Linear Least Squares Fitting: Special Cases

There are two special cases of the least squares fit which we may need: The case in which all uncertainties in \( y \) (the \( \sigma_i \)) are the same, and the case in which the fit is required to pass through the origin (\( A = 0 \)).

**All uncertainties the same:** This is the case derived in Chapter 8 of Taylor.\(^1\) The \( \sigma_i \)'s just divide out of the solutions for \( A \) and \( B \) in the general case.

Suppose we have \( N \) points \((x_i, y_i)\) that we wish to fit to a line of the form

\[
y = A + Bx
\]

If we define the sums

\[
\Sigma_x = \sum_{i=1}^{N} x_i \quad \Sigma_y = \sum_{i=1}^{N} y_i \\
\Sigma_{xy} = \sum_{i=1}^{N} x_i y_i \quad \Sigma_{xx} = \sum_{i=1}^{N} x_i^2
\]

then the best-fit values for \( A \) and \( B \) are given by

\[
B = \frac{N \Sigma_{xy} - \Sigma_x \Sigma_y}{\Delta} \quad A = \frac{\Sigma_{xx} \Sigma_y - \Sigma_x \Sigma_{xy}}{\Delta}
\]

where

\[
\Delta = N \Sigma_{xx} - \Sigma_x \Sigma_x
\]

and the uncertainties in \( B \) and \( A \) are

\[
\sigma_B = \sigma_y \sqrt{\frac{N}{\Delta}} \quad \sigma_A = \sigma_y \sqrt{\frac{\Sigma_{xx}}{\Delta}}
\]

**Uncertainties the same, line through origin:** In this case, the best fit value for \( B \) is

\[
B = \frac{\Sigma_{xy}}{\Sigma_{xx}}
\]

and its uncertainty is

\[
\sigma_B = \sigma_y \sqrt{\frac{1}{\Sigma_{xx}}}
\]

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