Asymmetric Deviation of the Cross Section from the Lorentzian Around Ly Alpha

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Introduction

Quasar Absorption Systems

1) Ly alpha Forest $\sim N_{\text{HI}} < 10^{17} \text{ cm}^{-2}$
2) Lyman Limit Systems $N_{\text{HI}} > 10^{17} \text{ cm}^{-2}$
3) Damped Ly alpha Systems $N_{\text{HI}} > 10^{20.2} \text{ cm}^{-2}$
4) Metal Systems $\sim$ C, N, O Lines
5) Broad Absorption Line Systems $\sim$ Quasar Wind
1. Lyman alpha forest may correspond to residual neutral hydrogen forming filametary structures with characteristic length scale of tens of kiloparsecs.

2. Damped Lyman alpha systems are galaxies that lie on the line of sight in front of the quasar.

\[ f(N) = 4.9 \times 10^7 N^{-1.46}, \quad 12.3 < \log N < 14.5 \]
1. First stars and galaxies emit strong UV radiation.

2. More and more part of intergalactic medium gets ionized.

3. First stars are believed to be very massive due to lack of coolants during their star formation.
Gunn-Peterson Troughs

1. Gunn-Peterson Troughs: Due to neutral hydrogen in early universe, blueward of Lyman alpha is heavily absorbed.

2. Accurate atomic physics will be helpful in order to extract useful information from profiles.
Voigt Profile Fitting Analysis

1. Voigt function is given by convolution of a Gaussian and a Lorentzian.

2. A Lorentzian function represents a natural profile near a resonance transition in the rest frame of the atom.
1. A resonance line is treated as a damped simple harmonic oscillator driven by external oscillating E-fields.

2. This leads to cross section given by a Lorentzian in \( \omega \) (frequency) space.

3. Lorentzian presented in \( \lambda \) (wavelength) space is asymmetric even in classical treatment.

4. A real atom is regarded as a complex spring characterized by infinitely many spring constant \( k \).
1. Matrix elements of dipole operator inversely weighted by energy difference comprise the probability amplitude.

2. In the case of hydrogen, all the matrix elements are known analytically.

3. Annihilation of the incoming photon, creation of an outgoing photon, excitation to and de-excitation from np state.

4. The contribution from np state is weighted inversely by the difference of the photon energy and level spacing between 1s and np.

5. The dominant contribution is made from 2p state, leading to Lorentzian cross section.
Analogy of Multislit Experiment

1. The issue of energy conservation is avoided due to the uncertainty principle.

2. We do not disturb the system in order to check which ‘np’ state an electron is excited into during the interaction.

3. A similar analogy is provided from a multislit experiment. We do not care which slit a given incident particle passes through.

4. The contribution from np state is weighted inversely by the difference of the photon energy and level spacing between 1s and np.
Matrix Elements

\[ \tau_{nl}^{n'l-1} = \int_0^\infty R_{n'l-1}(r)R_{nl}(r)r^3 \, dr \]

\[ = \frac{2^{2l}}{(2l-1)!} \left[ \frac{(n+l)!(n'+l-1)!}{(n-l-1)!(n'-l)!} \right]^{1/2} \times (nn')^{l+1} \left( n+n' \right)^{-n-n'} \left( n-n' \right)^{n-2-l} (n'-n)^{n'-l} \times \left\{ F(-n+l+1, l-n', 2l, a_{nn'}) - \left( \frac{n-n'}{n+n'} \right)^2 F(-n+l-1, l-n', 2l, a_{nn'}) \right\} \]

\[ R_{n'l=1} = \frac{2[1+n'^2]^{1/2}}{[1-e^{-2\pi n'}]^{1/2}} \frac{\Gamma^2}{2\pi} \int e^{-2ir\xi/n'} (\xi + \frac{1}{2})^{-in'-2} (\xi - \frac{1}{2})^{in'-2} \, d\xi \]
1. No atom is not a two-level system. Need to consider the contribution from other levels.

2. One can expand the Kramers–Heisenberg formula in terms of $\frac{\Delta \omega}{\omega_{\text{Ly}\alpha}}$ in omega space. The leading term is Lorentzian, but the next term provides asymmetric deviation of cross section.

3. In lambda space, expansion in terms of $\frac{\Delta \lambda}{\lambda_{\text{Ly}\alpha}}$ introduces an additional term enhancing asymmetry.

4. This effect is important in high column system including DLAs.
Analysis in Omega Space

Cross section blueward of line center is smaller that redward in the first order approximation!
Interpretation

1. Tug of war between 2p and higher p states.

2. ‘2p’ state gives a blueward contribution whereas higher np states contributes to redward of Lyman alpha.

3. The two contributions are similar but ‘np’ contributions are slightly larger resulting in red asymmetry.

4. One rough measure is the oscillator strength.

\[ \sigma = \sigma_T \left( \frac{\omega_{21}}{\Delta \omega} \right)^2 \left| A_0 + A_1 \left( \frac{\Delta \omega}{\omega_{21}} \right) + A_2 \left( \frac{\Delta \omega}{\omega_{21}} \right)^2 + \cdots \right|^2 \]

where the coefficients \( A_k \) are obtained through the following relations:

\[ A_0 = \frac{3}{2} \omega_{21} < x >_{12} = \frac{1}{2} f_{12} = 0.2081 \]

\[ A_1 = \frac{1}{2} \sum_{n=3}^{\infty} \frac{\omega_{21}^2 \omega_{n1}}{\omega_{n1}^2 - \omega_{21}^2} < x >_{1n}^2 = -0.1865 \]
Analysis in Lambda Space

1. Observational spectroscopy is often presented in wavelength space rather than in frequency space.

2. Additional asymmetry is introduced due to the inverse relation between omega and lambda.

\[ \frac{\Delta \omega}{\omega_\alpha} = -\frac{\Delta \lambda}{\lambda} = \sum_{n=1}^{\infty} \left(-\frac{\Delta \lambda}{\lambda_\alpha}\right)^n \]
Result (Cross section)

Red Part:
Cross section (solid) is Higher than Lorentzian (dotted)

Blue Part
1. The line center of damped wing profile appears redward of true Ly α center.

2. For $N_{\text{HI}}=5 \times 10^{21} \text{ cm}^{-2}$, the apparent shift is 0.4 Å. Overestimation of the redshift by $0.4/1218=0.0003$ or 100 km/s

3. For $N_{\text{HI}}=4 \times 10^{22} \text{ cm}^{-2}$ the apparent shift is 1 Å, which is measurable.
Asymmetry near Ly beta

In a similar way we may expand K-H formula in the vicinity of Ly beta.

An additional scattering channel that gives rise to H alpha emission.

More significant and opposite deviation from Lorentzian.
Asymmetry near Ly beta

- Rayleigh channel: strong blue asymmetry
- Competition between (2p+3p) and higher np states.
- Contributions of (2p+3p) dominates to result in blue asymmetry

- Raman channel: weak red asymmetry
- Competition between 3p and other np states with n≠3.
- Absence of 2p contribution leads to slight red asymmetry

In total we obtain strong blue asymmetry.

\[
\sigma_{\text{tot}}(\lambda) \approx \sigma_\beta \left( \frac{\lambda_\beta}{\Delta \lambda_2} \right)^2 \left[ 1 - 24.63 \left( \frac{\Delta \lambda_2}{\lambda_\beta} \right) \right].
\]
Fig. 4. — Absorption profiles around Ly$\alpha$ (upper panel) and Ly$\beta$ (lower panel) for various neutral hydrogen column densities.
Cross Section Asymmetry around Ly $\alpha$ and $\beta$
Cross Section Asymmetry
Asymmetric Profile of DLA

\[ \text{Ly}\beta \]

\[ \text{Ly}\alpha \]

\( f(N_{\text{HI}})_\lambda \)

\( \lambda [\text{\AA}] \)

\( N_{\text{HI}} = 10^{20} \)

\( N_{\text{HI}} = 10^{21} \)

\( N_{\text{HI}} = 10^{22} \)

\( N_{\text{HI}} = 10^{22.5} \)

\( N_{\text{HI}} = 10^{23} \)
Asymmetric Profile of DLA

\[ N_{\text{HI}} = 10^{20} \]

\[ N_{\text{HI}} = 10^{21} \]

\[ N_{\text{HI}} = 10^{22} \]

\[ N_{\text{HI}} = 10^{23} \]
Asymmetric Profile of DLA

\[ N_{HI} = 10^{20} \]

\[ N_{HI} = 10^{21} \]

\[ N_{HI} = 10^{22} \]

\[ N_{HI} = 10^{23} \]

Classic Mechanics Solution
Quantum Mechanics Solution
Spectra of DLA in Noterdaeme et al. 2015
Gunn-Peterson Troughs
Discussion

1. Asymmetry may be observable in an absorbing system with $N_{\text{HI}} > 4 \times 10^{22} \text{ cm}^{-2}$

2. Can be important in precise determination of kinematics associated with metal lines in DLA’s with high HI column densities.

3. Can be applied to Gunn–Peterson Troughs and Red Lyman alpha profiles in the Reionization Era.