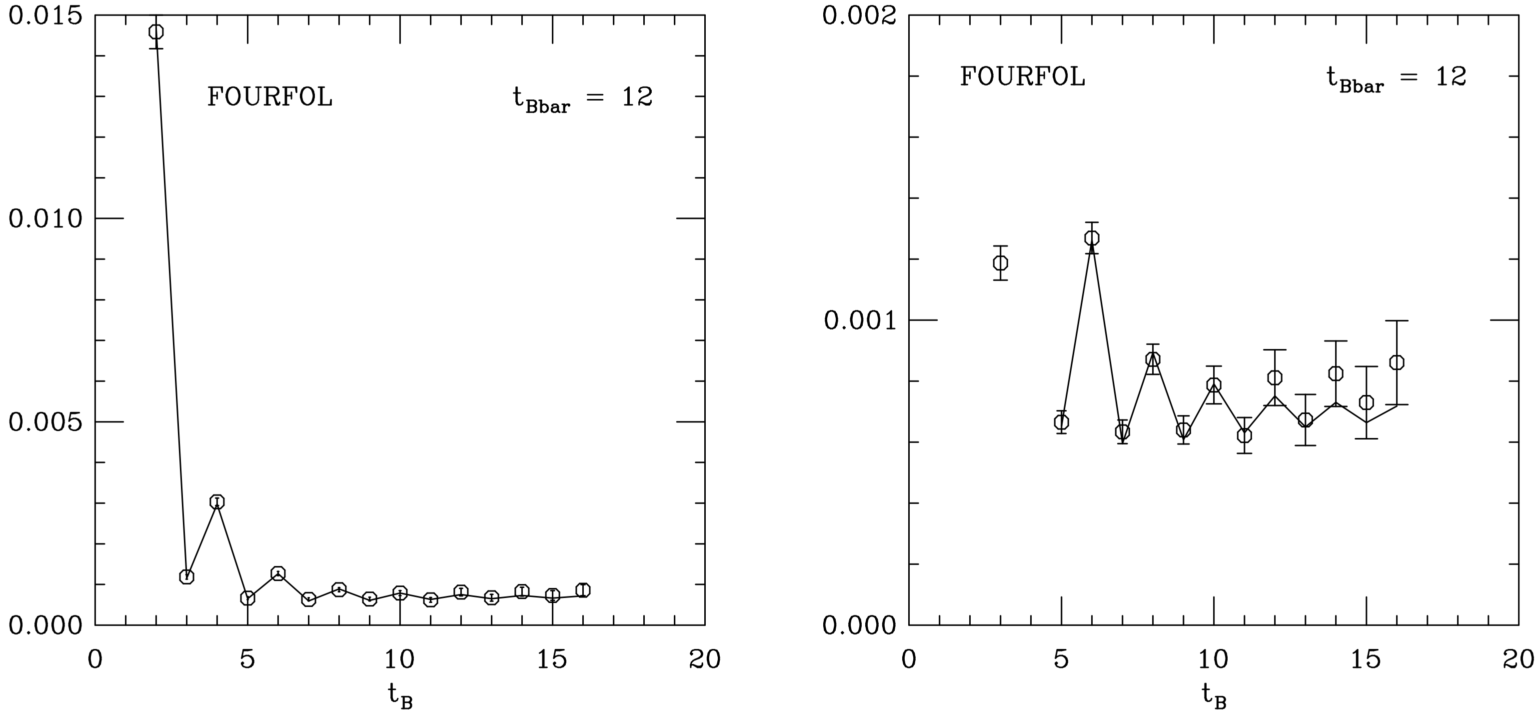
FOURFOL
t_{Bbar} = 4
0.006
0.004
0.002
0.000
0
5
10
15
20
t_B

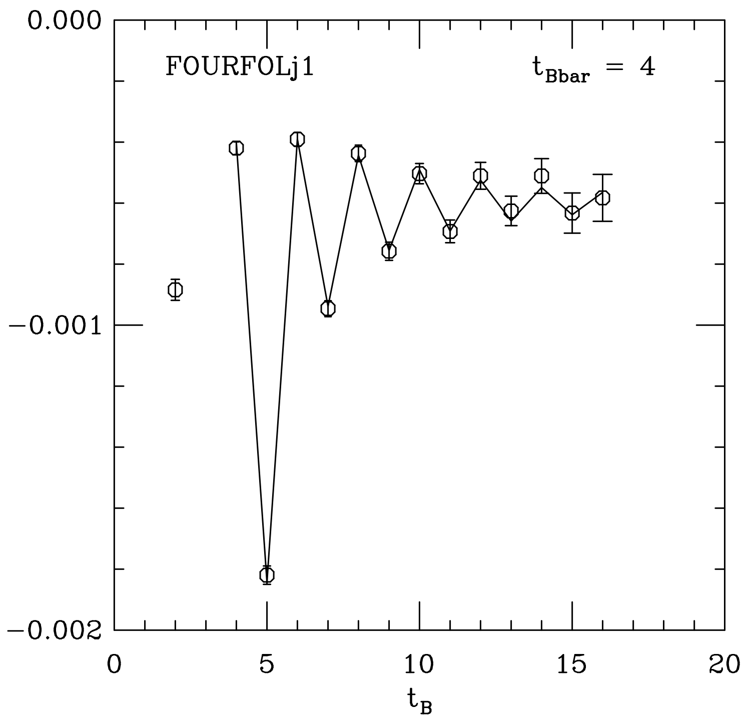


0.015
0.010
0.005
0.000
FOURFOL
t_{Bbar} = 8
0
5
10
15
20
t_B

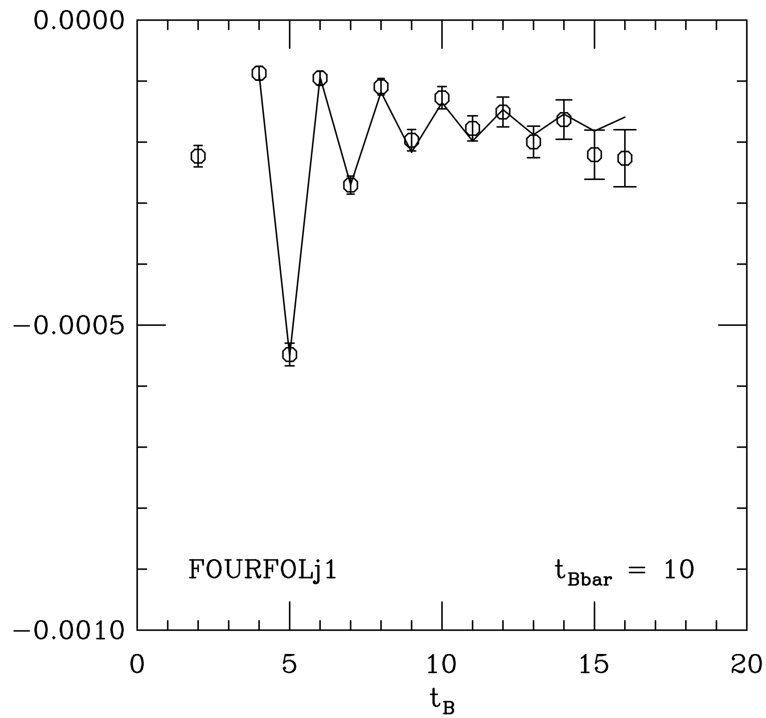


0.002
0.001
0.000
FOURFOL
t_{Bbar} = 8
0
5
10
15
20
t_B


FOURFOL
t_{Bbar} = 12
0.015
0.010
0.005
0.000
0
5
10
15
20
t_B
FOURFOL
t_{Bbar} = 12
0.002
0.001
0.000
0
5
10
15
20
t_B



FOURFOLj1
t_{Bbar} = 4
0.000
−0.001
−0.002
0
5
10
15
20
t_B



FOURFOLj1
t_{Bbar} = 10
0.0000
−0.0005
−0.0010
0
5
10
15
20
t_B

Effect of 1/M Correction

	OL	OS	$Q3$
$\langle OXj1\rangle_{eff}/\langle OX\rangle_{eff}$ (before power law subtraction)	-22%	19%	16%
$\langle OXj1\rangle_{(sub)}/\langle OX\rangle_{eff}$ (after power law subtraction)	-13%	11%	7%
$\langle OXj1\rangle_{(sub)}/[full\ (1/M)^0]$	-13%	8%	4%

Main Errors in $f_{B_s}^2 B_{B_s}$

(preliminary error analysis)

Statistical + Fitting	9 %
Higher Order matching	9 %
Discretization	4 %
Relativistic	3 %
Scale (a^{-3})	5 %
Total	15 %

The statistical + systematical errors are still substantial. Nevertheless, this error is $<$ error for $f_{B_s}^2 \approx 2\times(10-12)\% = (20-24)\%$

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Main Results to Date

(preliminary)

	$m_f/m_s = 0.25$	$m_f/m_s = 0.50$
$f_{B_s}\sqrt{\hat{B}_{B_s}}$ [GeV]	0.281(21)	0.289(22)
$f_{B_s}\sqrt{B_{B_s}}$ [GeV]	0.227(17)	0.233(17)
$f_{B_s}\frac{\sqrt{B_S}}{R}$ [GeV]	0.295(22)	0.301(23)
$f_{B_s}\frac{\sqrt{\tilde{B}_S}}{R}$ [GeV]	0.305(23)	0.310(23)

$(\mu = m_b)$

Light sea quark mass dependence smaller than current errors. We will use the $m_f/m_s = 0.25$ results in the following comparison with experimental data.

Comparison with Experiment : ΔM_s

CDF Measurement :

$$\Delta M_s = 17.31^{+0.33}_{-0.18} \pm 0.07 \ \ ps^{-1}$$

The Standard Model expression for the mass difference is :

$$\Delta M_s = \frac{G_F^2 M_W^2}{6\pi^2} |V_{ts}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_s} f_{B_s}^2 \hat{B}_{B_s}$$

where $x_t = m_t^2/M_W^2$. η_2^B is a QCD correction factor and $S_0(x_t)$ is the Inami-Lim function.

Taking $|V_{ts}^* V_{tb}| \approx |V_{cs}^* V_{cb}| \approx 4.1 \times 10^{-2}$ and using our 0.281(21)GeV number for $f_{B_s}\sqrt{\hat{B}_{B_s}}$ one gets

$$\Delta M_s(theory) = 20.3(3.0)(0.8) ps^{-1}$$

Conversely, $|V_{ts}^* V_{tb}| = 3.8(3)(1) \times 10^{-2}$.

Comparison with Experiment : $\Delta\Gamma_s/\Gamma_s$

Unofficial World Average (R.v.Kooten, FPCP, Vancouver, April 2006)

$$\Delta\Gamma_s = 0.097^{+0.041}_{-0.042}\, ps^{-1} \longrightarrow \left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} \approx 0.15 \pm 0.06$$

Use (preliminary) NLO formula of Lenz & Nierste [A.Lenz, LHC Workshop, June 2006]

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{245\,MeV}\right)^2 \left[0.170\, B_{B_s} + 0.059\, \tilde{B}_S - 0.044\right]$$

$$\rightarrow \left(\frac{1}{0.245}\right)^2 \left[0.170\,\left(f^2_{B_s} B_{B_s}\right) + 0.059\, R^2 \left(f^2_{B_s}\frac{\tilde{B}_S}{R^2}\right) - 0.044\, f^2_{B_s}\right]$$

Inserting HPQCD's $f_{B_s} = 0.260(29)$GeV and $R^2 = 0.652$ (see later) and numbers from previous Table of Results (in GeV) one gets,

$$\boxed{\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} (theory) = 0.16(3)(2)}$$

The Bag Parameters

In order to compare with other work we need to convert our results into numbers for B_{B_s}, B_S and $\tilde{B}_S$. This requires values for f_{B_s}, $\overline{m}_b$ and $\overline{m}_s$. We use the central values

$f_{B_s} = 0.260 GeV$ (HPQCD, M.Wingate et al.)

$\overline{m}_b = 4.25 GeV$, $\overline{m}_s = 85 MeV$ (Lenz & Nierste).
$\rightarrow 1/R^2 = 1.534$.

Comparison with Other Work

	$m_f/m_s = 0.25$	$m_f/m_s = 0.50$	JLQCD
			($N_f = 2$)
B_{B_s}	0.76(11)	0.80(12)	
B_{B_s} (no 1/M)	0.88(13)	0.92(14)	0.85(6)
$\hat{B}_{B_s}$	1.17(17)	1.23(18)	1.30(9)
			Hashimoto et al.
			(quenched)
$\frac{B_S}{R^2}$	1.29(19)	1.34(20)	1.24(16)
$\frac{\tilde{B}_S}{R^2}$	1.38(21)	1.42(21)	
			Becirevic et al.
			(quenched)
B_S	0.84(13)	0.87(13)	0.84(2)(4)
$\tilde{B}_S$	0.90(14)	0.93(14)	0.91(3)(8)

The Bag Parameters (cont'd)

Recall

$$\langle OL \rangle^{\overline{\text{MS}}} \equiv \frac{8}{3} f_{B_s}^2 B_{B_s} M_{B_s}^2$$

$$\langle OS \rangle^{\overline{\text{MS}}} \equiv -\frac{5}{3} f_{B_s}^2 \frac{B_S}{R^2} M_{B_s}^2$$

$$\langle Q3 \rangle^{\overline{\text{MS}}} \equiv \frac{1}{3} f_{B_s}^2 \frac{\tilde{B}_S}{R^2} M_{B_s}^2$$

where $\frac{1}{R} = \frac{M_{B_s}}{(\overline{m}_b + \overline{m}_s)}$.

Results for the bag parameters in previous Table indicate that "Vacuum Saturation" works at the $10 \sim 24\%$ level.

Summary and Future Work

- Results are presented for the B_s meson mixing parameters $f^2_{B_s} B_{B_s}$, $f^2_{B_s} \frac{B_S}{R^2}$ and $f^2_{B_s} \frac{\tilde{B}_S}{R^2}$ using the MILC Collaboration $N_f = 2 + 1$ configurations, NRQCD b-quarks and AsqTad s-quarks.

- Standard Model predictions using these mixing parameters are then consistent with recent experimental determinations of ΔM_s and within large errors also with $(\Delta\Gamma/\Gamma)_{B_s}$

- Using $f_{B_s} = 0.260(29)$GeV, the extracted bag parameters B_{B_s}, B_S, $\tilde{B}_S$ are consistent with previous $N_f = 2$ and quenched results.

Summary and Future Work (cont'd)

In addition to completing checks of our current results, we plan to :

- repeat calculations with smaller light valence quark masses and determine $\longrightarrow [f^2_{B_s} B_{B_s}]/[f^2_{B_d} B_{B_d}]$

- work on finer lattices

- work on higher order matching

- explore different smearings and better fitting approaches