B meson decays from moving-NRQCD on MILC fine lattices

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Outline

1. $|V_{ub}|$: $B \to \pi l \nu$ semileptonic form factors $f_+(q^2),\ f_0(q^2)$
2. lattice simulations with ***NRQCD***
3. advantages of ***moving-NRQCD***
4. determination of ***renormalization constants*** from high-β (weak coupling) simulations
5. preliminary results on *B* meson decays

Semileptonic decay $B \to \pi l \nu$

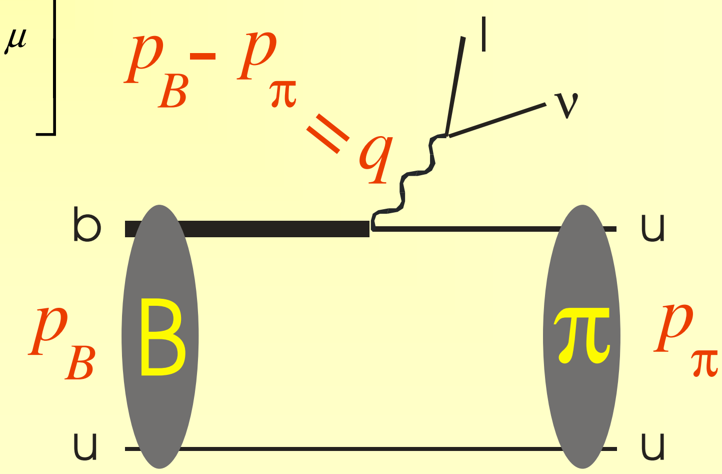
- CKM matrix elements … new physics?
- $|V_{ub}|$ from semileptonic process $B \to \pi l \nu$

$$\frac{d\Gamma}{dq^2}(B \to \pi l \nu) = \frac{G_F^2}{24\pi^3} p_\pi^3 |V_{ub}|^2 |f_+(q^2)|^2$$

- ***form factors $f_+(q^2)$, $f_0(q^2)$ from lattice QCD***

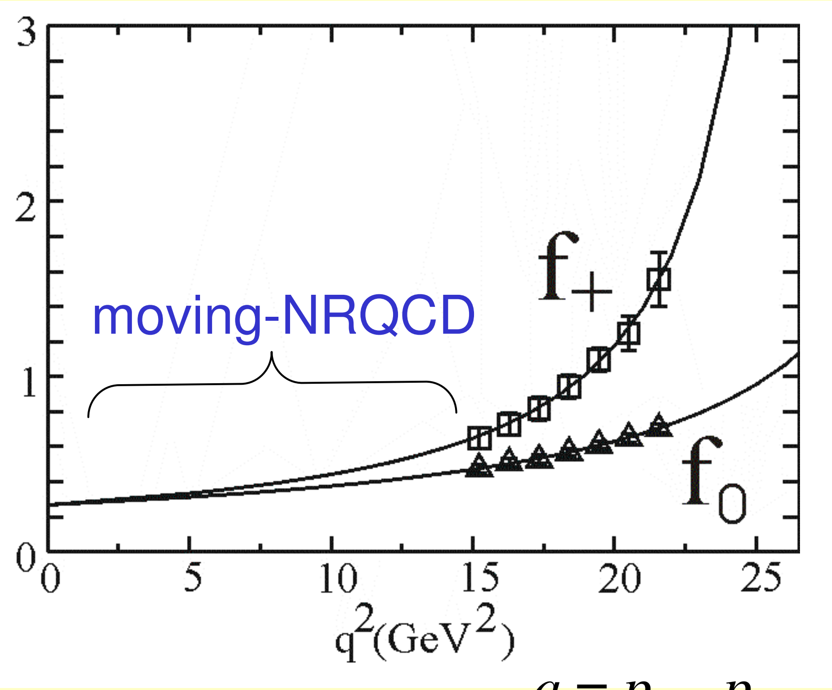
$$\langle \pi | V^\mu | B \rangle = f_+(q^2) \left[p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu$$

large $q^2 \to$ small recoil mom.
small $q^2 \to$ large recoil mom.



$B \to \pi l \nu$ with NRQCD

- ***NRQCD for the b quark***
- E. Gulez et al. HPQCD Collaboration. PRD**73**:074502 (2006)



$q = p_B - p_\pi$

- π can have very ***large recoil momentum*** (small q^2); but difficult to access this region in lattice calculations

1. discretization errors ~ $(ap)^2$

2. statistical errors grow exponentially as kinetic energy of the hadrons grows

Moving-NRQCD

- last transparency: ***B at rest*** + entire recoil momentum channeled into π
- ***solution***: B and π move in opposite direction
- ***problem***: large $(ap)^2$ errors from NRQCD
- ***moving-NRQCD***
  [J.H. Sloan hep-lat/9710061; K.M. Foley & G.P.Lepage hep-lat/0209135]

$$p_b = m_b u + k\,; \quad u = 4\text{-velocity of B} = p_B / m_B = \gamma(1, \vec{v})$$

treat the average momentum $m_b u$ exactly (like M_0 in NRQCD),
discretize the residual momentum k ***~ small***

Moving-NRQCD action

- action (boost the NRQCD action)

$$G_{x,t+1} = \left(1-\frac{\delta H}{2}\right)\left(1-\frac{H_0}{2n}\right)^n U^{\dagger}_{x,\hat{t}}\left(1-\frac{H_0}{2n}\right)^n\left(1-\frac{\delta H}{2}\right)G_{x,t}$$

$$H_0 = -iv\cdot\Delta^{\pm} - \frac{\Delta^{(2)}}{2M_0\gamma}$$

$$\begin{aligned}\delta H = &-\frac{1}{2m\gamma}\sigma\cdot B_m + \left(\frac{1}{2m\gamma}+\frac{1}{4n}\right)\left(v\cdot\Delta^{\pm}\right)^2 \\ &+\frac{1}{24m\gamma}\sum_i \Delta_i^{(4)} - \frac{1}{6m\gamma}\left(v\cdot\Delta^{\pm}\right)\left(\sum_i v_i\Delta_i^{+}\Delta_i^{\pm}\Delta_i^{-}\right) \\ &+\frac{i}{6}\sum_i v_i\Delta_i^{+}\Delta_i^{\pm}\Delta_i^{-} - \frac{i}{4mn\gamma}\left(v\cdot\Delta^{\pm}\right)\Delta^{(2)}\end{aligned}$$

complete through $O(1/m)$

$$B_m = \gamma\left(B - v\times E - \frac{\gamma}{\gamma+1}v(v\cdot B)\right)$$

Renormalization constants

- dispersion relation ***on the lattice***

$$E = E_0 + \sqrt{(Z_p p_0 + k)^2 + m_{kin}^2}$$

$$E_0 = c_1\alpha_s + c_2\alpha_s^2 + \ldots$$

$$Z_p = 1 + c_1\alpha_s + c_2\alpha_s^2 + \ldots$$

$$m_{kin} = Z_m m_0 = (1 + c_1\alpha_s + c_2\alpha_s^2 + \ldots)m_0$$

- diagrammatic perturbation theory

  1^{st} order calculations finished

  A. Dougall, C.T.H Davies, K.M. Foley & G.P.Lepage (lattice 2004)
  also checked by R. Horgan *et al*. (University of Cambridge)

- ***numerical method (high-β simulations)***

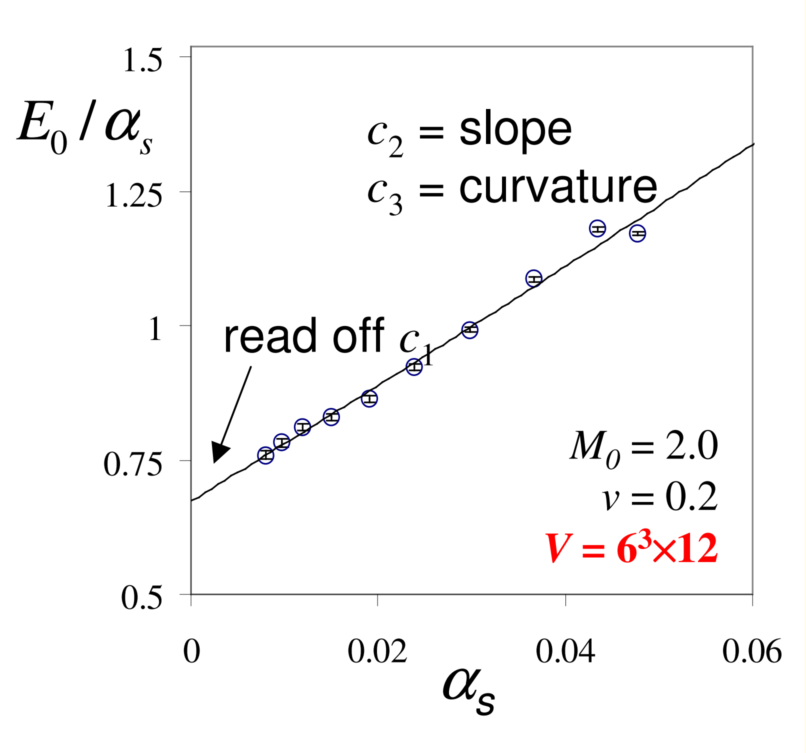
  do several simulations at high-β (weak couplings) and fit the results to an expansion in α_s to extract the perturbative coefficients

High-β simulations

1. simulations at weak couplings
2. measure E_0 (from dispersion relation) in each case
3. fit the results to

$$E_0 = c_1\alpha_s + c_2\alpha_s^2 + \ldots$$

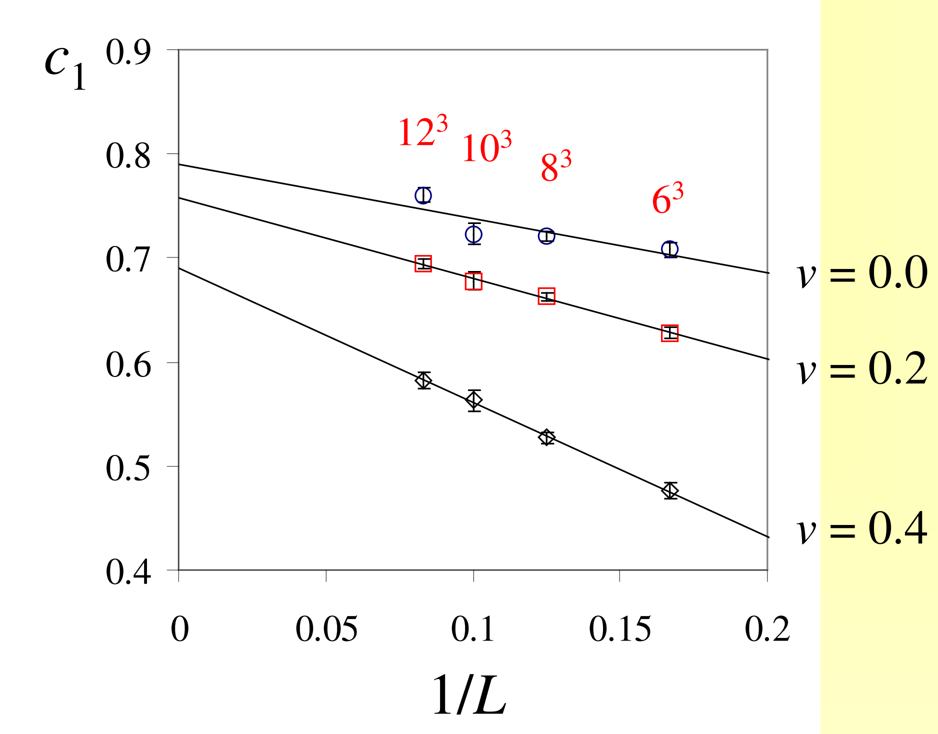
4. ***read off PT coefficients***



quenched + H_0

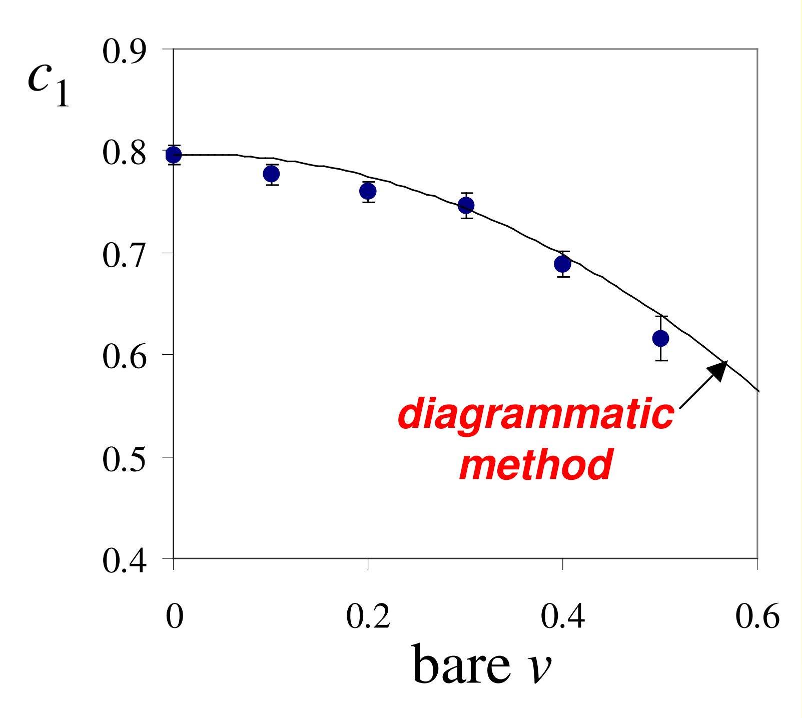
Difficulties

- difficulty #1 accurate measurements of observable
- difficulty #2 finite volume effects

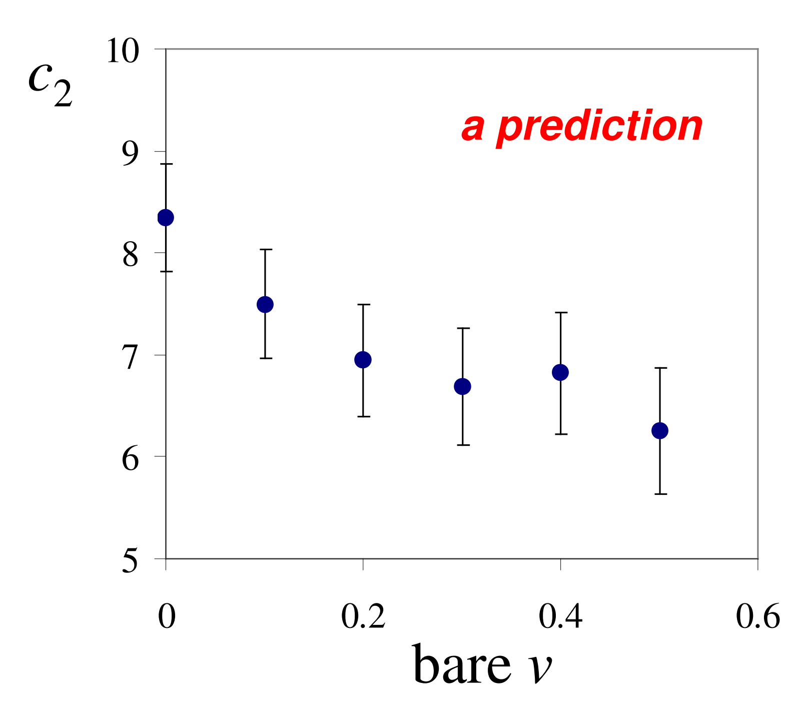


Results for E_0

1st order c_1

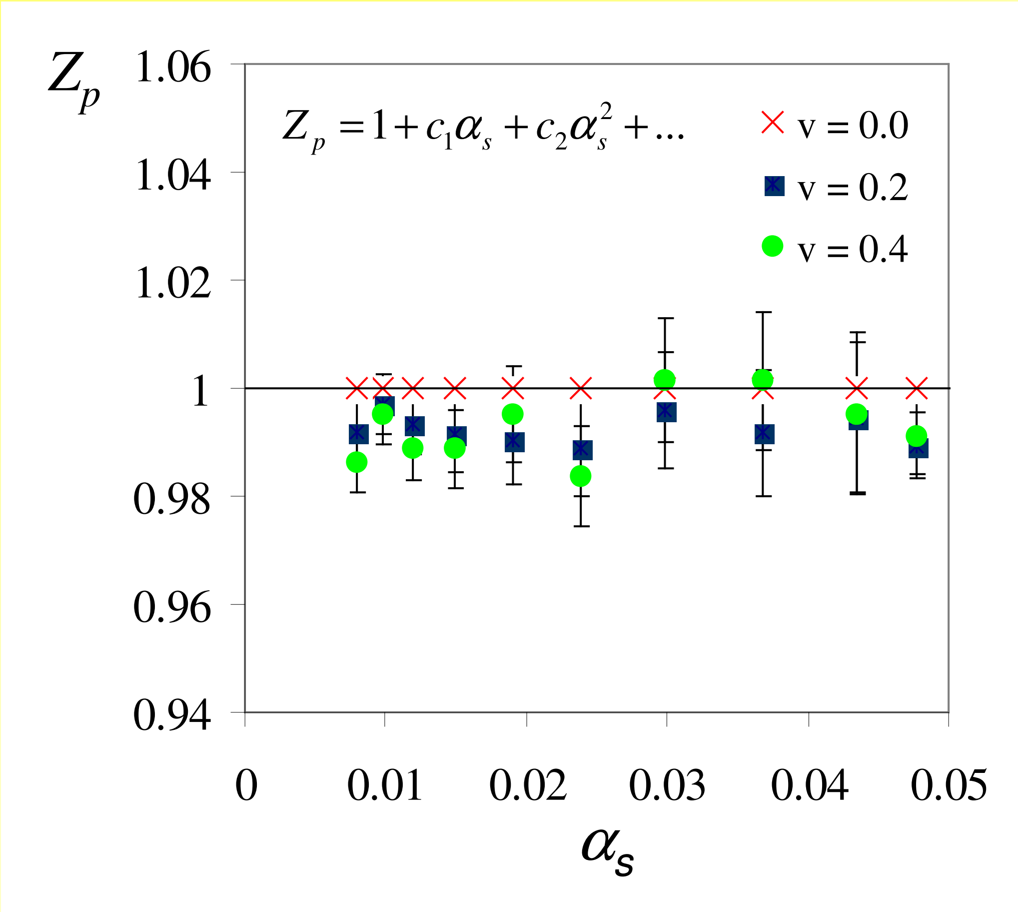


2nd order c_2



- high-β: ***much simpler*** compared to diagrammatic method ***at high orders***

Results for Z_p



- error bars too large, difficult to do fits
- ***good***: shows little renormalization, due to re-parameterization invariance

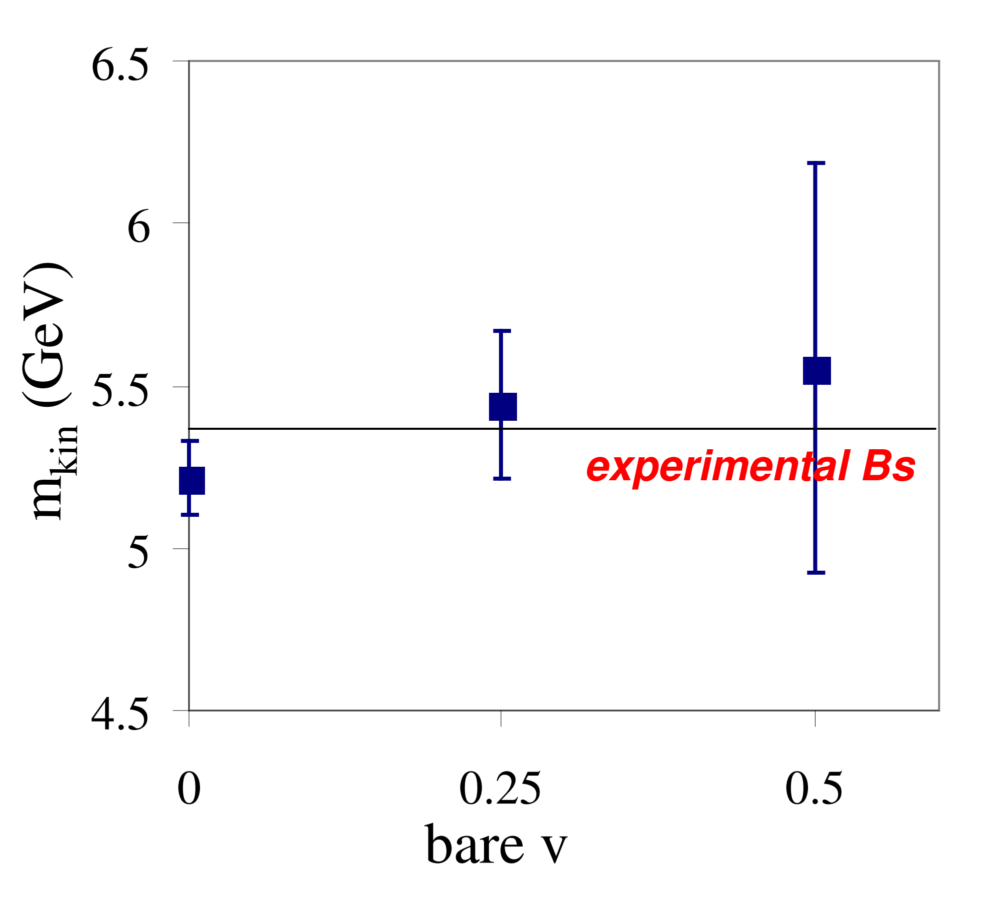
Simulations

- decay of B meson (***2pt correlation function***)
- first study on ***MILC fine lattices*** ($a^{-1} \approx 2.258\text{GeV}$, $am_f = 0.0062$)
- asqtad light quark propagators ($am_q = 0.031$ ~ ***Bs***)
- aM_0 = 1.95 (see later)
- $v = (v_x, 0, 0)$; $v_x = 0, 0.25, 0.5$
- ***we want to know***: how to set the parameters for the 3pt runs

  dependence of m_{kin} on v; need to tune M_0?

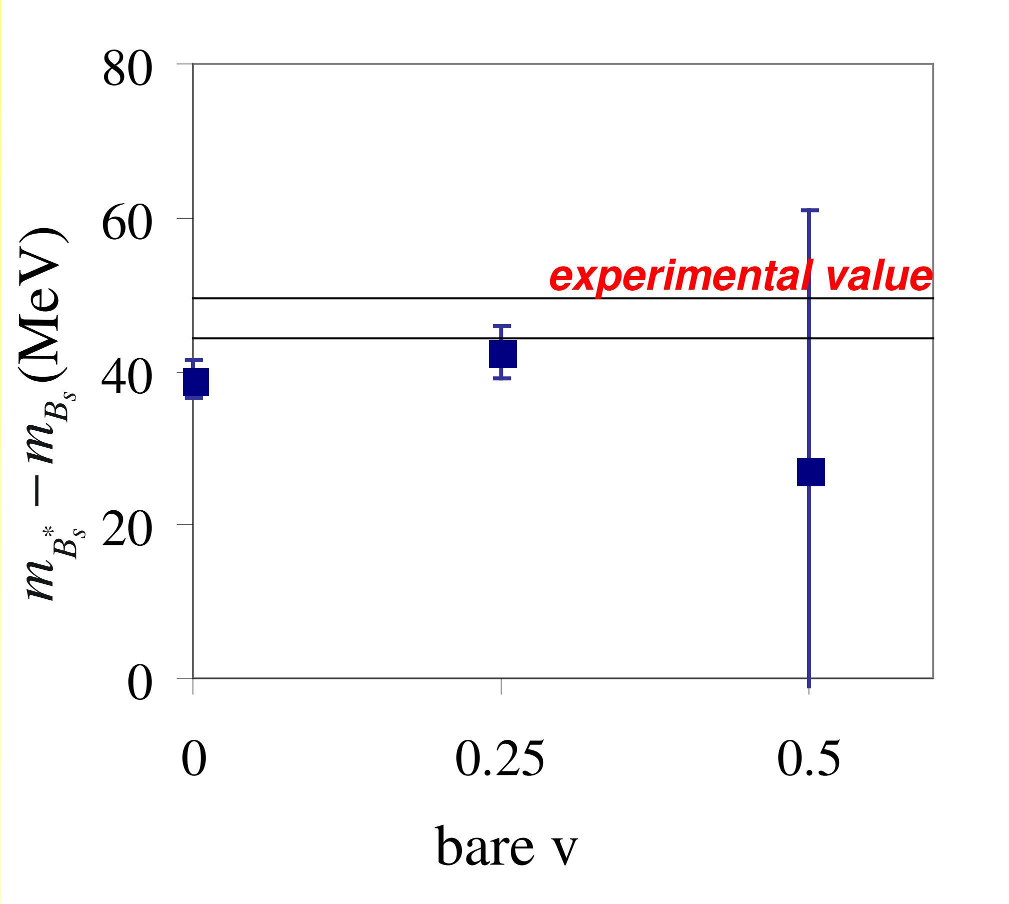
  dependence of Z_p on v; need to tune the bare v?

m_{kin}

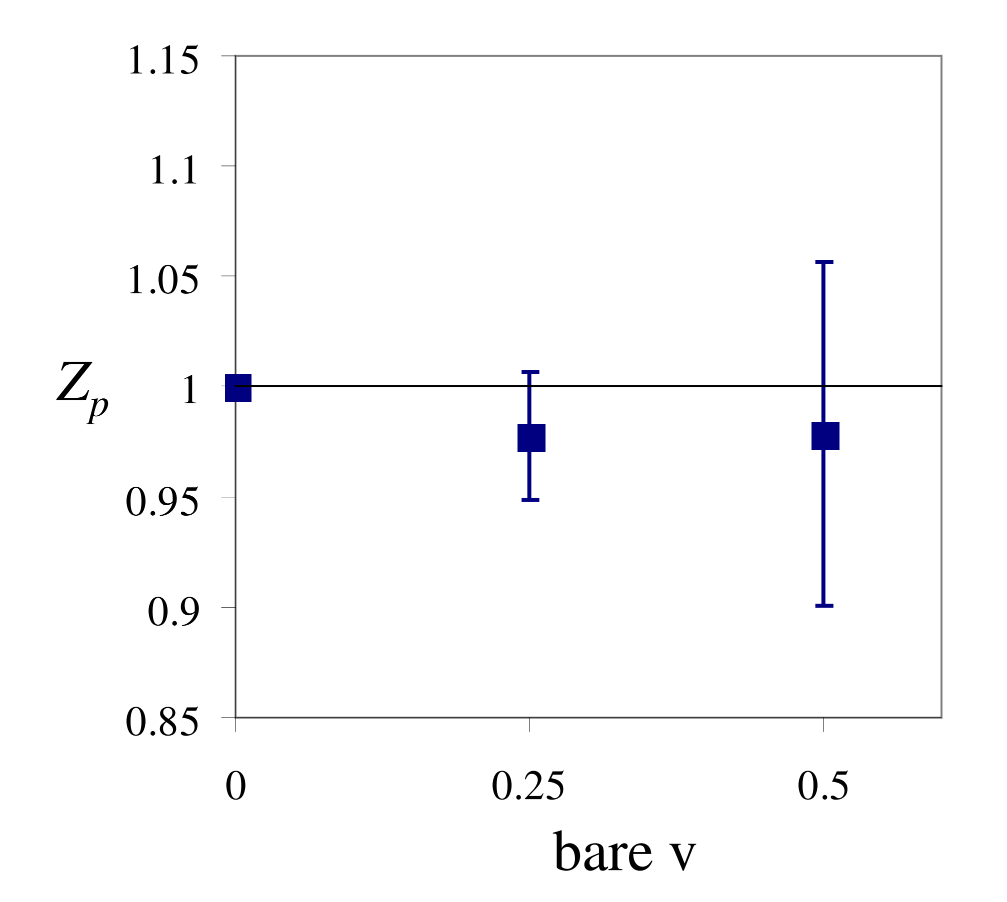


- input bare mass aM_0 = 1.95
- ***good***: m_{kin} is quite stable as v increases $\rightarrow$ ***no need to tune the bare mass***

$m_{B_s^*} - m_{B_s}$



Ext. Momentum Renormalization Z_p



- ***good***: shows little renormalization (agree with PT) $\rightarrow$ ***no need to tune the input velocity***
- small renormalization due to re-parameterization invariance

Conclusions

- ***moving-NRQCD*** $\rightarrow$ semileptonic form factors $f_+(q^2),\ f_0(q^2)$ at ***high recoil momentum (small*** q^2***)***
- results for B decays on MILC fine lattices look good
- 3pt correlation function is in progress
- need renormalization constants: high-β simulations

goal: high-precision determination of $f_+(q^2),\ f_0(q^2)$ and $|V_{ub}|$