C++ Functions

ftype fname (arguments)
{
   ...
   return [ftype];
}

vtype vname...
Newton (or Newton-Raphson) Method for Finding Roots
Problem: given $f(x) = 0$, solve for $x$.

Example: consider a balloon rising underwater, subject to the equation:

$$h(t) = t^2 - 4$$

Make a guess $t_0$, for the time at which the balloon reaches the surface. Expand the function in a Taylor series about $t_0$:

$$h(t) = h(t_0) + (t - t_0) \cdot \frac{dh(t_0)}{dt} + \cdots$$

Keep only the first two terms. It is then straightforward to find the time $t_1$ at which this linear approximation to $h(t)$ equals zero:

$$h(t_0) + (t_1 - t_0) \cdot \frac{dh(t_0)}{dt} = 0$$

or, rearranging

$$t_1 = t_0 - \frac{h(t_0)}{dh(t_0)/dt}$$

$t_1$ is an improved guess as to the true zero of our initial function $h(t)$.

For our example, suppose we guess $t_0 = 1$. Then we have

$$t_1 = t_0 - \frac{h(t_0)}{dh(t_0)/dt} = 1 - \frac{(1 - 4)}{2} = 2.5$$

which is closer to the true root of $h(t)$ than $t_0$. Iterating this procedure, we can make successively better guesses as to the root of $h(t)$ as follows:

$$t_2 = t_1 - \frac{h(t_1)}{dh(t_1)/dt} = 2.05$$

$$t_3 = t_2 - \frac{h(t_2)}{dh(t_2)/dt} = 2.000609756$$

$$\cdots$$

$$t_{k+1} = t_k - \frac{h(t_k)}{h'(t_k)}$$
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