Determining the Acceleration Due to Gravity with a Simple Pendulum

(Your name)

Advanced Undergraduate Lab, Department of Physics and Astronomy, University of Utah, Salt Lake City, Utah 84112

This is an example of a lab report associated with obtaining the acceleration due to gravity ($g$) and applying mathematical models. This example serves as a template to assist you in writing lab reports for PHYS 3719. The bolded text represents wording or ideas that could be stated or included in an actual handout. The unbolded text provides guidance, tips, and other information necessary to carry out this experiment. But use your own words when you write your lab reports. Remember that a paper (your lab report in this case) has to be complete, so everything (text, graphs, results, tables, etc.) has to be tied together. Remember, keep your lab reports short and to the point.

Abstract

Using a simple pendulum and the model that the square of the period of a pendulum is proportional to its length, we report that the model is [or is not] supported by our data because the relationship between the two parameters is [or is not] linear based on a $\chi^2$ value of _____ for ___ degrees of freedom, calculated from our fit. The value of $g$ we find, based on measurements taken in Salt Lake City, is _____±____ m/sec$^2$. We calculate that the correction to our measurement from the finite angular displacement of the bob is ____%. The measured value of $g$ is within _____ standard deviations of the accepted value of _____m/sec$^2$.

This abstract is just an example. Write it using your own words including the following information: purpose/goal of your work; what technique(s), method(s) and model(s) you used; what results you obtained (value of $g$ with corresponding uncertainty); comparison with an accepted value for Salt Lake City.

Introduction

Importance of this experiment (Motivation). History of the experiment.

Of the four forces of nature, the first to be understood quantitatively was gravitation. Newton formulated his theory of universal gravitation, including the $1/r^2$ force law, in about 1666. Often the angular frequency of a pendulum is written $\omega^2 = g/l$, where $g$ is the acceleration of gravity on the surface of the earth and $l$ is the length of the pendulum. This
equation omits effects such as the variation of $\omega$ with the amplitude of the motion. In this report we will determine $g$ including several corrections, and compare it with the accepted value at the location of Salt Lake City.

Studying the motion of a simple pendulum achieves several goals for the physics undergraduate: (1) It allows us to actually determine the value of $g$, a common parameter often encountered in undergraduate physics problems, to greater precision than is available from textbooks; (2) it allows us to test a mathematical model of a physical system, namely that the square of the period of a pendulum is proportional to its length; and (3) it gives a transparent introduction to sources of errors and their propagation through an experimental calculation and into the result of the experiment. This experiment has been described in great detail in Nelson and Olsson [1].

Here we also collect all the equations that will be used in the analysis of the data. Derivations should not be included.

When we solve Newton’s 2nd law for a simple pendulum, in the small angle approximation, we get

$$T_0^2 = \frac{4\pi^2}{l}g, \quad (1)$$

where $T_0$ is the period and $l$ the length of the pendulum. The complete solution to the equation of motion of the pendulum without the small angle approximation is given by Nelson and Olsson [1]:

$$T(\theta_{\text{max}}) = T_0 \left(1 + \frac{1}{4} \sin^2 \frac{\theta_{\text{max}}}{2} + \frac{9}{64} \sin^4 \frac{\theta_{\text{max}}}{2} + \cdots \right) \quad (2)$$

**Apparatus**

Describe apparatus and setup very briefly. Attach pictures and sketches if appropriate.

![Figure 1. Experimental setup of pendulum. (a) Pendulum with protractor. (b) Close up of some measuring tools.](image)
Data Collection

Describe timing method and equipment if relevant. Here we include, for the student’s benefit, the procedure to follow for the experiment.

Measurement of time:
The purpose of this preliminary study is to learn the optimum way of measuring the period of a regularly occurring process (like a clock’s ticks or a pendulum’s swing). This exercise should not be part of your lab report.
- You will want to reference the precision of your time interval measurements to an “atomic clock”: use the stopwatch to measure a 1 sec interval on the atomic clock.
- Repeat 20 times. Calculate the mean and standard deviation.
- Repeat for a 100 sec time interval.
  - Which set of measurements gives you a better gauge of the accuracy of the stopwatch?
  - What is the uncertainty in your averaged time measurements? This will be relevant for measurements of the pendulum period.

Measurement of length:
Think about the issue of how you measure the length of the pendulum, e.g. note that the pivot point, the top of the arm, is fixed by pinching the string with a screw. Which is more accurate: a tape or meter stick? What is the significance of the fact that the hook on the end of the measuring tape moves about 1 mm? What different ways can you use to minimize the relative error in measuring the length? How can you use statistical analysis to minimize the relative error in the length; i.e. how many different ways can you think of to generate independent measurements of the length of the pendulum which can then be averaged to reduce error? Very important experimental design consideration, resulting from the fact that $g$ is extracted from a fit to the slope of your plot: Will your experimental errors be less if you only take data with the maximum length of pendulum or should you include some data sets with shorter pendula? This is very important; it is a potential source of a huge error in your result; think it through carefully. In the “Data Collection” section, list the range of values of length over which you took data; in your “Results and Conclusion” section discuss why you made the choices you did and how they relate to your final accuracy and precision.

The “Data Collection” section in your report should address the experimental aspects of the data collection process. Here is a continuation of instructions for the student.

- Connect the bob to a string by means of the hook attached to the bob. (Alternatively there is one bob with no hook: the string is glued directly into the hole intended for attaching the hook).
- Pass the string through a hole in a stout aluminum bar. The wing screw and washer on top lock the string at a fixed length; the screw on front not only holds the protractor but also pins the string, fixing a precise point of oscillation and length. Be sure that the pendulum can swing freely. Record all data in the provided lab notebook (“Bluebook”) and in a software spreadsheet (Origin is preferable to Excel).
- With the stopwatch in one hand and bob in the other and a length of approximately 20 cm, time the first period as you release the pendulum. Use these measurements to calculate one value of $g$.
- Repeat, but time the second swing of the pendulum.
• Repeat, but average the measured period over 20 complete oscillations not including the first one. Repeat this procedure 10 times. Calculate the average of those 20 oscillation periods, standard deviation and standard deviation of the mean.

• Repeat after increasing the length of string and with an amplitude which is “quite small”.

• Calculate $g$ for each of the above cases and compare to the “accepted value”.

• Think about how you can get statistical information on the length of the pendulum which allows you to use statistical methods to reduce the error in length measurements, similar to what you are doing with the time (period) measurements. This is not trivial. It is an important part of your effort to get maximum precision and accuracy in your measurement of $g$.

• How can you use a mirror to minimize parallax in the length measurement?

Here are instructions for collecting your main data:

• For ten (or more) different lengths $l$ of the pendulum (choose the range of lengths, based on your systematic error considerations and considerations discussed above) measure and record the time $t_{20}$ for 20 complete oscillations by using a stopwatch. Repeat 10 times for each pendulum length. Keep $\theta_{\text{max}}$ at or below 20°.

• For one length of pendulum, measure 1 set of 20 periods at $\theta_{\text{max}} = 5°, 10°, 20°, 30°, 40°$. Compare your results to the model, $T(\theta_{\text{max}}) = T_0 \left(1 + \frac{1}{4} \sin^2 \frac{\theta_{\text{max}}}{2} + \frac{9}{64} \sin^4 \frac{\theta_{\text{max}}}{2} + \cdots\right)$, keeping the third term, at least initially. Note that here you are testing a non-linear model of the behavior of the system. Do your data support this model?

You may describe here any additional details of your technique which may be of importance. Did any “tricks” help you to get more repeatable measurements? Any pitfalls that you became aware of as you were taking data?

Data Analysis

All graphs should be in this section. If you attach your raw data, it should be in an appendix.

Plot your data.

To determine the best value of $g$, use all your data together by fitting. Make a plot of period squared (on the y-axis) vs. length (on the x-axis), using averages of the 20 swings with the 10 different pendulum lengths. Determine the uncertainties in period and include them on the plot. Remember that $d(T^2) = 2TdT$. Fit these data to a straight line using $\chi^2$ minimization. Do not use Excel for the fit. Determine $g$, propagate uncertainties, and quote your result with its fitting uncertainty.

Residual plot.

A useful tool to check the goodness of your fit is the residual graph [2]. The residuals $R_i$ are the differences between the $T_i^2$ values of your data points and the “predictions” of your best fit line. Make a plot of $R_i$ versus $l$. Use the uncertainties in $T_i^2$ as your error bars for this graph. To have a good fit for a linear model your plot should show a random pattern and about 2/3 of your data points’ error bars should pass through zero.

Include a caption underneath your figure, such as:

Figure 3. Pendulum period squared as a function of pendulum length (or vice versa depending on their relative errors). The open circles (or another kind of symbol, color, etc.) represent the experimental data and the solid line (or another kind of line, color, etc.) represents the least-squares fit.
Include both plots in your report. Remember: the axes must have titles and units.

**Corrections to g:**
Plot your data for the pendulum period as a function of \( \theta_{\text{max}} \). Superimpose on this plot the theoretical curve (Equation 2). Comment on the agreement [or lack thereof].

**Results and Conclusions**

First state your answer. You wrote this report to describe your measurement of a physical quantity; state your answer clearly, in the form \( g = \ldots \pm \ldots \text{m/sec}^2 \). State the accepted value of \( g \) for Salt Lake City (see below) and describe how you found this value. State the difference between your measurement and the accepted value, in standard deviation units; i.e., \([g(\text{measured}) - g(\text{accepted})]/dg(\text{measured})\). This experiment is usually in agreement with the accepted value within one or two standard deviations. If your result is not within this margin, and you have more time in lab, try to see where the difference lies.

Of the various corrections to the answer that one might make (finite \( \theta_{\text{max}} \), damping, etc. See Appendix B) the only one that is big enough to matter is that for \( \theta_{\text{max}} \). Do your results for the measurement of period as a function of \( \theta_{\text{max}} \) for a given length agree with the proposed model? Explain. Quantify.

Calculate an “accepted” value of \( g \). Hint: see [http://www.ngs.noaa.gov/cgi-bin/grav_pdx.prl](http://www.ngs.noaa.gov/cgi-bin/grav_pdx.prl). In this section compare your calculated value to that from the NOAA website, for instance, and to your measurement.

Have your efforts tested a model? What model? Do your results support or refute the model? Quantify your support. How do your values for \( g \) in compare to the “correct value”. If your values are not within the error of your calculated “correct value”, try to explain unexplored sources of error, starting with inadequate estimates of the errors in your measurements. Was your method of testing the accuracy of the stopwatch adequate; the ability of your reflexes to start and stop the watch? Meter stick and tape measure, both their inherent accuracy and your ability to read them? If your value of \( g \) differs from the “correct value” by more than two standard deviations of the mean, try to think of plausible explanations, a.k.a. “models”. How would you test your models?

Write a one-short-paragraph conclusion to your paper. Your conclusion should recap your major numerical result(s), and your main inferences. Begin with a statement of the model you are testing. Then address the question: Do the data support the model you are testing, or not? Use a discrepancy criterion. How certain are you of this claim? Certainty in physics is established by the number of significant digits you can justify. Was the experiment you performed a good way of testing the model? How could you improve your method/experiment?

**References**

Cite all references you consulted in the writing of this report. References must be formatted using the conventions of the AIP style manual: [http://www.aip.org/pubservs/style/4thed/toc.html](http://www.aip.org/pubservs/style/4thed/toc.html)
References (cited in this handout)

Appendix A

Include here the tables with your data and the corresponding uncertainties.

Appendix B

Other considerations you may take into account to explain your results:
Up to now, we have neglected several other factors that will affect our calculated value of $g$. We address some of those now. You can find a more detailed description in Nelson and Olsson’s work [1].

*Extensibility of the String*
Consider the simple pendulum shown in the Figure 2. Take the pendulum to be a point-like bob of mass $m$, attached to a string that doesn’t stretch, which has length $l$. How will you determine the extent (quantify, of course) to which the string stretches as the tension in it changes during the course of a period? How can you determine the extent to which the string stretches? How can you arrange other parameters to minimize the effects of a finite spring constant in the string? How does the stretchiness of the string factor into your measurements?

*Damping Contribution* — Air resistance will damp the amplitude of the oscillation over time, slowing the bob and increasing the period of the oscillations. The actual period for a damped oscillator is given by [1]:

$$T_d = \frac{T_0}{\sqrt{1-\xi^2}}$$

(5)

where $\xi = T_0 / 2\pi\tau$, $T_0$ is the period in a vacuum and $\tau$ is the time for the oscillations to decrease in magnitude by a factor of $1/e$.

Using your setup, estimate $\tau$. Determine the relative systematic error of the measured period $T_d$ compared to $T_0$. Does damping contribute significantly to the error in your experiment? Note that, as was with the case with $T(\theta_{max})$, $T_d$ gives a correction to be applied to your measurements.

*Rotational Inertia Contribution* — A key difference between a “real world” pendulum and the simple pendulum is the fact that any real pendulum bob will not be point-like. The total energy of the system must be modified to include a rotational inertia term:

**Point-like bob:**

$$E_{tot} = \frac{1}{2} m (\dot{\theta})^2 + mgl(1 - \cos \theta) \quad (KE + PE)$$

(6)

**Physical bob:**

$$E_{tot} = \frac{1}{2} m (\dot{\theta})^2 + \frac{1}{2} I \dot{\theta}^2 + mgl(1 - \cos \theta)$$

(7)

where $I$ is the moment of inertia of the bob around its center of mass.
(a) From these equations, show that, for a spherical bob, the effect of the rotational term is to modify the period of oscillation by a factor

\[ T_{\text{physical}} = T_{\text{simple}} \times \sqrt{1 + \frac{2a^2}{5l^2}} \] (8)

where \( a \) is the radius of the bob. As with the others, this is another correction to your measurement of the period.

(b) Measure the mass and radius of the bob used in our experiment. For what values of the length of the string, \( l \), will this effect produce a 1\% change in the period? 0.1\%? What is the contribution of the moment of inertia of the bob to your measurements?