1. (8 pts) Please give short answers to the following questions.

(a) (4) Describe what is meant by the term *differential cross section* $d\sigma/d\Omega$ in the context of scattering experiments. What is $d\sigma$? What is $d\Omega$? What are the units of each?

\[ \frac{d\sigma}{d\Omega} \text{ is a mapping of an infinitesimal area of the scattering target (d}d)\text{ onto the solid angle (d}\Omega\text{) into which the phase is scattered. d}d\text{ has units of area, typically "barns" = (}10^{-24}\text{ m}^2\text{). d}\Omega\text{ has units of steradians or solid angle.} \]

(b) (4) Explain what is meant by the term *form factor*. When and how is a form factor used?

*Form factor is the Fourier transform of the charge distribution in momentum transfer space. It modifies the Lorentz differential cross section in the case of an extended target as:* \[ \frac{d\sigma}{d\Omega \text{ extended}} = |F(q)|^2 \frac{d\sigma}{d\Omega \text{ point}} \]

2. (6 pts) Fill in the table below. Use a “+” sign to indicate that the particle can interact via the given force. Use a “−” sign to indicate that the particle *does not* interact via the given force.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Strong</th>
<th>Electromagnetic</th>
<th>Weak</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0$ boson</td>
<td>−</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>neutral pion $\pi^0$</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>muon</td>
<td>−</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>electron antineutrino</td>
<td>−</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>photon</td>
<td>−</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>top quark</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>
3. (4 pts) Particles which decay via the strong interaction have extremely short lifetimes, typically of order $10^{-24}$ seconds. Even at the speed of light, such particles will travel a distance less than the diameter of a proton in that time. Explain then how we are able to determine their lifetimes.

\[ \Delta E \Delta t = \Delta (mc^2) \geq \frac{\hbar}{\Delta E} \]

By observing the width of the mass distribution for a short-lived particle, we may infer its lifetime $\Delta t$.

4. (6 pts) Parity violation in the weak interaction can be equated with the statement that “all neutrinos are left-handed”.

(a) (3) Explain what that statement means. A figure might be useful.

The spin of a neutrino, by the right-hand rule, always points opposite the direction of its momentum.

\[ \downarrow \rightarrow \vec{p}_\nu \]

\# Opposite for an antineutrino

\[ \uparrow \rightarrow \vec{p}_\bar{\nu} \]

(b) (3) Explain how the left-handedness of neutrinos results in a parity-violating effect in $^{60}\text{Co}^{27}$ decays.

$^{60}\text{Co}^{27}$ loses a unit of spin when it decays to $^{60}\text{N}_\text{e}^{28} + \text{e}^- + \bar{\nu}_\text{e}$.

The anti-neutrino’s spin must point in the direction of the lost unit, therefore its momentum must be in this direction.

The electron to conserve momentum must therefore be emitted in a direction opposite that of the $^{60}\text{Co}^{27}$ spin.
5. (12 pts)

(a) (4) Define, in words or an equation, the term binding energy.

"Binding Energy" is the difference between the observed mass of a nucleus and the sum of the masses of its constituents.

\[
BE(A, \chi^2) = M(A, \chi^2) - Z \times m_p - (A-Z) \times m_n
\]

(b) (4) Make a sketch of the "Curve of Binding Energy", that is, of the binding energy per nucleon \((-BE/A)\) versus the nucleon number \(A\) of a nucleus. On this curve, indicate the location of the most stable nucleus. What is the most stable nucleus? Indicate the range of nuclei which will give off energy if induced to undergo nuclear fission.

(c) (4) Is it possible for a spontaneous fission reaction to have a positive \(Q\)-value? If so, give an example. If not, explain why.

A spontaneous fission reaction **must** have a positive \(Q\)-value.

\[
Q = (M_{\text{initial}} - M_{\text{final}})c^2
\]

If the final mass is greater than the initial mass, the decay cannot occur spontaneously.
6. (8 pts) Consider the plutonium isotopes \(^{239}\text{Pu}\) and \(^{240}\text{Pu}\).

(a) (4) The table below lists names of terms in the Bethe-Weizsäcker semi-empirical mass formula. For each term, write “239” if the term tends to make the \(^{239}\text{Pu}\) isotope heavier, write “240” if it tends to make the \(^{240}\text{Pu}\) isotope heavier, and “=” if the term is equal for the two isotopes.

<table>
<thead>
<tr>
<th>Term</th>
<th>Heavier?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
<td>239</td>
</tr>
<tr>
<td>Surface</td>
<td>240</td>
</tr>
<tr>
<td>Coulomb</td>
<td>239</td>
</tr>
<tr>
<td>Asymmetry</td>
<td>240</td>
</tr>
<tr>
<td>Pairing</td>
<td>239</td>
</tr>
</tbody>
</table>

\[
M(A, Z) = Zm_p + (A-Z)m_n - a_v A + a_s A^{2/3} + a_c Z^2/A^{1/3} + a_d (N-Z)^2/A - 101 + a_p/A^{3/4}
\]

(b) (4) Which of the two plutonium isotopes is capable of sustaining a fission chain reaction (i.e. which is “fissile”)? Explain this on the basis of your answer to part (a) above.

\(^{239}\text{Pu}\) is fissile. Because it is odd-odd, it can absorb a neutron, becoming even-even, and increasing the BE (make it more negative). Energy released can initiate further reactions.
7. (16 pts) $^{10}\text{C}^6$ is unstable with a half life of 19 seconds.

(a) (4) Into what will it decay, and through what process? List all particles and nuclei participating in this reaction, in an equation of the form $A + B + \ldots \rightarrow C + D + \ldots$.

It is neutron poor, and hence will decay via electron capture into $^{10}\text{B}^5$:

$$e^- + ^{10}\text{C}^6 \rightarrow ^{10}\text{B}^5 + \nu_e$$

(b) (6) What particle(s) will be emitted from the nucleus in the course of this decay? What is the maximum kinetic energy of the particle(s) which will be emitted from the nucleus?

An electron will be captured and an electron neutrino emitted.

$$Q = \left[M_{\text{init}} - M_{\text{final}}\right]c^2 = KE$$

$$= \left[(9,327.573 + 0.511) - (9,324.436)\right]c^2$$

$$= 3.648 \text{ MeV} \rightarrow \text{max KE of Neutrino}$$

(c) (6) Draw a quark diagram for this decay. Only show the nucleon(s) which undergo change, plus any other elementary particles involved in the reaction. Include any mediating bosons involved in the reaction.

```
\begin{center}
\[ P \left\{ \begin{array}{c}
  \bar{u} \\
  d \\
  u \\
\end{array} \right\} \xrightarrow{W} \left\{ \begin{array}{c}
  u \\
  d \\
  \bar{u} \\
\end{array} \right\} n \\
\end{center}
```

$$e^- \quad \nu_e$$
8. (20 pts) *Photoproduction* is the creation of a short-lived resonance by bombarding particles with high-energy photons, *i.e.* gamma rays. For example, consider the reaction

\[ \gamma + X \rightarrow \Delta(uud) \rightarrow n + \pi^+ \]

(a) (4) What is particle \( X \)? Explain how you know this.

*Particle \( X \) interacts electromagnetically with a photon. The EM force cannot change quark flavors, so \( X \) must have same quark content as \( \Delta \), uud.*

\[ \Rightarrow \ X \ is \ a \ proton \]

(b) (4) Via what fundamental force does the \( \Delta \) decay? Again, explain how you know this.

\[ \Delta^{uud} \rightarrow n^{udd} + \pi^+ \]

\[
\begin{array}{cccc}
\text{Baryon #} & 1 & 1 & 0 \\
I_3 & \frac{1}{2} & -\frac{1}{2} & 1 \\
\end{array}
\]

*Conservation all quantities \( \Rightarrow \) Strong decay*
(c) (4) Draw a simple quark-line diagram for the $\Delta$ decay.

\[
\Delta \left\{ \begin{array}{c}
  u \\
  d \\
  u
\end{array} \right\} u \left\{ \begin{array}{c}
  u \\
  d \\
  d
\end{array} \right\} n
\]

\[
\left\{ \begin{array}{c}
  d \\
  u \\
  d
\end{array} \right\} \bar{d} \left\{ \begin{array}{c}
  d \\
  u
\end{array} \right\} \pi^+
\]

(d) (8) In a photoproduction experiment, the $\Delta$ decays into a neutron with momentum 10,300 MeV/c and a pion with momentum 809 MeV/c, with an opening angle of 12.3$^\circ$ between them. Calculate the mass of the $\Delta$.

\[
E_n = \sqrt{10,800^2 + 939.565^2} = 10,343 \text{ MeV}
\]

\[
E_\pi = \sqrt{809^2 + 139.570^2} = 821 \text{ MeV}
\]

\[
m_\Delta^2 = E^2 - p^2 = (E_n + E_\pi)^2 - (p_n^2 + p_\pi^2 + 2p_n p_\pi \cos \theta)
\]

\[
M_\Delta = 1265 \text{ MeV/}c^2
\]

(Note: Accuracy $m_\pi = 1232$ MeV, changed for exam purposes)
9. (20 pts) The $B^0$ meson ($\bar{d}b$) is the lightest particle containing a "bottom" quark.

(a) (4) Without knowing anything about the observed decay modes of the $B^0$, make an argument that the $B^0$ must decay via the weak interaction.

It is the lightest particle containing a $b$-quark, so whenever it decays into will not contain a bottom quark. Quark flavor will be lost in the reaction, only weak force can change quark flavor.

(b) (4) One observed decay mode is $B^0 \rightarrow K^+ \pi^-$. Draw the simplest quark-line diagram for this decay.
(c) In the decay above, the momentum of either daughter particle in the center-of-mass frame is $|\vec{p}_{\text{com}}| = 2614.5$ MeV/c. What is the mass of the $B^0$?

$$m_B = E_k + E_{\pi^-} = (m_{K^+}^2 + p_{K^+}^2)^{\frac{1}{2}} + (m_{\pi^-}^2 + p_{\pi^-}^2)^{\frac{1}{2}}$$

$$= (493.667^2 + 2614.5^2)^{\frac{1}{2}} + (139.570^2 + 2614.5^2)^{\frac{1}{2}}$$

$$= 2660.7 \text{ MeV} + 2618.2 \text{ MeV}$$

$$m_B = 5278.9 \text{ MeV}$$

(d) If the $B^0$ has a speed $v = 0.999c$ in the lab when it decays, what is the minimum opening angle between the $K^+$ and $\pi^-$ in the lab?

Minimum opening angle will occur when daughter momenta 1 TO Z-AXIS in $B^0$ rest frame:

Energy-momentum conserved Transforms:

$$P_x' = P_x, \quad p_z' = \gamma(p_z - \beta E), \quad E' = \gamma(E - \beta p_z)$$

$$\rightarrow$$
\( \beta = 0.999 \)

\[
\sigma = \frac{1}{\sqrt{1-\beta^2}} = 22.366
\]

**1st K\(\alpha\)**

\( E_k = 2660.7 \text{ MeV (above)} \)

\( P_{kx} = P_{com} = 2614.5 \text{ MeV/}\mu \)

\( P_{kz} = 22.366 \left( 0 + 0.999 \times 2660.7 \right) = 59.450 \text{ MeV/}\mu \)

\( \theta_k = \arctan \left( \frac{P_{kx}}{P_{kz}} \right) = \arctan \left( \frac{2614.5}{59.450} \right) = 2.518^\circ \)

**2nd Pion**

\( E_\pi = 2618.2 \text{ MeV (above)} \)

\( P_{\pi x} = P_{com} = 2614.5 \text{ MeV/}\mu \)

\( P_{\pi z} = 22.366 \left( 0 + 0.999 \times 2618.2 \right) = 58.501 \text{ MeV/}\mu \)

\( \theta_\pi = \arctan \left( \frac{P_{\pi x}}{P_{\pi z}} \right) = \arctan \left( \frac{2614.5}{58.501} \right) = 2.559^\circ \)

Opening angle = \( \theta_k + \theta_\pi = 2.518^\circ + 2.559^\circ = 5.077^\circ \)
<table>
<thead>
<tr>
<th>A</th>
<th>Z</th>
<th>Name</th>
<th>Mass (MeV)</th>
<th>BE</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>002</td>
<td>$^{10}$He</td>
<td>9,362.728</td>
<td>30.339</td>
</tr>
<tr>
<td>10</td>
<td>003</td>
<td>$^{10}$Li</td>
<td>9,346.458</td>
<td>45.315</td>
</tr>
<tr>
<td>10</td>
<td>004</td>
<td>$^{10}$Be</td>
<td>9,325.503</td>
<td>64.977</td>
</tr>
<tr>
<td>10</td>
<td>005</td>
<td>$^{10}$B</td>
<td>9,324.436</td>
<td>64.751</td>
</tr>
<tr>
<td>10</td>
<td>006</td>
<td>$^{10}$C</td>
<td>9,327.573</td>
<td>60.320</td>
</tr>
<tr>
<td>10</td>
<td>007</td>
<td>$^{10}$N</td>
<td>9,350.163</td>
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<td>10,254.018</td>
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<td>79.575</td>
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<td>006</td>
<td>$^{12}$C</td>
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<td>12</td>
<td>007</td>
<td>$^{12}$N</td>
<td>11,191.689</td>
<td>74.041</td>
</tr>
<tr>
<td>12</td>
<td>008</td>
<td>$^{12}$O</td>
<td>11,205.888</td>
<td>58.549</td>
</tr>
</tbody>
</table>

Masses and binding energies for various nuclei, from [http://www.einstein-online.info](http://www.einstein-online.info).

| $\hbar$ | $6.582119.28 \times 10^{-22}$ MeV·sec |
| $m_e$    | $0.510998928$ MeV/$c^2$ |
| $m_n$    | $939.565378$ MeV/$c^2$ |
| $m_{\pi^+}$ | $139.57018$ MeV/$c^2$ |
| $m_{K^+}$ | $493.667$ MeV/$c^2$ |

Table of constants