Lecture 01: 1D Kinematics

Physics 2210
Fall Semester 2014
Announcements

- 78 students registered on smartPhysics as of this morning.
- You should have done Prelecture 1 and Checkpoint 1 by now.
- Policy Change: Attendance at discussion sections is not mandatory but strongly encouraged. Students who participate in discussion sections consistently perform better in Physics 2210.

Alles Leben ist Problemlösen
- Karl Popper
Definitions of Kinematic Quantities

Displacement: \( x(t) \)

Velocity: \( v(t) \equiv \frac{dx(t)}{dt} \) Velocity is the time rate of change of displacement.

Acceleration: \( a(t) \equiv \frac{dv(t)}{dt} \) Acceleration is the time rate of change of velocity.

Obtaining Displacement and Velocity from Acceleration

Displacement: \( x(t_f) - x(t_i) = \int_{t_i}^{t_f} v(t) \, dt \) Displacement is the integral of the velocity over time.

Velocity: \( v(t_f) - v(t_i) = \int_{t_i}^{t_f} a(t) \, dt \) Velocity is the integral of the acceleration over time.

Special Case: Motion with Constant Acceleration

The displacement is obtained by integrating the velocity over time.

The velocity is obtained by integrating the constant acceleration over time.

\[ x = x_0 + v_0 t + \frac{1}{2} at^2 \]

\[ v = v_0 + at \]

\( a = \text{constant} \)
“Supplemental Readings” link on course webpage:

Equation Sheet

**Kinematics**
\[ g = 9.81 \text{m/s}^2 = 32.2 \frac{\text{ft}}{\text{s}^2} \]
\[ \ddot{v} = \dot{a} \]
\[ \ddot{x} = x_0 + \dot{v}_0 t + \frac{1}{2} a t^2 \]
\[ v^2 = v_0^2 + 2a(x - x_0) \]

**Uniform Circular Motion**
\[ a = \omega^2 r \]
\[ v = \omega r \]

**Dynamics**
\[ \vec{F}_{\text{net}} = m \vec{a} = \frac{d\vec{v}}{dt} \]
\[ \vec{F}_{\text{friction}} = -\mu F_N \vec{v} \]
\[ F = mg \text{ (near Earth's surface)} \]
\[ F_{\text{gravity}} = G \frac{m_1 m_2}{r^2} \text{ (in general)} \]

**Friction**
\[ f = \mu F_N \text{ (kinetic)} \]
\[ f = \mu N \text{ (static)} \]

**Work & Kinetic Energy**
\[ W = \int \vec{F} \cdot d\vec{l} \]
\[ W = \vec{F} \cdot \Delta \vec{r} = F \Delta r \cos \theta \]
\[ W_{\text{grav}} = -mg \Delta y \]
\[ W_{\text{spring}} = -k \left( x_2 - x_1 \right) \]
\[ K = \frac{1}{2} mv^2 = \frac{\vec{p}^2}{2m} \]

**Potential Energy**
\[ U_{\text{grav}} = mgy \text{ (near Earth)} \]
\[ U_{\text{grav}} = -G \frac{Mm}{r} \text{ (general)} \]
\[ U_{\text{spring}} = \frac{1}{2} kx^2 \]
\[ \Delta E = \Delta K + \Delta U = W_{\text{NC}} \]

**System of Particles**
\[ \vec{R}_{\text{CM}} = \frac{\sum m_i \vec{r}_i}{\sum m_i} \]
\[ \vec{V}_{\text{CM}} = \frac{\sum m_i \vec{v}_i}{\sum m_i} \]
\[ \sum \vec{F}_{\text{ext}} = M \vec{a}_{\text{CM}} \]
\[ K_{\text{system, lab}} = K_{\text{relative to CM}} + K_{\text{CM}} \]

**Momentum**
\[ \vec{p}_{\text{total}} = M \vec{v}_{\text{CM}} \]
\[ \frac{d\vec{p}_{\text{total}}}{dt} = \vec{F}_{\text{net}} \]
\[ \int \vec{F}_{\text{net}} dt = \Delta \vec{p} = \vec{p}_{\text{final}} - \vec{p}_{\text{initial}} \]

**Elastic collisions**
\[ K_{\text{system}} = \sum \frac{1}{2} m_i v_i^2 \] is conserved

**Rotational kinematics**
\[ s = R\theta, \ v = R\omega, a = R\alpha \]
\[ \theta = \theta_0 + \omega t + \frac{1}{2} \omega^2 t^2 \]
\[ \omega = \omega_0 + \alpha t \]
\[ \omega^2 = \omega_0^2 + 2\alpha (\theta - \theta_0) \]

**Rotational Dynamics**
\[ I = \sum m_i r_i^2 \]
\[ I_{\text{parallel}} = I_{\text{CM}} + MD^2 \]
\[ I_{\text{loop}} = MR^2 \]
\[ I_{\text{link}} = \frac{1}{2} MR^2 \]
\[ I_{\text{rod-CM}} = \frac{1}{12} ML^2 \]
\[ I_{\text{solid-sphere}} = \frac{4}{5} \frac{M}{2} \]
\[ I_{\text{solid-rod}} = \frac{1}{4} ML^2 \]
\[ I_{\text{hollow-sphere}} = \frac{2}{5} \frac{M}{2} \]
\[ \vec{\tau} = \vec{r} \times \vec{F} \]
\[ \vec{\tau} = I\vec{\alpha} \]

**Statics**
\[ \sum \vec{F} = 0, \sum \vec{\tau} = 0 \text{ (any axis)} \]

**Angular Momentum**
\[ \vec{L} = \vec{r} \times \vec{p} \]
\[ \vec{L}_{\text{total}} = \vec{L}_{\text{CM}} + \vec{\tau} \]
\[ \frac{d\vec{L}_{\text{total}}}{dt} = \vec{\tau}_{\text{net}} \]

**Simple Harmonic Motion**
\[ x(t) = A \cos(\omega t + \phi) \]
\[ v(t) = -A \omega \sin(\omega t + \phi) \]
\[ a(t) = -A \omega^2 \cos(\omega t + \phi) \]
\[ \omega = \sqrt{\frac{k}{m}} \text{ (mass-spring)} \]
\[ \omega = \sqrt{\frac{k}{c}} \text{ (simple pend.)} \]
\[ \omega = \sqrt{\frac{k}{s}} \text{ (physical pend.)} \]
\[ \omega = \sqrt{\frac{k}{I}} \text{ (tension pend.)} \]

**General Harmonic Transverse Waves**
\[ y(x, t) = A \cos(kx - \omega t) \]
\[ k = \frac{2\pi}{\lambda} \]
\[ \omega = \frac{2\pi}{T} = 2\pi f \]
\[ v = \lambda f = \frac{\lambda}{T} \]

**Waves on a String**
\[ \frac{d^2y}{dx^2} = \frac{1}{v^2} \frac{d^2y}{dt^2} \]
\[ v_{\text{wave}} = \sqrt{\frac{T}{\mu}} \]
\[ K_{\text{max}} < \omega^2 A^2 \]
# Help Lab Schedule (JFB Rotunda)

<table>
<thead>
<tr>
<th>Day</th>
<th>Time</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>09:00 – 11:00 AM</td>
<td>Amaya</td>
</tr>
<tr>
<td>Tuesday</td>
<td>12:00 – 01:00 PM</td>
<td>Nick</td>
</tr>
<tr>
<td>Wednesday</td>
<td>09:30 – 11:30 AM</td>
<td>Jason</td>
</tr>
<tr>
<td>Thursday</td>
<td>12:00 – 01:00 PM</td>
<td>Nick</td>
</tr>
<tr>
<td></td>
<td>02:00 – 03:00 PM</td>
<td>Mei</td>
</tr>
<tr>
<td>Friday</td>
<td>02:00 – 03:00 PM</td>
<td>Mei</td>
</tr>
</tbody>
</table>

Also will be posted to course web page.
Unit 1 Feedback

• Thanks for your comments!

• Main points:
  • Definition of acceleration, esp. negative acceleration.
  • Interpreting graphs
  • Checkpoint #2
  • Calculus practice
  • Examples

I would really like to have a breakdown of the concepts into singular elements during lecture. I feel like that would be really helpful since the broad concepts are pretty much ingrained in most of the particulars, but the underlying principles live in an unnecessarily hazy world. Even those principles could be broken down into smaller pieces. I think, basically, I'd like to be taught from the ground up, not from the ethereal cloud of concept down to the minutia in a long tangents that's caused me to forget the original question.

I find the ball rolling down to be the hardest because I've used seeing a equation of this and the being able to take a derivative and figure out time, maybe more clarity around this question would help me.

It was all easy.

How to breakdown physics questions into simpler parts. More what we need to find and what we don't know.

I would like to see the formula existing displacement, velocity, and acceleration without time as a variable again.

Well, I started off the ball rolling question by trying to do $0 = V_0C (change in t)$, but after mailing a bit, that wouldn't be a sound calculation since the velocity was not constant. The hard part for me was breaking past my "High School Physics Zombie Mode. Typical Calculation Machine" type of thinking. Hopefully it was worth it and I actually got the answer correct, or this will be embarrassing. I had trouble mostly with starting to integrate (his pain intended) Calculus into my Physics.

Examples of applying the kinematics equations to different scenarios would be helpful to understand what problems to use what equation.

I found the integration of the acceleration and the displacement equation a bit confusing, I'm not quite sure how doing $2a(x) = V_2 = V_0 + at$ as an equation is more helpful than just finding the $v(x), v(t)$ and the $x(t)$ as separate equations.

I want to talk more about the concept of how "slowing down" and acceleration is related.

If anything, a quick overview of the basic formulas we use to solve for derivatives and integrals would be nice just because I haven't used them in a while.

I wish there was a summary of equations.

Note:
Talk about how to find the velocity and acceleration by differentiating by using examples.

Simulate the examples.

Although the calculus presented is not difficult I find myself questioning how to proceed at times.

I found reading the graphs to be the hardest. As well as understanding the differences between derivatives and integrals and when to use them.

I would like to cover constant acceleration,

when does the object speeds up or down. Relationship between acceleration and velocity.

Can we talk a little more about acceleration. I was confused on the pre-lecture slide about velocity being negative while acceleration is positive. I was confused about the signs and the reasons for them.

I understand the pattern of $+/+$ velocity and $+/+$ acceleration with respect to a graph. However, I would love some more examples to make it easier to visualize. Also, some more mathematical examples for velocity and acceleration problems. Thanks.

I am a bit confused on interpreting the various kinds of graphs.

Please show examples of acceleration down a ramp. I am trying to fully understand the concepts.

What is the exact definition of magnitude? I recognize the word, but can't remember the definition, which made it hard to follow.

I find reading the graphs of velocity to find acceleration hard, and also the positive and negative acceleration slide??? That one lost me. Need more explanation on that and reading those graphs please.

Specific examples, not a re-invention of the broad concepts from the pre-lecture or checkpoint.

Graph on question one was a bit confusing as the x-axis was defined as time, but the y-axis wasn't specified next to it. Also, the question was confusing before reading the question carefully to determine what the axis was and what it wanted me to find. Otherwise the lecture was fine, lot of these past the formulas quickly which would have been confusing had I not already understood what I was looking at from my Calculus classes.

I wrote you an email and was unable to attend Monday's lecture and with this due by tomorrow morning. I figured I would at least say that you were very informative and prompt with responding to my email earlier this week.

What I found hardest to understand at first was how velocity related to displacement over time. I know that velocity is equal to the derivative of displacement, but the way the guy in the videos explained it was confusing.

Now I think that being taught online instead of by a teacher is dumb. Youtube is free and uses does a better job.

In the equation for displacement with constant acceleration, I don't understand how $0$ is calculated for the value of $v_0$ (negative) $t$. In other words, I don't understand why the middle of the equation on the right zero's out in the ball rolling example when computing the displacement at $t = 2$; $x = v_0 t + v_0 t^2 + \frac{1}{2} a t^2$. Also, further discussion of demonstrating the acceleration from displacement and velocity graphs would be helpful.

I didn't find any concepts to be too difficult, but I would like to see more graph examples and the relationships between the displacement, velocity, and acceleration.

The relationship between displacement and velocity when you remove the time from the equations, resulting in the $2a(x) = v_0 - v_0$ equation.

Slowing down vs. speeding up when acceleration is constant (velocity is a straight line)

I had trouble wrapping my head around the backwards work. Such as, how to get the change in displacement from the velocity and how to get change in velocity from acceleration. I would like to review the position from velocity (1.4) section most.

The concepts were really easy to pack back up knowing that it was simple calculus, but what was difficult was remembering how to set up the graphs relationship with displacement, velocity, and acceleration. I was hoping that a quick review on how to draw the graphs with $v(t)$ as also could be done in lecture.

I would like a confirmation of the ramp problem of Checkpoint 2, as my attempts to test it have not borne the same results consistently.
Kinematics: \textit{n.} The branch of mechanics that studies the motion of a body or a system of bodies without consideration given to its mass or the forces acting on it.

\texttt{www.thefreedictionary.com}

Today's Concepts:

a) Displacement, Velocity, Acceleration

b) 1-D Kinematics with constant acceleration
Displacement and Velocity in One Dimension

**Average Velocity**

\[ v_{\text{avg}} \equiv \frac{\Delta x}{\Delta t} \]

**Instantaneous Velocity**

\[ v(t) \equiv \frac{dx(t)}{dt} \]

**Displacement, time interval finite**

**Displacement, time interval infinitesimal**
Flashcard Questions
Displacement and Velocity in One Dimension

Are the plots shown at the left correctly related?

A) YES
B) NO
Displacement and Velocity in One Dimension

Are the plots shown at the left correctly related?

A) YES
B) NO
The velocity vs. time plot of some object is shown to the right.

Which diagram below could be the Displacement vs. time plot for the same object?
The velocity vs. time plot of some object is shown to the right.

Which diagram below could be the Displacement vs. time plot for the same object?

A  B  C  D
Acceleration

Displacement \( x(t) \)

Velocity \( v(t) \equiv \frac{dx(t)}{dt} \)

Acceleration \( a(t) \equiv \frac{dv(t)}{dt} \)
Checkpoint 1:
For the Displacement and Velocity curves shown on the left, which is the correct plot of acceleration vs. time?
The object slowed down (negative acceleration) and at the midpoint sped up (positive acceleration).

Acceleration is negative while the velocity decreases and it is positive as the velocity increases.
**Constant Acceleration**

\[ v(t_f) - v(t_i) = \int_{t_i}^{t_f} a(t) \, dt \]

\[ x(t_f) - x(t_i) = \int_{t_i}^{t_f} v(t) \, dt \]

**constant**  
\[ a(t) = a \]

\[ x = x_o + v_o t + \frac{1}{2} at^2 \]

\[ v = v_o + at \]

\[ v^2 - v_o^2 = 2a(x - x_o) \]
Example

A velocity-time graph for an object moving along the x axis is shown in the figure.

a) Plot a graph of the acceleration vs time
b) Determine the average acceleration for $5 \, \text{s} < t < 15 \, \text{s}$ and
c) for $0 < t < 20 \, \text{s}$. 
Suggested Problem Solving Template

1) Keep calm, and...
2) Read the problem carefully.
3) Draw a diagram w/ coordinates.
4) List knowns.
5) List unknowns.
6) Find equation linking knowns & unknowns.
7) Solve the equation.
8) Repeat as needed.
At $t = 0$ a ball, initially at rest, starts to roll down a ramp with constant acceleration. Suppose it moves 1 foot between $t = 0$ sec and $t = 1$ sec.

How far does it move between $t = 1$ sec and $t = 2$ sec?

A) 1 foot  B) 2 feet  C) 3 feet  D) 4 feet  E) 6 feet
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A) 1 foot  B) 2 feet  C) 3 feet  D) 4 feet  E) 6 feet
Acceleration is one foot per second, using displacement equation with accel as a constant gives us \( x = 0 + 0 + .5 \times 1 \times 2^2 = 2 \)

The ball has an acceleration of 1ft/s\(^2\). So, for every second that the ball is moving, it gains another 1ft/s of velocity.
The ball is now rolled up the same ramp. It is given an initial speed of 10 feet/second. How far will it roll before stopping, and then rolling back down the ramp?

Ans: 25 feet
A hare and a tortoise race over a straight course, 1.00 km long. The tortoise crawls at a speed of 0.200 m/s toward the finish line. The hare runs at a speed of 8.00 m/s toward the finish line for 0.800 km and then stops while the tortoise passes. The hare then resumes running at 8.00 m/s, and the hare and the tortoise cross the finish line at the same time.

a) How far is the tortoise from the finish line when the hare resumes running?
b) For what time interval was the hare stationary?

1) Keep calm, and...
2) Read the problem carefully.
3) Draw a diagram w/ coordinates.
4) List knowns.
5) List unknowns.
6) Find equation linking knowns & unknowns.
7) Solve the equation.
8) Repeat as needed.

a) 25 meters
b) 4,875 seconds
Example

A ball is thrown upwards with an initial speed of 3.0 meters/second. How far will it rise before it comes to rest, and then falls back down?

(Assume the acceleration due to gravity $g = 9.81 \text{ m/s}^2$, and negligible air resistance.)

0.459 meters