$B \to D^* l\nu$

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Outline

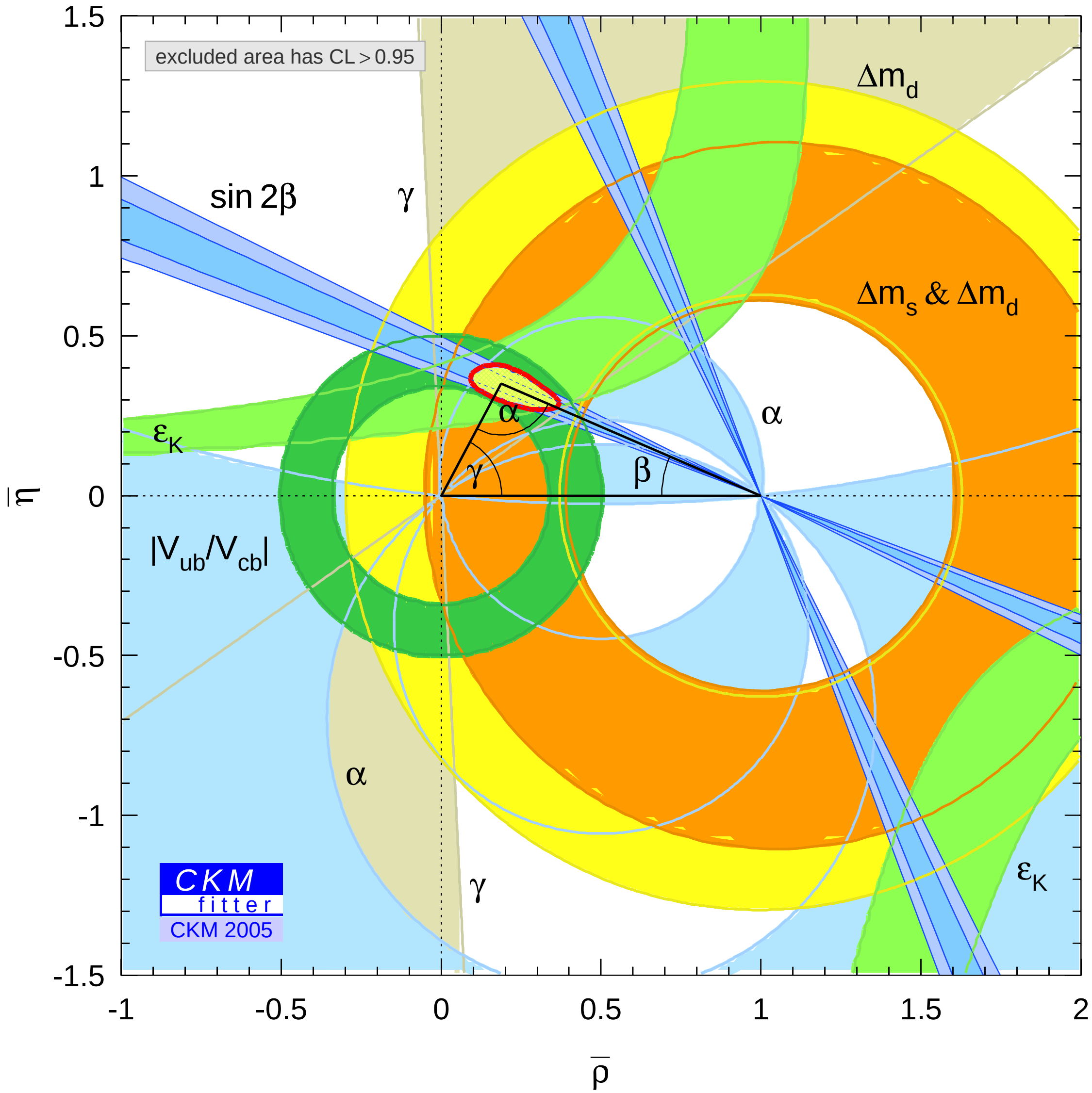
I. Brief Motivation for calculating $|V_{cb}|$

II. The lattice calculation using heavy quark effective theory

III. Numerical results and chiral extrapolation

IV. What remains to be done?

Constraining the Unitarity Triangle



Importance of $|V_{cb}|$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \quad (1)$$

$|V_{cb}|$ is needed to constrain the apex of the unitarity triangle from kaon mixing (along with B_K). Given that

$$A = \frac{|V_{cb}|}{\lambda^2} \quad (2)$$

has $\approx 2\%$ error, we see that this contributes a 9% error to ϵ_K because it appears in the formula below to the fourth power.

$$|\epsilon_K| = C_\epsilon B_K A^2 \overline{\eta}\{-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\overline{\rho})\}$$

Methods for extracting $|V_{cb}|$

- Inclusive $b \to c\ell\nu$ can be calculated perturbatively, but is ultimately limited by the breakdown of local quark-hadron duality. Difficult to estimate systematics.
- Exclusive $B \to D\ell\nu$ is theoretically cleaner but is experimentally more difficult because of phase space suppression.
- Exclusive $B \to D^*\ell\nu$ is experimentally cleaner but more challenging for the lattice. Still can be done as this talk will demonstrate.

Obtaining V_{cb} from $\overline{B} \to D^* l \overline{\nu}_l$

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 \sqrt{w^2 - 1}\, \mathcal{G}(w) \times |V_{cb}|^2 |\mathcal{F}_{B\to D^*}(w)|^2 \quad (3)$$

where $\mathcal{G}(w)$ is a kinematic factor and $\mathcal{F}_{B\to D^*}$ is a nonperturbative matrix element. $w = v'\dot{v}$ is the velocity transfer from initial (v) to final state (v').

Lattice calculation

- Done on MILC lattices with improved staggered (asqtad) sea quarks
- Heavy quarks are treated using the Fermilab action (and heavy quark effective theory)
- Light valence quarks are also asqtad staggered
- Many MILC lattice ensembles exist. This work only makes use of the MILC coarse lattices ($a \approx .12$ fm). We will add the fine ($a \approx .09$) soon.

Fermilab Action

The Fermilab method uses the SW (clover) action

$$S = \sum_{x,f} \overline{\psi}^f_x \psi^f_x - \sum_{x,y,f} \kappa_f \overline{\psi}^f_x M_{xy} \psi^f_y + \frac{i}{2} c_{\mathrm{SW}} \sum_{x,f} \kappa_f \overline{\psi}^f_x \sigma_{\mu\nu} F_{\mu\nu} \psi^f_x$$

with

$$am_{0f} = \frac{1}{u_0}\left(\frac{1}{2\kappa_f} - \frac{1}{2\kappa_{\mathrm{crit}}}\right) \qquad (4)$$

The bare mass, am_0 and clover coupling c_{SW} are adjusted so that the leading effects of the heavy quark expansion are correctly accounted for.

Calculating $B \to D^*$

$$\mathcal{F}_{B\to D^*}(1) = h_{A_1}(1), \tag{5}$$

$$\langle D^*(v)|\mathcal{A}^\mu|\overline{B}(v)\rangle \quad = \quad i\sqrt{2m_B 2m_{D^*}}\overline{\epsilon'}^\mu h_{A_1}(1). \tag{6}$$

$h_{A_1}(1)$ is constrained by heavy quark symmetry:

$$h_{A_1}(1) \quad = \quad \eta_A \left[1 - \frac{l_V}{(2m_c)^2} + \frac{2l_A}{2m_c 2m_b} - \frac{l_P}{(2m_b)^2}\right] \tag{7}$$

Double ratio method

Hashimoto et al

$$\frac{\langle D^*|\bar{c}\gamma_4 b|\overline{B}^*\rangle\langle\overline{B}^*|\bar{b}\gamma_4 c|D^*\rangle}{\langle D^*|\bar{c}\gamma_4 c|D^*\rangle\langle\overline{B}^*|\bar{b}\gamma_4 b|\overline{B}^*\rangle} = |h_1(1)|^2 \,. \tag{8}$$

$h_1(1)$ is constrained by heavy quark symmetry:

$$h_1(1) \quad = \quad \eta_V \left[1 - l_V \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2\right],$$

(η_V is a perturbative factor matching HQE theory to QCD.)
Note that statistical and systematic errors cancel in the ratio, including non-perturbative renormalization factors.

Double ratio method

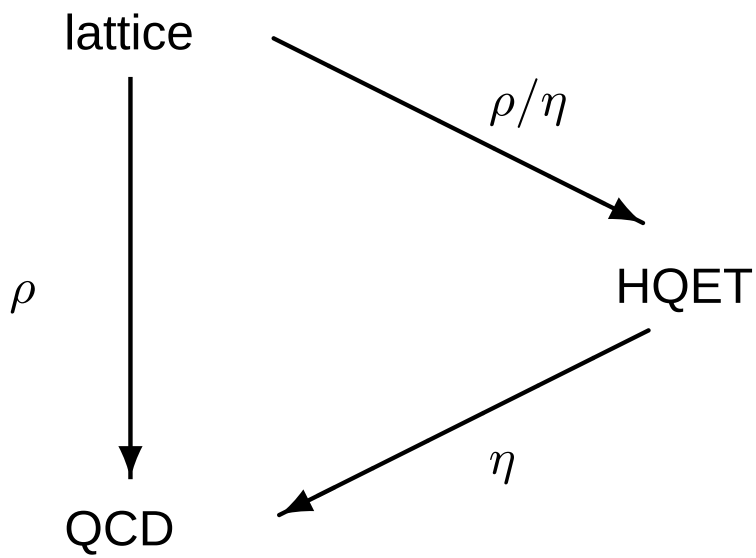
$$\frac{\langle D|\bar{c}\gamma_0 b|\overline{B}\rangle\langle\overline{B}|\bar{b}\gamma_0 c|D\rangle}{\langle D|\bar{c}\gamma_0 c|D\rangle\langle\overline{B}|\bar{b}\gamma_0 b|\overline{B}\rangle} = |h_+(1)|^2 \,. \tag{9}$$

$$\frac{\langle D^*|\bar{c}\gamma_j\gamma_5 b|\overline{B}\rangle\langle\overline{B}^*|\bar{b}\gamma_j\gamma_5 c|D\rangle}{\langle D^*|\bar{c}\gamma_j\gamma_5 c|D\rangle\langle\overline{B}^*|\bar{b}\gamma_j\gamma_5 b|\overline{B}\rangle} = \left|\check{h}_{A_1}(1)\right|^2 \,. \tag{10}$$

$h_+(1)$ and $\check{h}_{A_1}(1)$ are constrained by heavy quark symmetry:

$$\begin{aligned} h_+(1) &= \eta_V \left[1 - l_P\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2\right], \\ \check{h}_{A_1}(1) &= \check{\eta}_A \left[1 - l_A\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right)^2\right]. \end{aligned} \tag{11}$$

Matching

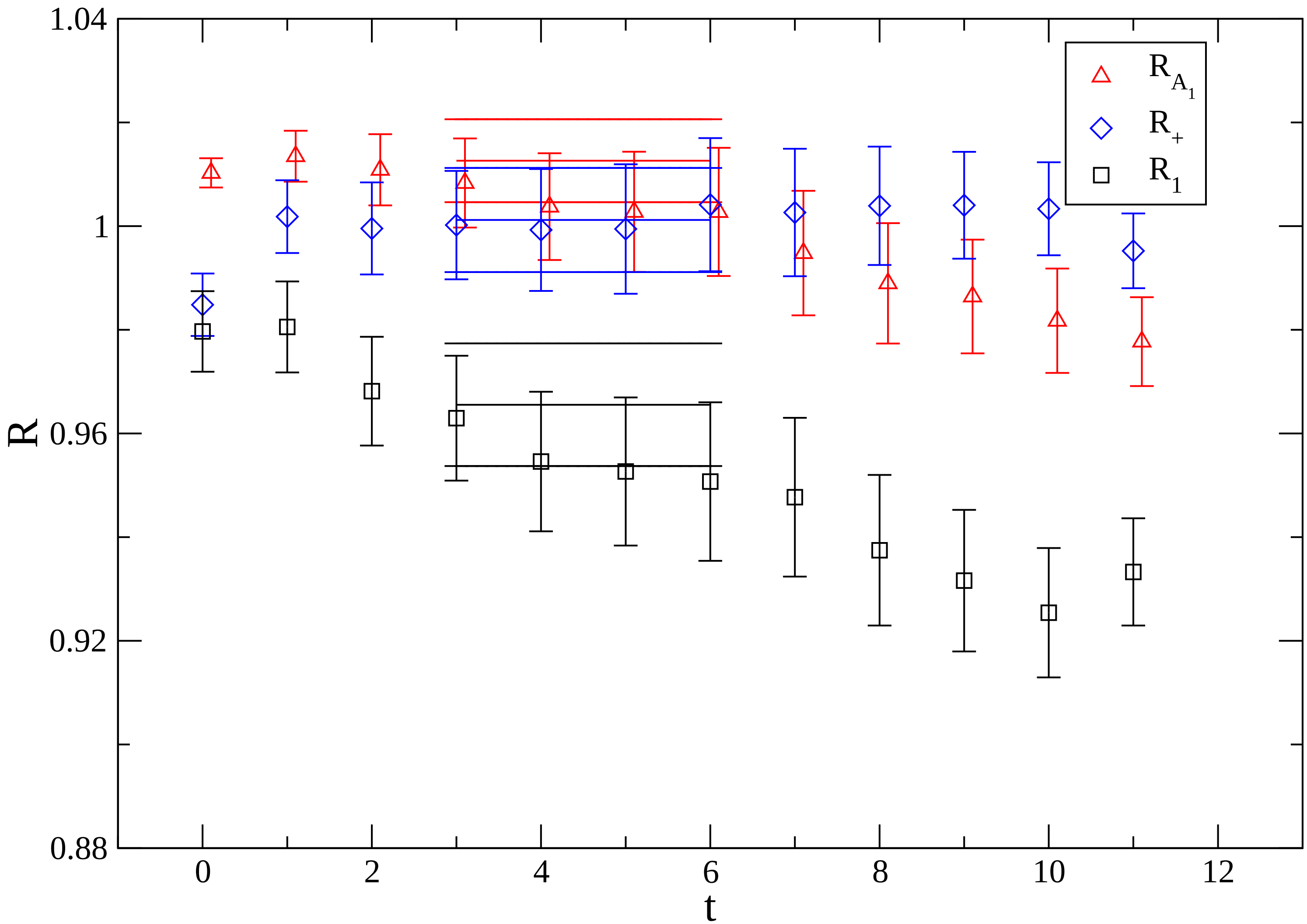


This diagram illustrates the factors that match lattice gauge theory to HQET and QCD.

Numerical Results

- In the following, the η and ρ factors are set equal to 1. The ρ factors matching the lattice to the continuum have been calculated by Nobes, and will be incorporated into the analysis in the very near future.
- The data has been analyzed for three light dynamical mass points using a single time source with $m_{light} \approx m_s/7$ for the lightest light quark mass.

Double Ratio Plots



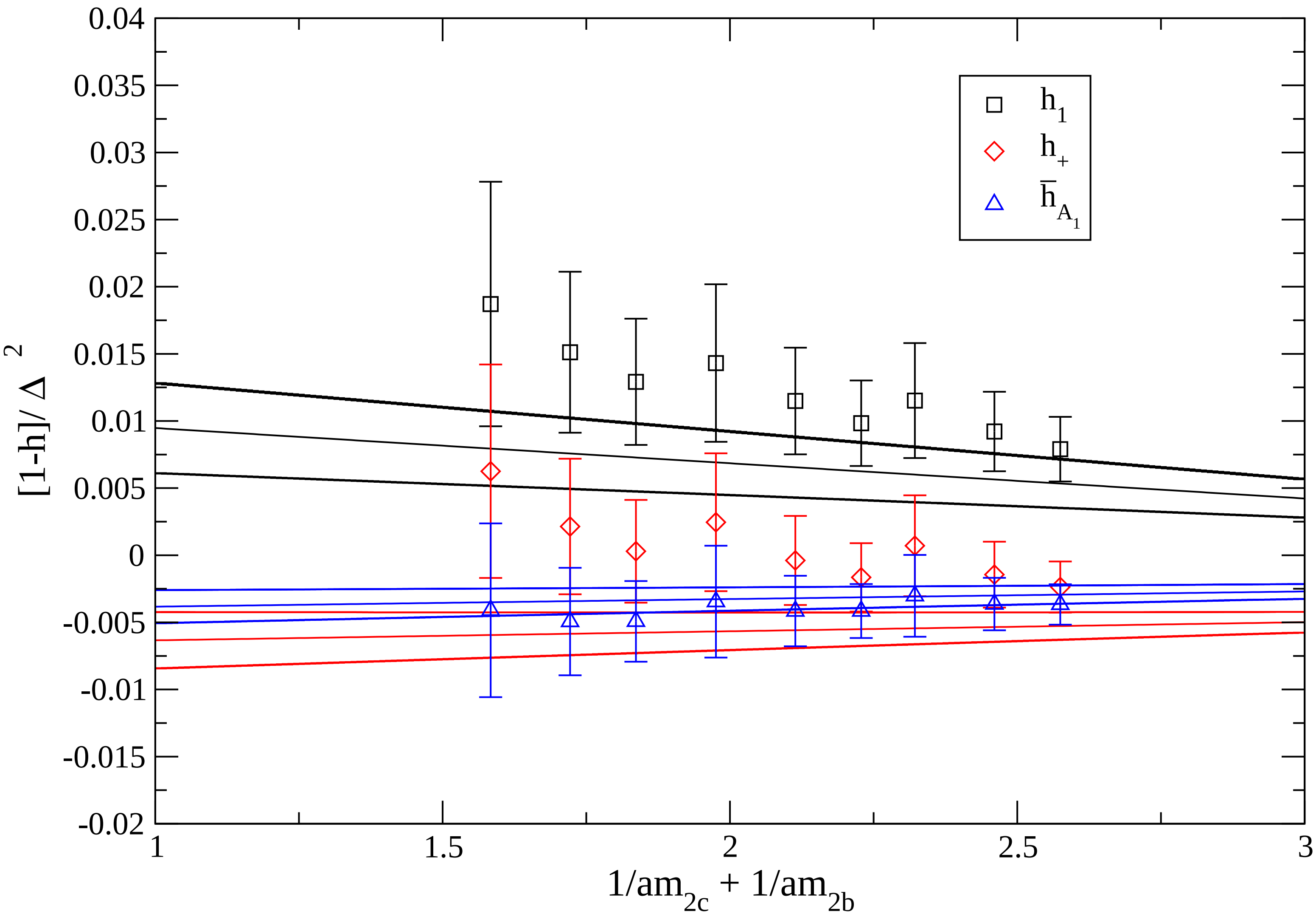
Heavy Quark Dependence

$$\frac{\rho\sqrt{R}}{\eta} = \frac{h}{\eta} = 1 - \frac{1}{4}\Delta^2(c^{(2)} + \frac{1}{2}c^{(3)}\Sigma), \tag{12}$$

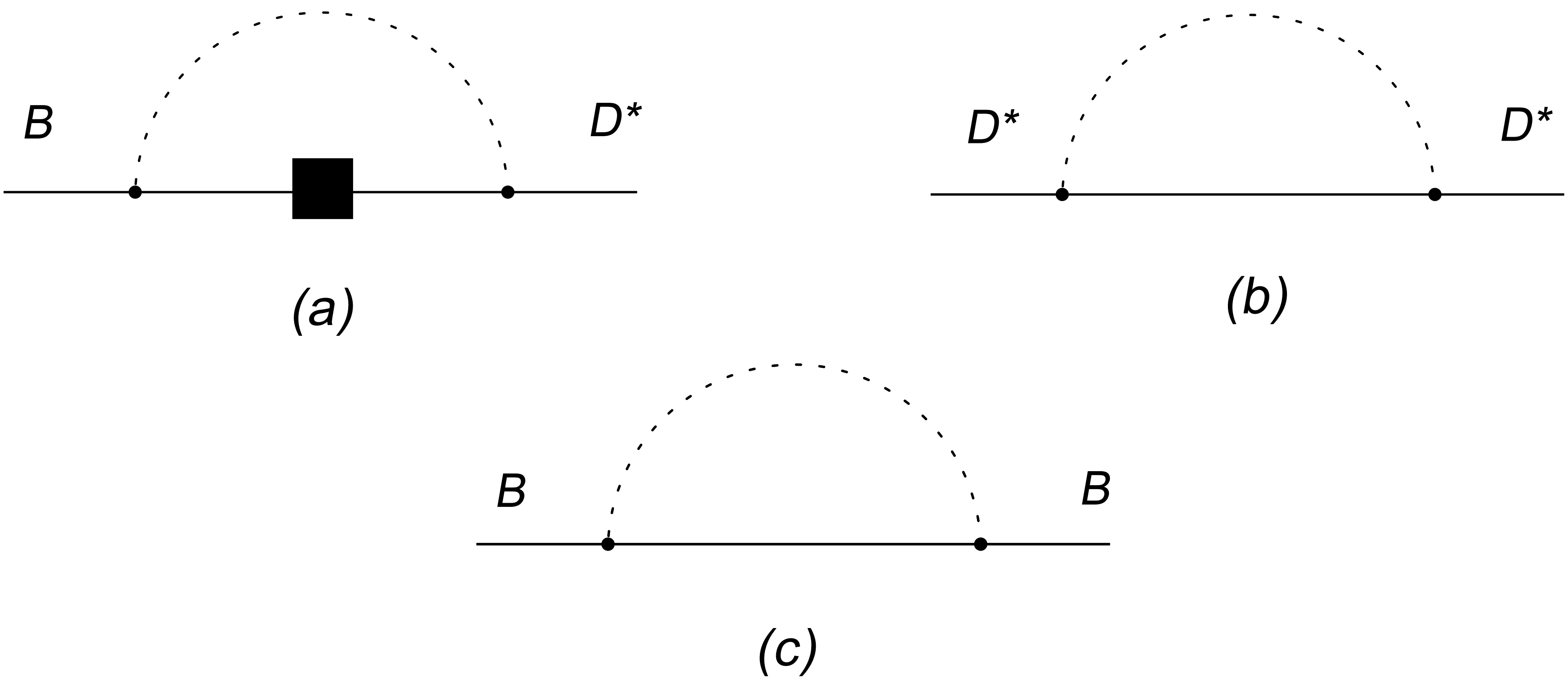
$$\Delta = \frac{1}{am_{2c}} - \frac{1}{m_{2b}}, \tag{13}$$

$$\Sigma = \frac{1}{am_{2c}} + \frac{1}{m_{2b}}. \tag{14}$$

Heavy Quark Dependence



Diagrams contributing to $B \to D^*$ ChPT

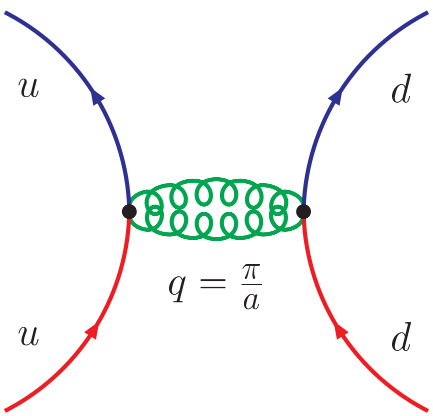


One-loop diagrams that contribute to $B \to D^*$. The solid line represents a meson containing a heavy quark, and the dashed line represents light mesons. The small solid circles are strong vertices and contribute a factor of g_π. The large solid square is a weak interaction vertex. Diagram (a) is a vertex correction, and (b) and (c) correspond to wavefunction renormalization.

Taste Symmetry Breaking

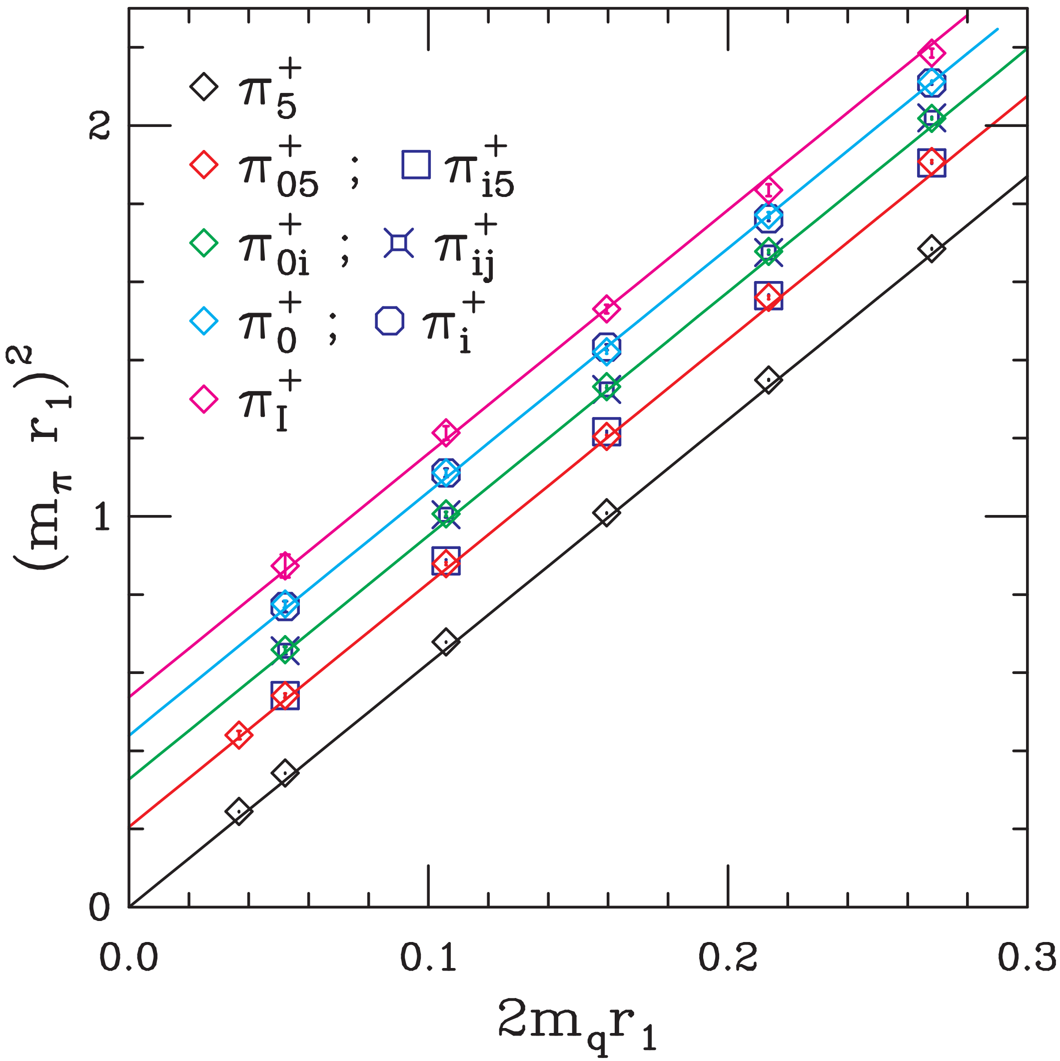
- Staggered quarks come in 4 tastes $\Rightarrow$ staggered mesons come in *16 tastes*
- Labeled by the *taste matrix* in the lattice operator: $\pi_T \equiv \overline{Q}_i(\gamma_5 \otimes \xi_T)Q_j$

1 Singlet – ξ_I	1 Goldstone – ξ_5	
4 Vector – ξ_μ	4 Axial – $\xi_{\mu 5}$	6 Tensor – $\xi_{\mu\nu}$



- On the lattice, quarks of one taste can turn into another by exchanging high-momentum gluons

Taste Splittings

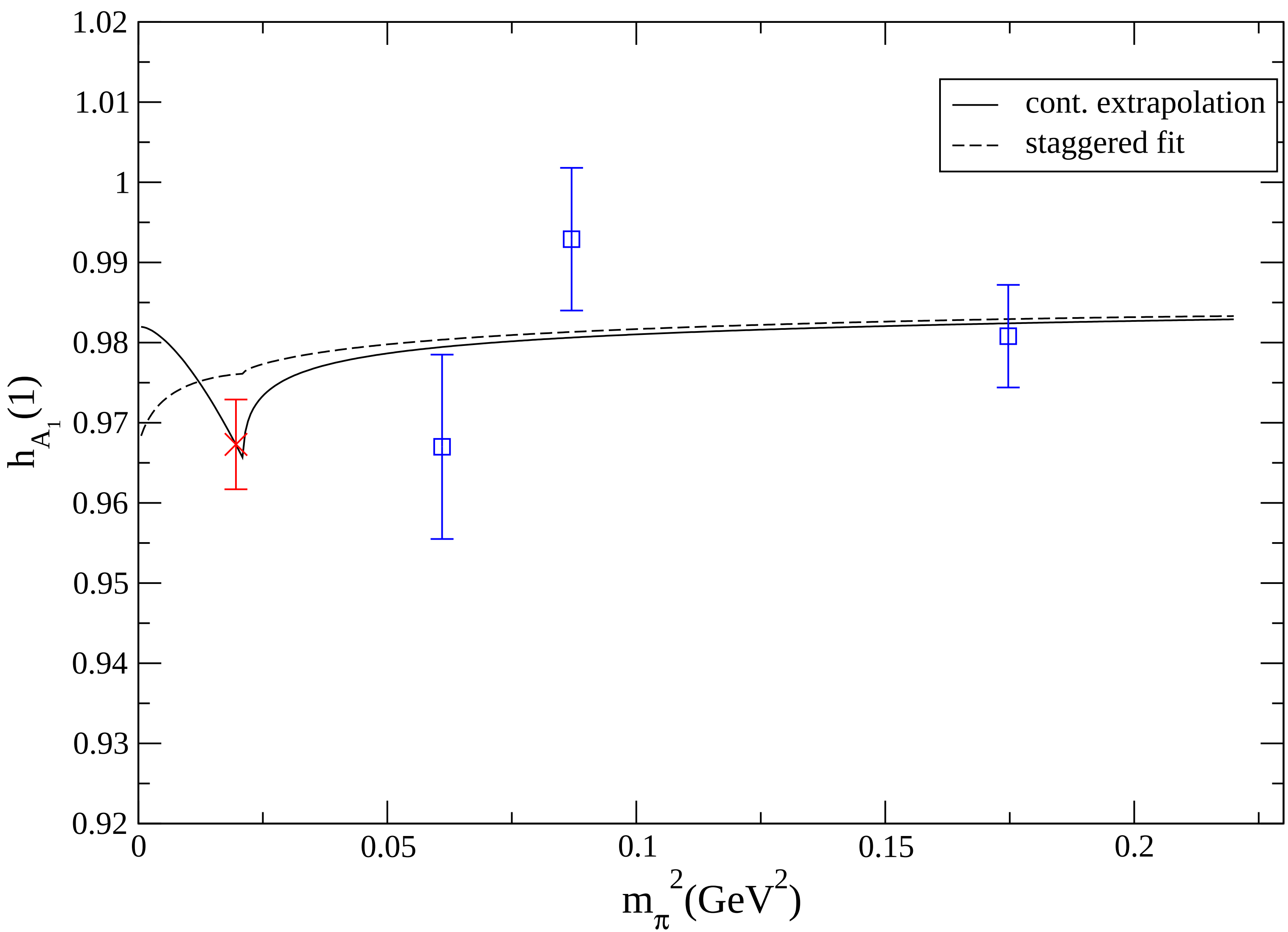


Staggered ChPT formula

$$
\begin{aligned}
h_{A_1}^{2+1}(1) \quad = \quad & 1 + X_A + \frac{g_\pi^2}{48\pi^2 f^2}\left[\frac{1}{16}\sum_B (2\overline{F}_{\pi_B} + \overline{F}_{K_B}) - \frac{1}{2}\overline{F}_{\pi_I} + \frac{1}{6}\overline{F}_{\eta_I}\right. \\
& + a^2\delta'_V \left(\frac{m_{S_V}^2 - m_{\pi_V}^2}{(m_{\eta_V}^2 - m_{\pi_V}^2)(m_{\pi_V}^2 - m_{\eta'_V}^2)}\overline{F}_{\pi_V} \right. \\
& + \frac{m_{\eta_V}^2 - m_{S_V}^2}{(m_{\eta_V}^2 - m_{\eta'_V}^2)(m_{\eta_V}^2 - m_{\pi_V}^2)}\overline{F}_{\eta_V} \\
& \left.\left. + \frac{m_{S_V}^2 - m_{\eta'_V}^2}{(m_{\eta_V}^2 - m_{\eta'_V}^2)(m_{\eta'_V}^2 - m_{\pi_V}^2)}\overline{F}_{\eta'_V} \right) + (V \to A) \right],
\end{aligned}
\tag{15}
$$

where a is the lattice spacing, δ'_V, g_π and X_A are constants, and $\overline{F}$ is a complicated function involving logs.

Chiral Extrapolation



Conclusions

Still to do:

- More statistics by exploiting large time extent on MILC lattices using multiple time sources
- Running on additional lattice spacings
- Including matching factors in the analysis
- Careful enumeration of all systematic errors