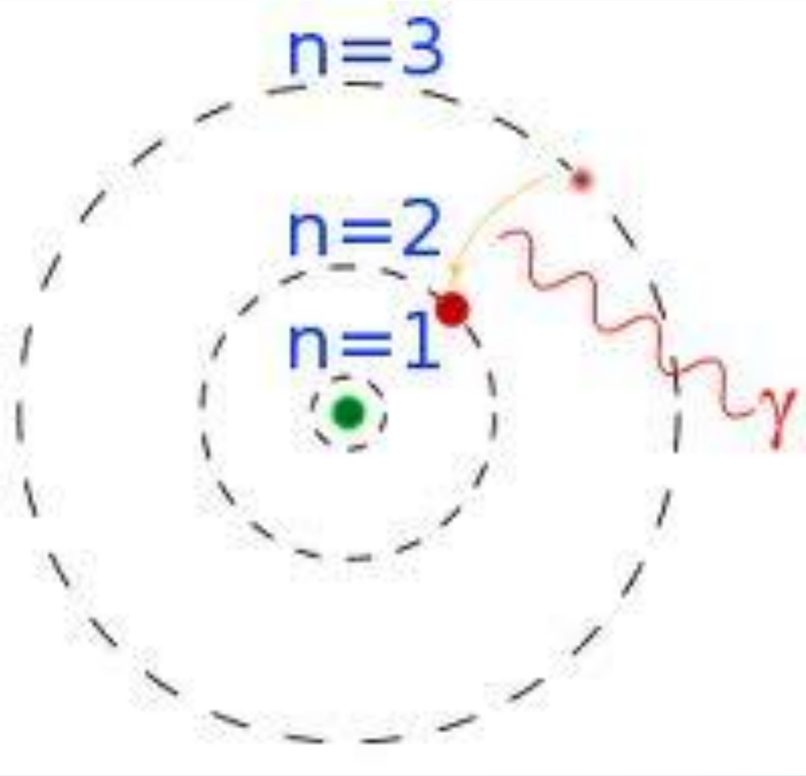
The Measurement of The Rydberg Constant

By Alex Gibbs

Background

- In 1913, Niels Bohr proposed his "Bohr Model"; a working model of the hydrogen atom.
- Supported his model by deriving the Rydberg Formula: $\frac{1}{\lambda} = R\left[\left(\frac{1}{nf^2}\right) - \left(\frac{1}{ni^2}\right)\right]$
- What is the Rydberg Constant R?



Theoretical Derivation of Rydberg Formula

- Electron contained in orbit around nucleus by a balance between Centripetal and Coulomb Forces:

$$\left(\frac{1}{4\pi\epsilon o}\right)\left(\frac{e^2}{r_n^2}\right)=\frac{m_e v_n^2}{r_n} \qquad (1)$$

- Angular momentum($L = m_e v r$) is quantized and is an integral multiple of $\frac{h}{2\pi}$ so:

$$m_e v_n r_n = n\left(\frac{h}{2\pi}\right) \qquad (2)$$

- Solving Equations (1) and (2) gives:

$$r_n = \varepsilon_0\left(\frac{n^2h^2}{\pi m_e e^2}\right) \quad \text{and} \quad v_n = \left(\frac{1}{\varepsilon_0}\right)\left(\frac{e^2}{2nh}\right)$$

Theory Continued

- In the Bohr Model Total energy is equal to Kinetic energy plus Potential energy:

$$E_n = KE_n + PE_n = \left(\frac{1}{2}\right) m_e v_n^2 + \left(-\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{e^2}{r_n}\right) = \left(\frac{1}{\varepsilon_0^2}\right)\left(\frac{m_e e^4}{8n^2h^2}\right) - \left(\frac{1}{\varepsilon_0^2}\right)\left(\frac{m_e e^4}{4n^2h^2}\right)$$

- Which reduces to:

$$E_n = -\left(\frac{1}{\varepsilon_0^2}\right)\left(\frac{m_e e^4}{8n^2h^2}\right)$$

- Since the principle quantum number n characterizes the orbit, the energy change due to orbit transitions is:

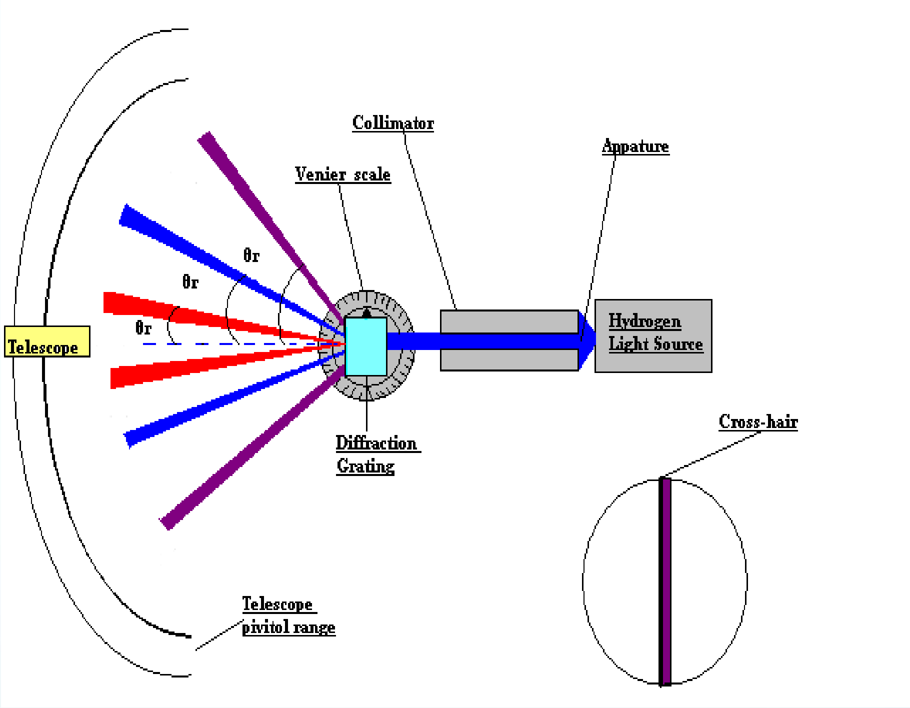
$$\Delta E = E_i - E_f = \left(\frac{m_e e^4}{8\varepsilon_0^2 h^2}\right)\left[\left(\frac{1}{n_f^2}\right) - \left(\frac{1}{n_i^2}\right)\right]$$

- Where the Rydberg Constant is:

$$R_H = \left(\frac{m_e e^4}{8\varepsilon_0^2 h^2}\right) = 1.09 * 10^7\ m^{-1}$$

Experimental Methods

- A Hydrogen Discharge lamp is used to excite atoms to make transitions between energy states producing light.
- The light is collimated and sent through a diffraction grating
- The diffraction grating separates the light into its spectrum
- A telescope is attached to view the spectrum and measure the angle of diffraction



Plan of Analysis

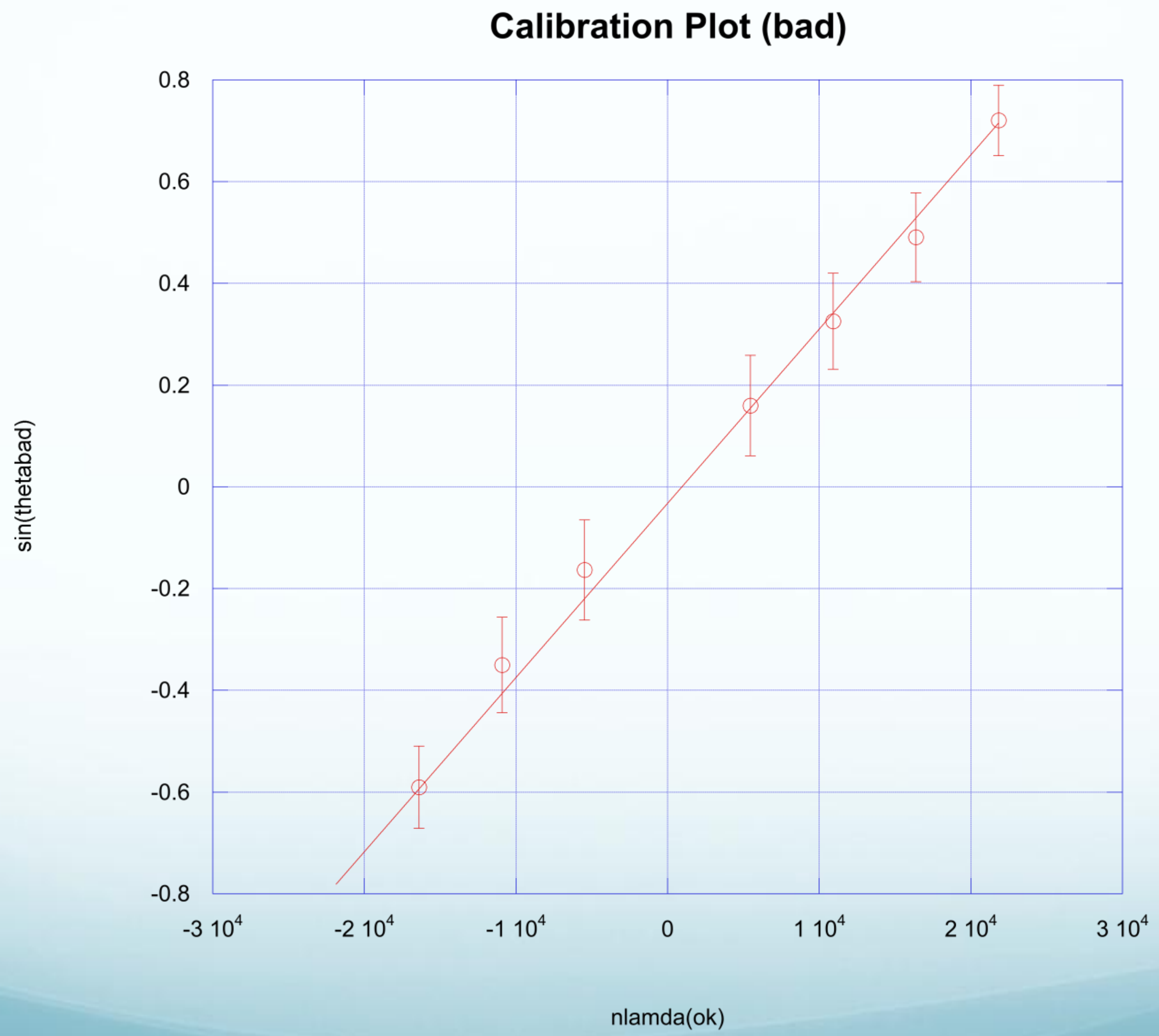
- Calibration of apparatus
- Hydrogen Visible Wavelengths Derivation
- Rydberg Constant Derivation

Calibration

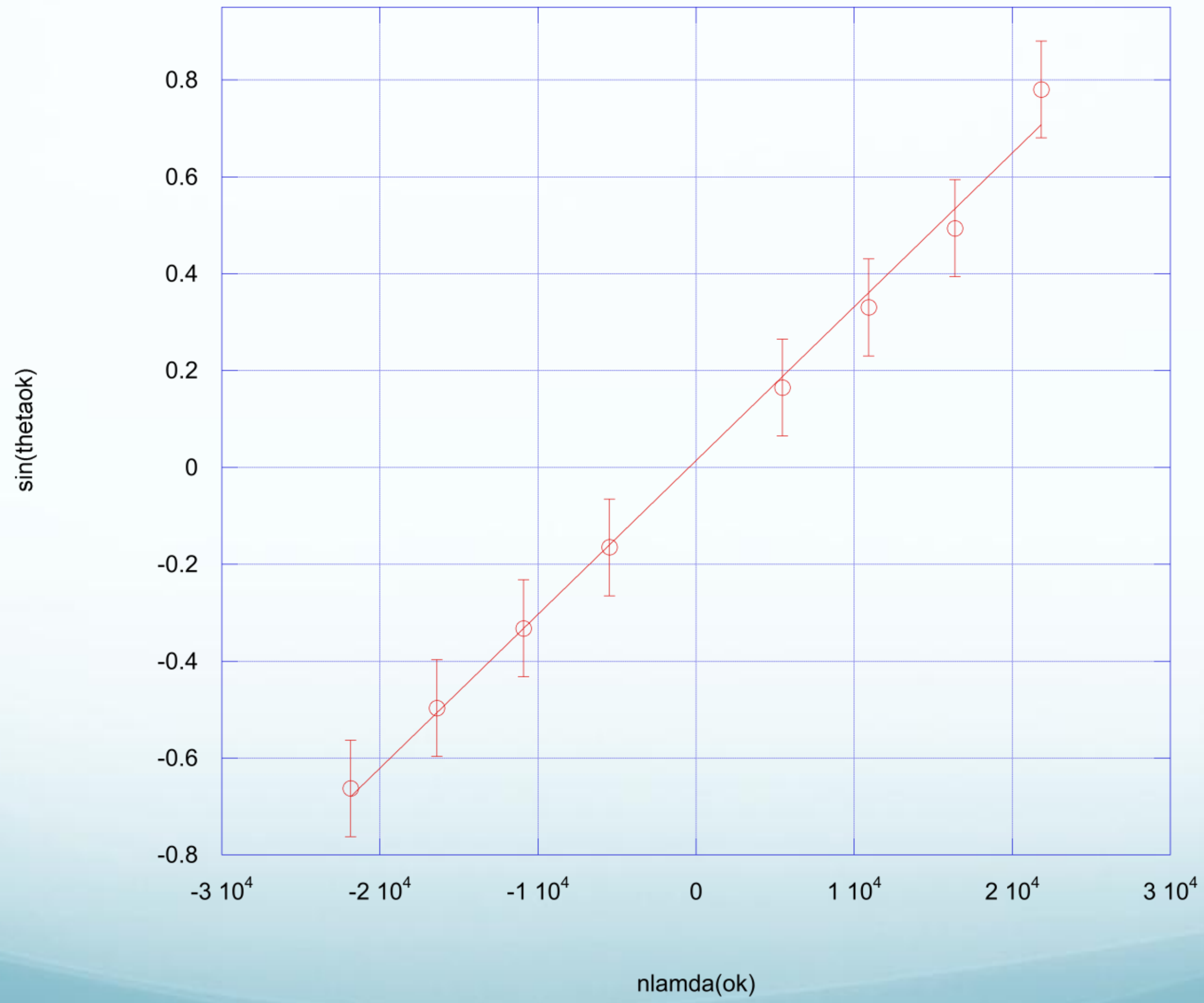
- A good calibration is crucial when conducting this experiment with this apparatus
- Diffraction angles are a “finger print” of the element so must be correct and not distorted
- Good calibration is achieved when diffraction angles are symmetric about θ_r=0
- Calibration is done using mercury lamp of known wavelength: $\boldsymbol{\lambda}\ (\text{green}) = 5460.74$ Å

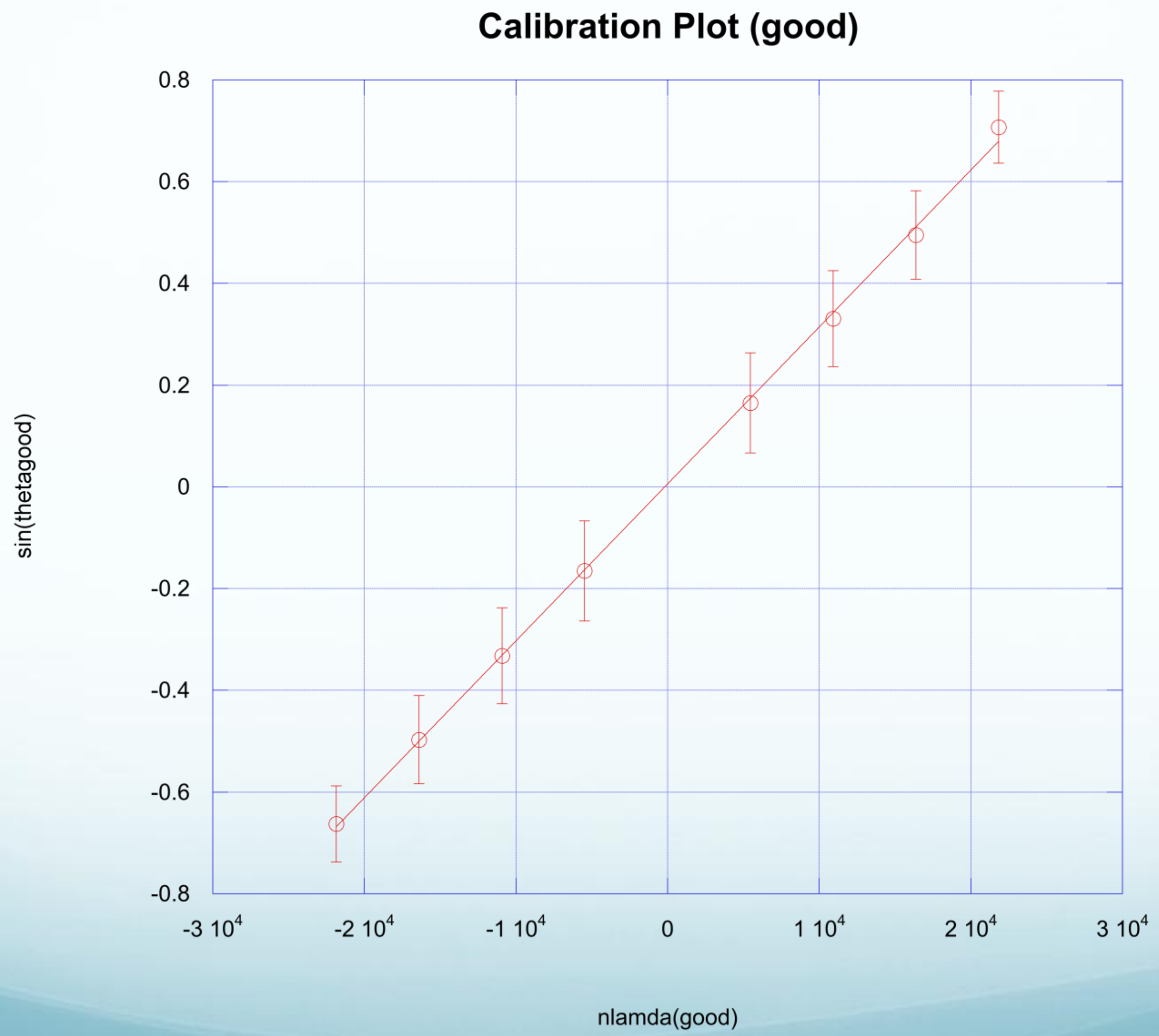
Calibration Data

n	Θr(bad)	Θr(ok)	Θr(good)	n*λ (angstoms)	sin[Θr(bad)]	sin[Θr(ok)]	sin[Θr(good)]
1	9.2±0.1	9.2±0.1	9.5±0.1	5460.74	0.16	0.165	0.165
2	19.0±0.1	19.3±0.1	19.3±0.1	10921.5	0.326	0.331	0.331
3	29.4±0.1	29.6±0.1	29.7±0.1	16382.2	0.491	0.494	0.495
4	46.1±0.1	51.3±0.1	45.0±0.1	21843	0.721	0.78	0.707
-1	-9.4±0.1	-9.5±0.1	-9.5±0.1	-5460.74	-0.163	-0.165	-0.165
-2	-20.5±0.1	-19.4±0.1	-19.4±0.1	-10921.5	-0.35	-0.332	-0.332
-3	-36.2±0.1	-29.8±0.1	-29.8±0.1	-16382.2	-0.591	-0.497	-0.497
-4	-58.9±0.1	-41.5±0.1	-41.5±0.1	-21843	-0.856	-0.663	-0.663


Calibration Plot (bad)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
sin(thetabad)
-3 10^4
-2 10^4
-1 10^4
0
1 10^4
2 10^4
3 10^4
nlamda(ok)

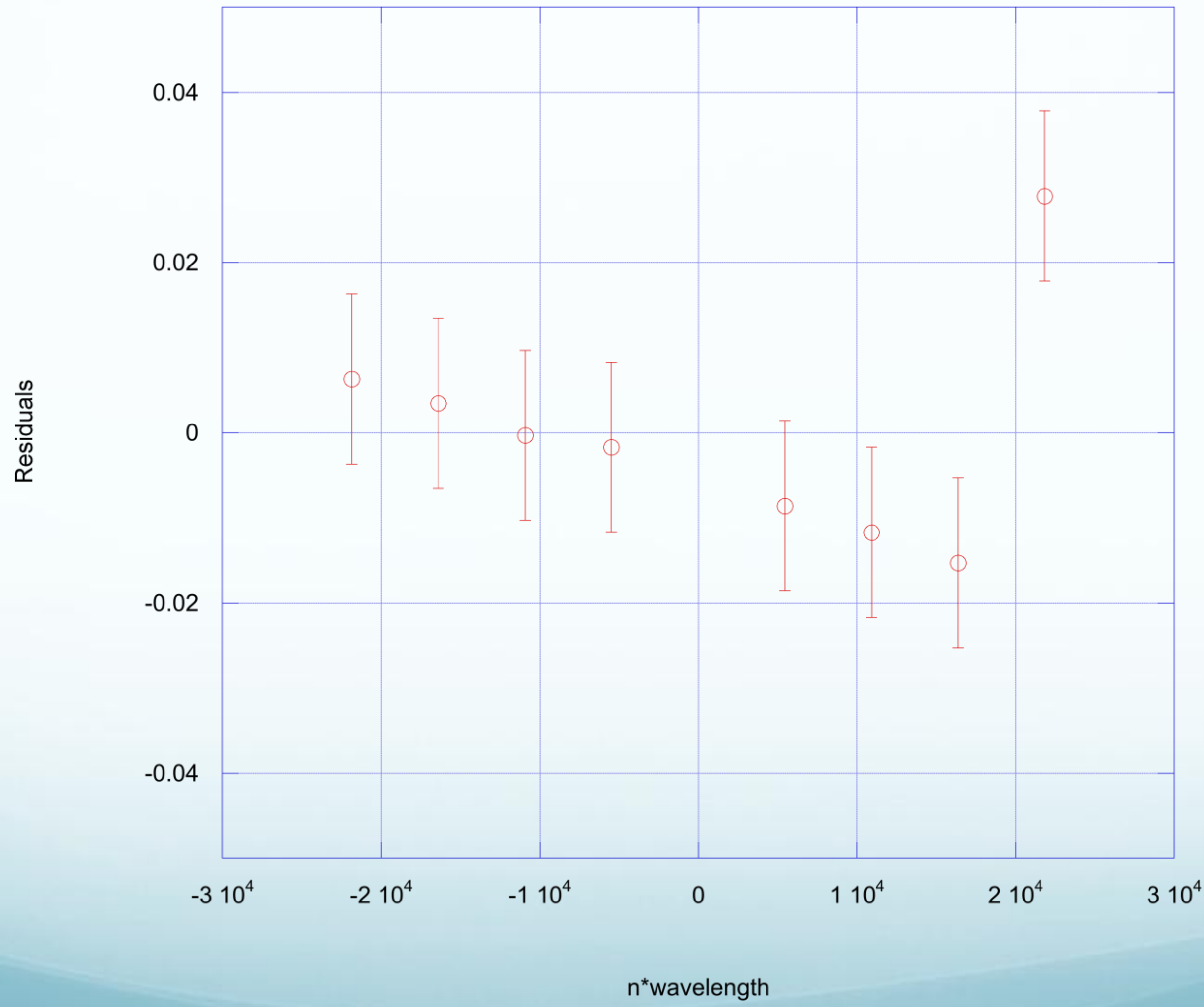
Calibration Plot (ok)





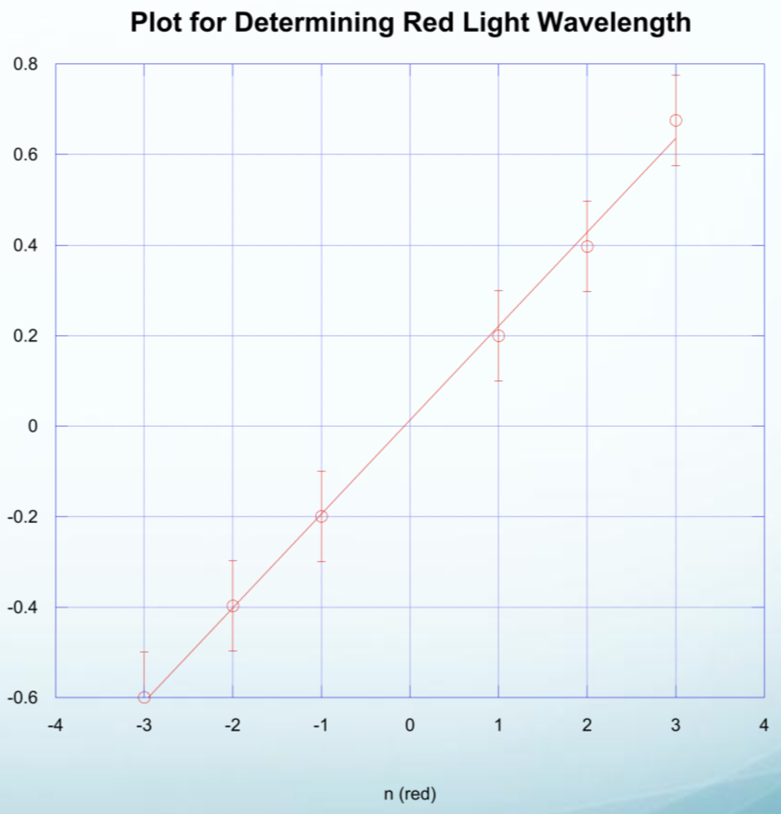
Calibration Plot (good)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-3 10^4
-2 10^4
-1 10^4
0
1 10^4
2 10^4
3 10^4
sin(thetagood)
nlamda(good)

Residual Plot (Good Cal.)



Determination of Visible Wavelengths

- Measure Θr at n= ±1, ±2, ±3 for the visible hydrogen wavelengths H(red), H(blue) and H(purple)
- Make a linear least squares fit from the data pairs
- Extract values of wavelengths from the slopes of the graphs
- λ(red)= 665 ± 21 nm,
- λ(blue)= 462 ± 9 nm,
- λ(purple)= 432 ± 4 nm



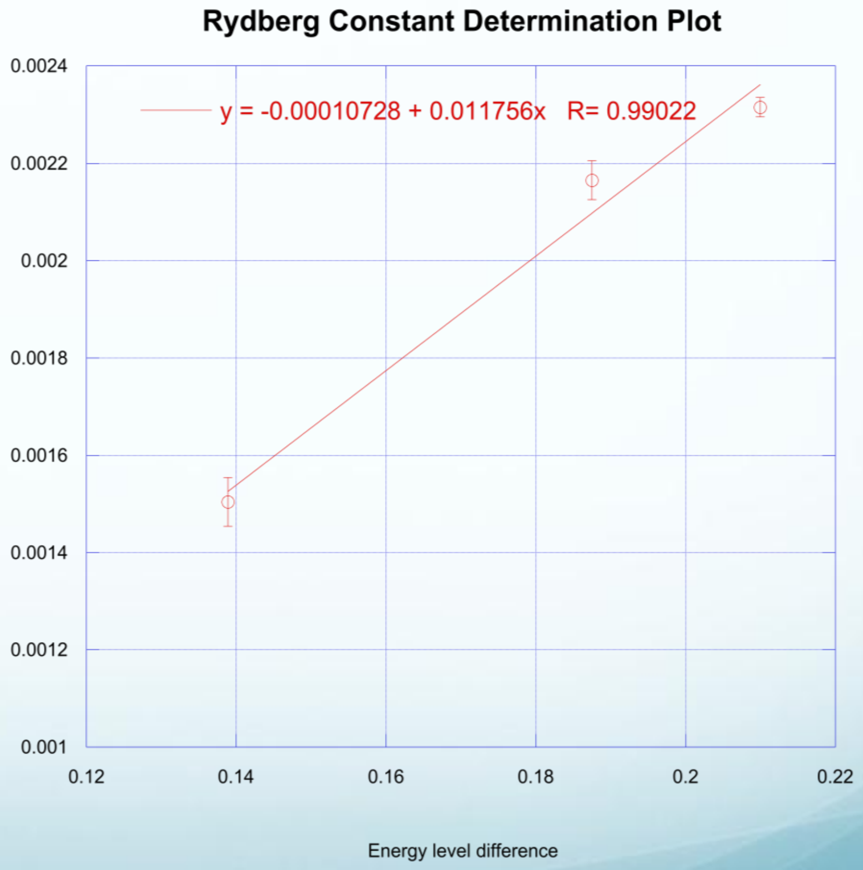
Determination of Rydberg Constant

- Make a linear least squares fit of the data pairs:

$$\left[\left(\frac{1}{2^2}\right)-\left(\frac{1}{n^2}\right)\right],\frac{1}{\boldsymbol{\lambda}}=(\mathbf{x},\mathbf{y})$$

- Determine Rydberg Constant from slope.
- Best Fit gives:

R_H=1.17x10^7 ± .03 m^-1



Final Result & Discussion

- Ok agreement between experimental and expected data. $\chi^2 = 2.5$
- Corresponds to a percentage probability of 8.2%.
- Greater than 5% level but not by much.
- 7.5% error relative to theoretical value.

Conclusion

- The hydrogen spectrometer can be used to obtain an ok value of the Rydberg constant,

R_H=1.17x10^7 ± .03 m^-1

- Further calibration can lead to a Really Good value for the Rydberg constant: Bad Calibration gives:

R_H=1.21x10^7 ± .05 m^-1

2.4% improvement in accuracy