ABSTRACT

A Search for the Decay $K^+ \rightarrow \pi^+\mu^+e^-$

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December 1997

We search for the lepton flavor number violating decay $K^+ \rightarrow \pi^+\mu^+e^-$ in Experiment E865 at the Alternating Gradient Synchrotron at Brookhaven National Laboratory. Data from the 1995 experimental run is analyzed. We find no evidence for this decay and establish an upper limit on its branching ratio of $2.2 \times 10^{-10}$ at the 90% confidence limit for a uniform phase-space density decay distribution. The experiment ran again in 1996, and took a factor of 4-5 more data. Another run is planned for 1998 which is expected to approximately double the amount of data taken in 1995 and 1996.
A Search for the Decay $K^+ \rightarrow \pi^+ \mu^+ e^-$

A Dissertation
Presented to the Faculty of the Graduate School
of
Yale University
in Candidacy for the Degree of
Doctor of Philosophy

By
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December 1997
Acknowledgements

I’ll begin by thanking my advisor, Michael Zeller. From him I learned how to be a physicist, and occasionally how not to be a physicist. His dedication and passion for physics were always an inspiration, and his analyses always insightful.

Next, I must thank the rest of the E865 collaboration, without which this work would have never happened. I must thank Hong Ma for his help in understanding most every aspect of the experiment, as well as his support in furthering my career. I must also thank Julia Thompson in that regard. The other graduate students: Juan Lozano, Hanh Do, Stefan Pislak, Wolfgang Menzel and Scott Eilerts helped to make life during runtime bearable, as did Heinz Kaspar, Don Lazarus, Jim Lowe, Dave Kraus, Robin Appel, Walid Majid, Horst Fischer, Christof Felder and Naipor Cheung. My Russian colleagues always provided a fresh perspective, I’d especially like to thank Vladimir Isakov, Andre Poblaguev and Grigor Atoyan.

When not at Brookhaven, it was my friends at Yale who kept me sane through the slings and arrows of outrageous fortune. John Lajoie, Randal Hans and Martin White saw me through from the very beginning; it seems I’m the last to go. All the people on the 5th floor who made my life easier, my lunches more interesting, and who distracted me when I was stuck on a problem: Abhay Deshpande, Rob Harr, Chris Kennedy, Steve Pappas, Andrew Wallace, Karen Ohl, Frank Rotondo, Brenda Naegel and Carole DeVore. Jean Belfonti always took care of whatever administrative problems I might have, as well as just being there to talk to.

I mustn’t forget the Happy Hour crew, even though the Happy Hour itself is now long defunct. Mark Vagins, Mel Michelson, Jim McCambridge and the rest. The volleyball and poker people: Dave Shiner, Paul and Beth Harkins, Chris Darling,
Jale Okay, Andy Beveridge, Jamie Nagle, Jeff Snyder, Kazim Yildiz, Kate Smith, Gary Schoenhals, Ron Dixon and Nick Bateman among many others.

There are three whom I must thank for putting up with me: Robin Totsch, who couldn’t stand it; Melissa Vollrath, who never had the chance to try it for long; and most importantly, Lucy Lifschitz, who’s agreed to do it for the rest of time. I very much doubt there’ll be enough of it for me to repay my debt of gratitude to her.

Finally, I thank my parents, Rolf and Marty Bergman, to whom I dedicate this dissertation. They’ve always supported me, emotionally and financial when necessary; allowing me to concentrate on the things that needed to be done. I thank them again, as I thank everyone, for getting me here and for helping me through, allowing me to transcend and move beyond.
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Chapter 1

Introduction & Motivation

This thesis describes an experiment to search for a rare, possibly forbidden, decay of the K meson. Experimental searches for rare or forbidden processes must be chosen with care. While such searches can be a good way to look for new physical phenomena, they can also be costly and frustrating disappointments. Thus, one wants to choose such a search with care, balancing the chance for spectacular finding with the probability of a null result. One wants to search for objects that probe the weakest points of theories, using elegant, simple and inexpensive methods. What’s more, one wants to search in areas where a null result is also a success, because of what one has shown does not happen.

Experiment E865 at Brookhaven National Laboratory’s Alternating Gradient Synchrotron is one such search. It is a search for the decay $K^+ \rightarrow \pi^+\mu^+e^-$ ($K_{\pi\mu\nu}$). This search embodies the criteria for worthwhile searches given above. While the Standard Model of Particle Physics forbids the decay, it does so only through the postulate of Lepton Flavor Number conservation. This postulate is an ad hoc addition to the Standard Model: never having been observed to have been violated, but not strongly theoretically motivated. In fact, most of the theories that extend the Standard Model explicitly violate this conservation law. The decay is also an example of a Flavor Changing Neutral Current, which is only suppressed, not forbidden, in the Standard Model. In fact, one might argue that Lepton Flavor Changing decays
are only suppressed, as are the Flavor Changing Neutral Currents. The only experimental difference between a process which is suppressed and one which is forbidden is that the forbidden process hasn’t been seen.

The decay $K_{\pi\mu e}$ also sheds light on one of the most perplexing problems with the Standard Model: the origin of mass. The masses of fermions are not determined in the Standard Model, rather they are put in by hand via the coupling to the Higgs sector. Many of the theories that extend the Standard Model address the origins of these coefficients. In these theories, it is often the case that the process that gives mass to particles also induces Lepton Family Number non-conservation. Chapter 2 describes the theoretical motivation for undertaking this search.

This is not an easy experiment. The previous limit on the branching ratio of this decay mode is $2.1 \times 10^{-10}$[1]. E865 was designed to reach a sensitivity of $3 \times 10^{-12}$, while this thesis describes an analysis of data from the very first running period, in the spring of 1995, and sets a limit of $2.2 \times 10^{-10}$. Sensitivity to such a rare decay requires one to strictly control any known process which can mimic the decay for which one is looking. Strict control of background processes motivates the design of the detector as described in Chapter 3. It also guides one through the process of Data Acquisition (Chapter 4) and Analysis (Chapter 5).

The data in hand needs to be normalized to the absolute number of kaon decays to which the detector was sensitive. This is done using a well known kaon decay with a similar signature to $K_{\pi\mu e}$. Two possibilities are $K^+ \rightarrow \pi^+\pi^+\pi^- (K_{\pi\pi\pi})$ and $K^+ \rightarrow \pi^+\pi^0, \pi^0 \rightarrow e^+e^-\gamma (K_{\text{Dal}}, \text{so called because of the Dalitz decay, } \pi^0 \rightarrow e^+e^-\gamma, \text{of the pion}). K_{\pi\pi\pi} \text{ has the advantage of consisting entirely of charged tracks, and can be completely reconstructed. } K_{\text{Dal}} \text{ has the advantage of having an electron track, so the efficiency of detecting the electron is the same for both signal and normalization. I use } K_{\pi\pi\pi} \text{ as the normalization in this thesis. One must also know the acceptances both of the possible signal and the normalization, as well as the effects of different reconstruction efficiencies. Both the acceptance calculation and corrections due to different efficiencies are discussed in Chapter 6.}

The final result is presented in Chapter 7, along with a discussion of backgrounds and future prospects.
Chapter 2

Theory and Phenomenology

$K^+ \rightarrow \pi^+ \mu^+ e^- (K_{\pi\mu e})$ is a Lepton Flavor Number (LFN) violating decay. LFN conservation is an *ad hoc* law incorporated into the Standard Model (SM) to explain the fact that interactions changing LFN have not been seen. It is consistent with the rest of the SM because massless, degenerate neutrinos allow both lepton and neutrino mass matrices to be diagonalized simultaneously. In the quark sector, where both weak-isospin components have mass, the two mass matrices cannot be diagonalized simultaneously and this leads to quark flavor mixing in weak interactions as parametrized by the Cabbibo-Kobayashi-Masskawa (CKM) Matrix.

The *ad hoc* nature of LFN conservation makes it a good place to look for new physics. Understanding the very low level of LFN violation requires an understanding of at least some of the following topics: the existence of three generations of fermions, because it is only with multiple generations that mixing can occur at all; the origin of mass, which is problematic in the SM and is related to the assumed masslessness and degeneracy of the neutrinos required for LFN conservation; and the difference between the quark and lepton sectors, because the mixing of generations is observed in the quark sector while it has not, to much lower levels, been observed in the lepton sector. Theories dealing with these topics are presented below in Section 2.1. They all allow $K_{\pi\mu e}$, but not all are equally constrained by any upper limit one may set. The constraints $K_{\pi\mu e}$ and other LFN violation limits set on these theories is discussed in Section 2.2.
2.1 Beyond the Standard Model

2.1.1 The Gauge-Hierarchy Problem

Two theories presented below as introducing LFN violating interaction, Supersymmetry (SUSY) and Extended Technicolor (ETC) were originally conceived as solutions to the Gauge-Hierarchy Problem in the SM. I will present this problem here by way of motivation for those theories below.

The SM[2, 3] seeks to describe all fundamental interactions, excluding gravity. It has been remarkably successful, with no confirmed discrepancies between its predictions and experimental results having been reported. It divides particles/fields into two categories: fermions and bosons. Fermions are the spin-$\frac{1}{2}$ constituents of matter. They are subdivided into quark and leptonic sectors, and three generations of weak-isospin doublets:

$$
\begin{pmatrix}
  u \\
  d \\
  \nu_e \\
  e
\end{pmatrix}
\begin{pmatrix}
  c \\
  s \\
  \nu_\mu \\
  \mu
\end{pmatrix}
\begin{pmatrix}
  t \\
  b \\
  \nu_\tau \\
  \tau
\end{pmatrix}
$$

(1)

Bosons are the integer spin force carriers. There are three forces included in the SM: the strong force, represented by the eight gluons; the weak force, represented by the $W^\pm$ and $Z$ bosons; and electromagnetism, represented by the photon.

The $W^\pm$ and $Z$ bosons are massive, and this is where one can begin to see a problem. The interactions of the SM are developed within the paradigm of gauge invariance. Requiring local gauge invariance of the fermionic constituents of matter leads to the introduction of the bosonic fields to keep the theory gauge invariant. There is one field introduced for each gauge group, $SU(3)_c \times SU(2)_L \times U(1)_Y$. However, these gauge fields must themselves be massless to maintain the gauge invariance. In addition, the weak force, mediated through the $W^\pm$ and $Z$ bosons interacts with only the left handed helicity state of any fermion. This disallows fermion masses as well, as massive particles don’t have a well defined, invariant helicity for the $SU(2)_L$ interaction.

Both boson and fermion masses can be generated dynamically but at the cost of
introducing a scalar particle into the theory and a number of arbitrary parameters. To give mass to the SU(2)L bosons, a scalar doublet field, \( \Phi \) is introduced with a potential,

\[
V(\Phi^\dagger \Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2
\]

For \( \mu^2 \) and \( \lambda \) positive, this has a minimum at

\[
\Phi^\dagger \Phi = \frac{\mu^2}{2\lambda}
\]

which determines a non-zero vacuum expectation value (VEV). The minimization specifies one constraint on the \( \Phi \) system, but three of the four degrees of freedom remain unconstrained. The physical ground state specifies these three degrees of freedom and thus spontaneously breaks the symmetry. If one works not with \( \Phi \), but with perturbations around the physical VEV, one finds terms in the Lagrangian corresponding to masses for the weak bosons. Small deviation along the three unconstrained degrees of freedom are manifest as pseudo-Goldstone bosons. These become the longitudinal components of the now massive weak bosons. Perturbations of the fourth, constrained degree of freedom become the physical, massive Higgs boson. In the spontaneous symmetry breaking process, the neutral SU(2)L and U(1)Y bosons mix to become the physical Z boson and photon. Fermion masses are introduced via a Yukawa coupling between each fermion and the Higgs field. Each fermion has its own coupling constant, proportional to its mass.

The Higgs VEV is constrained to be about 250 GeV in order to get the correct W and Z masses. The physical Higgs mass is not tightly constrained however, as it depends on both \( \mu \) and \( \lambda \). Small values for the Higgs' mass have been excluded experimentally. The current lower limit is 58.4 GeV[4]. An upper limit on the Higgs mass can be obtained by considering W-Z and Z-Z scattering[5]. The cross-sections for these processes passes the unitarity limit for large mass Higgs, with mass above about 1 TeV. This is not an absolute limit, as it means the Higgs becomes strongly interacting and the perturbation theory is no longer applicable. On the other hand, it is unclear if one can still speak of an elementary Higgs boson at that point.

The Gauge-Hierarchy Problem appears when one tries to calculate higher order
loop corrections to the bare Higgs mass. Like fermion mass corrections, these corrections diverge, meaning the integral over loop momenta is not finite. This is handled by instituting a cutoff energy in the integration. In physical terms this is the upper limit of the applicability of the theory. Unlike fermion mass corrections, which diverge logarithmically with the cutoff energy, scalar mass corrections diverge quadratically. The SM is generally assumed valid up to GUT (Grand Unified Theory) scales of \(10^{12}\) TeV. This is twelve orders of magnitude higher than the allowed Higgs mass, implying corrections of order \(10^{24}\) TeV\(^2\). One can set the bare Higgs mass at will, but in order to get an electroweak scale physical Higgs mass, one must match the bare Higgs mass to the corrections to 24 decimal places! There is no a priori reason to expect this particular Higgs mass, making the light Higgs seem unnatural. This is the source of the other names of the Gauge-Hierarchy Problem: The Fine-Tuning problem and the Naturalness Problem. The term Gauge-Hierarchy comes from the fact that the GUT gauge symmetry is broken at one scale, while \(SU(2)_L \times U(1)_Y\) is broken at another, much lower, scale. Fermion mass corrections, on the other hand, are all within a factor of 50 of the physical masses, so no such unnatural fine-tuning of the bare mass need be performed.

### 2.1.2 Extended Technicolor (ETC)

Technicolor\cite{6} (TC) is a theory that breaks the Electroweak Symmetry and gives masses to the weak bosons without introducing a fundamental scalar field. In this way it avoids the quadratic divergence of the Higgs in the SM.

TC is modelled on the strong interaction. In the absence of TC or a Higgs field, the chiral symmetry of the up and down quarks, \(SU(2)_L \times SU(2)_R\), is spontaneously broken down to \(SU(2)_V\), producing a VEV for scalar quark-antiquark bound states, related to the pion decay constant, \(f_\pi = 95\) MeV, and three massless pseudoscalar bound states, the three pions. The three pions become the longitudinal components of the weak bosons, breaking the electroweak sector in just the same way as for the Higgs. The only problem is that the \(W\) mass, \(M_W = \frac{1}{2} gf_\pi = 30\) MeV, is three orders of magnitude too small.
In TC, one imagines a new strong interaction, and a new set of fermions, technifermions, with a technipion decay constant \( F_\pi = 250 \) GeV, the electroweak symmetry breaking scale. Then the technipion bound state breaks the electroweak symmetry, giving the correct masses to the \( W \) and \( Z \) bosons. Note that unlike the Higgs’ Model, there are no degrees of freedom left over, so there is not even a physical scalar composite particle introduced.

To give masses to fermions, one combines ordinary fermions with technifermions in the same representation of a superset of the TC gauge group. This is Extended Technicolor\(^{[7, 8]}\) (ETC), which breaks down to TC at an energy scale \( \mu \). The ETC gauge boson, \( \epsilon \) connects ordinary fermions with technifermions and results in the effective four-fermion interaction

\[
\frac{1}{2} \left( \frac{g_{\text{ETC}}}{M_\epsilon} \right) (\bar{f}_L \gamma_\mu f_L)(\bar{f}_R \gamma^\mu F_R)
\]

When the technifermions get a VEV, this leads to a mass for the ordinary fermions

\[
m_f = \frac{1}{2\mu^2} < \bar{F} F >
\]

where \( < \bar{F} F > \) is of order 1 TeV. Thus one needs \( \mu \approx 30 \) TeV to get fermion masses at the GeV scale. This contribution to the fermion mass is depicted in Figure 1.

The addition of regular fermions to the technifermion representation is similar to the way leptons are added as a fourth color in leptoquark theories (see Sections 2.1.4).

\( \epsilon \) is an \( \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \) singlet, so one must introduce a set new of technifermions for each ordinary fermion to give it mass; that is eight separate ETC fermion multiplets for one generation of the SM. One does not necessarily need to add new multiplets for each generation, however. The hierarchical mass scheme of the SM
Figure 2: The s-channel diagram by which ETC contributes to $K_{\pi\mu\nu}$.

generation can be explained by a series of ETC symmetry breakings at different scales. One begins with an ETC multiplet containing the technifermions and all generations of a particular type of ordinary fermion. This large group is broken by steps to plain TC. Each step occurs at a different energy scale, and disconnects one generation of the SM fermions. This explains the hierarchy of fermion masses since the masses are a function of the ETC breaking scale. Having all the different generations in one group also allows LFN changing decays as leptons from different generations are now connected by off-diagonal elements of $\epsilon$. In fact, one now has to worry that these sorts of interactions occur at much too high a rate. Figure 2 shows how the ETC boson can contribute to $K_{\pi\mu\nu}$.

The SU(8)$_L \times$ SU(8)$_R$ chiral symmetry is broken as in QCD, producing an array of 63 pseudo-Goldstone bosons. Three of these (corresponding to the pions of QCD) give mass to the $W^\pm$ and $Z$ bosons, while the rest are physically observable states. Six of these are leptoquarks, having both lepton and baryon quantum numbers. They can contribute to $K_{\pi\mu\nu}$ as described below in Section 2.1.4.

ETC can thus explain the existence of masses in the SM without introducing a fundamental scalar. It also reduces the number of arbitrary parameters but only at the cost of introducing many new particles.

ETC predicts LFV at rather high rates. In fact, many of the early models[9, 10, 11, 12] are now ruled out as they predict a rate for $K_L \rightarrow \mu e$ well above the current limit of $3.3 \times 10^{-11}[13]$. The predicted rates can be lowered somewhat by considering a slowly varying coupling constant[14].
2.1.3 Supersymmetry (SUSY)

Supersymmetry[15] (SUSY) takes an alternate method of solving the Gauge-Hierarchy problem. Rather than eliminate the fundamental scalar from the theory, it removes the quadratic divergence. In fact, SUSY is a general theory incorporating fermionic degrees of freedom into the fabric of spacetime. For the experimental particle physicist, this boils down to adding a boson for every fermion in the theory and a fermion for every boson. These fermion-boson pairs are supposed to be degenerate, but SUSY is broken at a scale of about 1 TeV, and only the ordinary half of each pair has been detected.

SUSY is a solution to the Gauge-Hierarchy problem in that it introduces new loops into the mass corrections for the Higgs scalar. For every boson in a loop, we now have a fermion, and vice versa, and these two enter into the calculation with opposite signs. The effects of the two loops well above the SUSY breaking scale cancels and the Higgs mass corrections are limited to the order of the square of the difference in masses between SUSY fermion-boson pairs. Thus, if SUSY is to be a solution to the Gauge Hierarchy Problem, one expects the SUSY breaking scale to be about 1 TeV.

The contribution to LFN changing interactions, at least for $K_{\pi\mu\nu}$, is through the slepton (supersymmetric lepton) sector. Mixing in the lepton sector is not allowed in the SM because of the masslessness of the neutrinos. However, the sneutrinos have large masses and undetermined mixing angles, so one can imagine LFN changing in this sector limited by the mass of the wino (the SUSY partner to the $W$ boson, see Figure 3).

Unfortunately, SUSY contributions to LFN changing kaon decays are constrained
Leptoquarks arise out of theories attempting to explain the difference between quarks and leptons. In the theory of Pati and Salam[17], leptons are considered as a fourth color in an SU(4)$_c$ gauge group. SU(4)$_c$ breaks down to SU(3)$_c$ at some energy scale well above that of electroweak symmetry breaking. In the process, those gluons connecting the fourth color to the others acquire masses, decoupling the leptons from the strongly interacting quarks. Clearly, LFN is not conserved in these interactions, and decays such a $K_{\mu\nu\epsilon}$ occur via diagrams such as the one shown in Figure 4. Pati-Salam leptoquarks have the same coupling constant for all generations. Other models have leptoquarks that only interact with one generation. These models cannot contribute to $K_{\mu\nu\epsilon}$ without mixing in the lepton sector.

2.1.5 Horizontal Gauges

Horizontal Gauge[18, 19, 20] models try to explain the existence of generations in the SM by the introduction of a new gauge symmetry. Corresponding particles in different generations would constitute a representation of this symmetry and the new gauge bosons would mediate transitions between generations.

Horizontal Gauge models allow the natural introduction of a generation number,
$G$, which might be conserved in intergenerational interactions. The generation consisting of the up and down quarks, the electron and its neutrino might be assigned $G = 1$, while the generation consisting of the strange and charm quarks, the muon and its neutrino might be assigned $G = 2$. Antiparticles would have the corresponding negative values.

It is important to note that $K_{\mu e}$ conserves this generation number

$$K^+ \rightarrow \pi^+ \mu^+ e^-$$

$$-1 = 0 -2 +1$$

(6)

Other LFN violating decays do not conserve generation number, for example

$$\mu^- \rightarrow e^+ e^- e^-$$

$$-2 \neq 1 -1 -1 (\Delta G = 1)$$

(7)

or

$$\mu^- \rightarrow e^- \gamma$$

$$-2 \neq -1 0 (\Delta G = 1)$$

(8)

$K_{\mu e}$ with the charges of the leptons reversed also violates generation number,

$$K^+ \rightarrow \pi^+ \mu^- e^+$$

$$-1 \neq 0 +2 -1 (\Delta G = 2)$$

(9)

Thus, one can reasonably look only for the right sign $K_{\mu e}$ decay, and optimize the detector apparatus for the particular particles involved. This is in fact what was done in E865 (see Section 3.5).

Horizontal gauge bosons would also contribute to $K^0 \bar{K}^0$ mixing, and thus the $K_L - K_S$ mass difference. However, $K^0 \rightarrow \bar{K}^0$ has $\Delta G = 2$. If generation number is exactly conserved, these processes could not happen via the exchange of a Horizontal Gauge boson. Even if generation number is partially conserved, one expects a suppression of generation number violating processes.

### 2.1.6 Composite Models

Another way to explain the three generations and their hierarchy of masses is through composite models[21, 22, 23]. In these models, neither quarks or leptons are fundamental particles, rather both are made more fundamental particles often called preons.
The different generations are made from different configurations of the preons within the quark or lepton. Generation number may be a useful quantum number in these models as well, and one would expect a large effect on the $K_L-K_S$ mass difference without it.

### 2.1.7 Other Models

There are several other models which can contribute to $K_{\pi\mu\nu}$. The decay can occur in the SM if one allows neutrino masses and mixings, however, current mass limits and mixing angles limit $K_{\pi\mu\nu}$ to below $10^{-26}$[24]. A fourth generation of fermions would allow the decay at higher levels, since the fourth neutrino must be heavy. However, mixing between the light neutrinos and the heavy neutrino is expected to be small[24].

Left-right symmetric models[24, 25] suppose that a right-handed weak force accompanies the normal left-handed version, but with much heavier gauge bosons. Massive right-handed neutrinos in these models could contribute to $K_{\pi\mu\nu}$. The extended Higgs sector of such models also allows the physical Higgs particles to mediate LFN changing interactions.

All these models (except the extended Higgs sector) contribute to $K_{\pi\mu\nu}$ through box diagrams similar to that shown in Figure 3. The box itself is modified to fit the theory: Fourth generation and massive neutrinos have normal $W$ and quark lines with the heavy neutrino as the fourth side. Left-right symmetric models have a normal quark line with right-handed $W$ and neutrinos.

### 2.2 $K_{\pi\mu\nu}$ Phenomenology

All the theories above contribute to $K_{\pi\mu\nu}$ through one of the three diagrams shown: the $s$-channel, Figure 2 (ETC, Horizontal Gauge, Extended Higgs, Composite); the $t$-channel, Figure 4 (Leptoquarks, ETC); or the Box Diagram, Figure 3 (SUSY, massive neutrinos, Left-right Symmetric).

One can get an idea of the potential for discovering new physics with $K_{\pi\mu\nu}$, by
comparing it (modeled in the s-channel by a Horizontal Gauge boson) with the familiar $K_{\mu3}$ ($K^+ \to \pi^0 \mu^+ \nu_\mu$) decay[26]. Both decays, now with the spectator quark added, are shown in Figure 5. The QCD structure of both decays is the same, so we expect it to cancel in the ratio.

$$\frac{\text{BR}(K_{\pi\mu\epsilon})}{\text{BR}(K_{\mu3})} = \frac{16}{\sin^2 \theta_c} \left( \frac{g_X}{g} \right)^4 \left( \frac{M_W}{M_H} \right)^4$$  \hspace{1cm} (10)

The factor of 16 is due to an assumption of a pure vector interaction for the Horizontal Gauge boson, this would go to unity if the coupling were $V-A$ (vector minus axial vector) as it is for $K_{\mu3}$. One can then relate the mass of the Horizontal Gauge boson to the branching ratio

$$M_H \frac{g}{g_X} \approx 150 \text{ TeV} \left[ \frac{10^{-12}}{\text{BR}(K_{\pi\mu\epsilon})} \right]^{1/4}$$  \hspace{1cm} (11)

With an upper limit on $K_{\pi\mu\epsilon}$ of $2.1 \times 10^{-10}$, this implies a lower bound for $M_H \frac{g}{g_X}$ of 39 TeV. The same bound also applies to the ETC gauge boson causing LFN changing transitions. This derivation shows the connection between rare processes and high energies. One can explore physics at high energies without a huge collider by looking at rare processes in lower energy physics.

One can perform a similar analysis to study the mass reach for leptoquarks. In

Figure 5: Feynmann diagrams for $K_{\pi\mu\epsilon}$ and $K_{\mu3}$. 
this case[27]

\[
\frac{\text{BR}(K_{\pi\mu e})}{\text{BR}(K_{\mu 3})} = \frac{1}{8\sin^2\theta_c} \left( \frac{g_{LQ}}{g} \right)^4 \left( \frac{M_W}{M_{LQ}} \right)^4
\]  

(12)

which leads to

\[
M_{LQ} \frac{g}{g_{LQ}} \approx 500 \text{ TeV} \left[ \frac{10^{-12}}{\text{BR}(K_{\pi\mu e})} \right] \frac{1}{4}
\]

(13)

The limit on \( K_{\pi\mu e} \) implies a lower bound on \( M_{LQ} \frac{g}{g_{LQ}} \) of 131 TeV. This applies only to leptoquarks that couple equally to each generation.

An arbitrary s-channel process (not conserving generation number) will also contribute to the \( K_L - K_S \) mass difference[26]. The contribution can be approximated

\[
\Delta m_K \approx f_K^2 m_K \frac{g_X^2}{M_{H^0}^2}
\]

(14)

where \( f_K = 160 \text{ MeV} \) is the kaon decay constant and \( m_K \) is the mass of the neutral kaon. This is only an order-of-magnitude estimate. Since \( \Delta m_K \) is well explained by SM processes this serves as an upper limit on \( g_X^2 / M_{H^0}^2 \)

\[
\frac{g_X^2}{M_{H^0}^2} \leq 2.8 \times 10^{-7} \text{ TeV}^{-2}
\]

(15)

Plugging into equation 10, we find

\[
\text{BR}(K_{\pi\mu e}) \leq 2 \times 10^{-16}
\]

(16)

Thus, all s-channel processes are more tightly constrained by \( \Delta m_K \) than by LFN changing decays. Box diagram processes are similarly constrained. This constraint can be avoided if generation number is conserved, or partially conserved. Leptoquark processes cannot contribute to \( \Delta m_K \) and are not constrained. They also tend to conserve generation number automatically.

There are two generation number conserving kaon decays in which one can look for LFN violation: \( K_L \to \mu e \) and \( K^+ \to \pi^+ \mu^+ e^- \). The decays are quite similar and all the extensions to the SM given above contribute to both decays via the same diagrams. One might think the the \( K_L \) decay is the better place to look for LFN violation given its longer lifetime and the larger phase space available. However, the two decays are really complimentary, with \( K_L \to \mu e \) being sensitive to axial-vector
and pseudoscalar interactions (because of the pseudoscalar hadronic current), while $K_{\pi\mu\nu}$ is sensitive to vector and scalar interactions. A third LFN violating kaon decay, $K_L \to \pi^0\mu\nu$, is similar to $K_{\pi\mu\nu}$ in this respect, but has not been studied until very recently with the advent of high resolution crystal electromagnetic calorimeters.

It is unlikely that LFN violation will be seen first in the kaon system, unless it is via a generation number conserving process. Otherwise, one expects it to be very small — so as to not change $\Delta m_K$ — and more easily visible in muon decays because of the long lifetime and the lower level of backgrounds.
Chapter 3

Experimental Apparatus

3.1 Overview and Design Philosophy

Any search for rare decay modes places two conditions on the experimental apparatus:

1. The parent particle must be produced in copious amounts.

2. Backgrounds that mimic the decay must be reduced to a low enough level to allow a signal to be seen.

These conditions are often at odds with one another, as backgrounds sometimes scale quadratically or worse with beam rate for constant time resolution.

In searching for $K^+ \rightarrow \pi^+ \mu^+ e^- (K_{\pi\mu\nu})$ one needs a high flux source of kaons. This is provided by the Alternating Gradient Synchrotron (AGS) at Brookhaven National Laboratory (BNL). The AGS provides about $10^{13}$ protons in a one second pulse every three seconds. Kaons are produced by directing the protons at a copper target. The AGS is briefly described in Section 3.2.

The kaons are collected and transported to the detector via the E865 beamline described in Section 3.3. The beamline is designed to accept a narrow momentum band without introducing tertiary particles. Tertiary particles contribute to the overall trigger rate and can combine with other decays to form coincidental backgrounds.

Kaon decays from a designated Decay Volume are reconstructed in the E865 Detector which consists of a spectrometer (Section 3.4) and Particle Identification (PID)
Figure 6: A plan view of the E865 detector apparatus, showing the Decay Volume, spectrometer magnets (D5, D6), proportional chambers (P1-P4), Čerenkov counters (C1, C2), electromagnetic calorimeter, muon range stack and trigger hodoscopes (A–C). The beam traverses the detector from left to right in the figure.

systems (Section 3.5). The spectrometer and the rest of the E865 detector is shown in Figure 6.

Searching for $K_{\pi\mu e}$ at the level of $3 \times 10^{-12}$ requires us to be quite sure of the correct particle type identification. The most likely way to get a real pion, muon and electron from a kaon decay is via the chain, $K^+ \rightarrow \pi^+\pi^+\pi^-, \pi^+ \rightarrow \mu^+\nu_\mu, \pi^- \rightarrow e^-\nu_e$. This has a branching ratio of $6.9 \times 10^{-6}$, but is limited both by the fact that the pions have to decay within the apparatus (indeed, the negative pion needs to decay before the first Čerenkov counter) and the fact the the neutrinos carry off momentum and energy making it hard to reconstruct a vertex or to project the kaon back to the target. These requirements reduce the level of this background by $10^{-4}$ (without a mass cut), showing the great power of kinematic cuts on reducing backgrounds.

Particle misidentification is the real problem to be attacked in searching for $K_{\pi\mu e}$, putting the burden on the PID Systems. The two biggest backgrounds, allowing particle misidentification, are $K^+ \rightarrow \pi^+\pi^+\pi^- (K_{\pi\pi\pi},$ where one $\pi^+$ is identified as a $\mu^+$ and the $\pi^-$ is identified as an $e^-)$, and $K^+ \rightarrow \pi^+\pi^0, \pi^0 \rightarrow e^+e^-\gamma (K_{Dal},$ where
the $e^+$ is identified as a $\pi^+$ and the $\pi^+$ is identified as a $\mu^+$. It is difficult to get good separation between pions and muons since approximately 7% of pions decay to muons within the detector and some pions that don't decay will still make it through the muon stack to mimic muons. This shifts the burden to discriminating between pions and electrons. Electrons appear infrequently in three track kaon decays, the total rate being dominated by $K_{\pi\pi\pi}$ decays. When they do appear, it is often via the Dalitz decay of a $\pi^0$, where they are accompanied by a positron. The E865 detector uses this fact to reduce both the backgrounds above. The electron identification system is matched to the charge of the electron or positron: to reduce the $K_{\pi\pi\pi}$ background, the negative electron is sent to the left side of the detector where an efficient pion veto (but not so efficient electron identifier) is placed; to reduce the $K_{Dal}$ background, the positron is sent to the right side of the detector where there is an efficient positron veto.

Muons are much more penetrating than any of the other particle types, and are identified by looking for a track penetrating many interaction lengths of steel at the back of the detector. Pions and electrons will not penetrate to their predicted range in the steel.

Finally, detectors are needed to provide timing for the tracks and logic signals for the trigger decision. This is accomplished using scintillator hodoscopes along with the detectors in the PID Systems. The hodoscopes are described in Section 3.6.

### 3.2 The AGS

The Alternating Gradient Synchrotron (AGS) is the world's highest flux source of protons in the energy range of 15–30 GeV. The AGS primary beam momentum of 24 GeV is convenient for making secondary kaon beams at low momenta. This is useful for spectrometers, which have better resolution at lower momenta, and Čerenkov detectors which require that the heavier decay products species are not overly relativistic. For these two reasons, E865 is performed at the AGS.

The AGS complex consists of several parts: an $H^-$ source, an electro-static accelerator, two linear accelerators, the Booster, and finally the AGS itself. See Figure 7
Figure 7: The Alternating Gradient Synchrotron at Brookhaven National Laboratory. The proton beam originates at the front end of the Linac in the upper right. It is accelerated in the Linac, the Booster and finally the AGS before being extracted into beam lines in the building on the right (east) side of the main ring.

for the layout.

The beam starts with an $\text{H}^-$ source, not the proton ($\text{H}^+$) source one might expect. This allows the Booster to be filled in more than one turn as described below. The $\text{H}^-$ ions are first accelerated by a static electric field of 35 kV, then by a series of two linear accelerators. The first is a Radio Frequency Quadrupole, which accelerates the ions to 750 keV. The second is a drift tube linear accelerator, which accelerates the ions up to 200 MeV. Between the two linear accelerators, the beam is chopped into buckets to match the size of the RF buckets in the Booster and the AGS.

As the ions are injected into the Booster they are run through a foil to strip off the electrons. This allows injected ion beam and the pre-existing proton beam to be brought together by a single non-pulsed magnet. After the foil, the two beams
are indistinguishable. If one had begun with a proton source, a large kicker magnet would have been required to inject the beam for the linac into the Booster and one would no longer be able to add more particles to the existing beam.

The Booster is a synchrotron, 32 m in radius with 4 RF accelerating stations. It accelerates the protons from 200 MeV to 1.5 GeV, before sending them on to the AGS. The Booster operates at the same RF frequency as the AGS, but is exactly one quarter the size, allowing the Booster to completely fill the AGS in four cycles.

Finally, the AGS accelerates the protons to a final energy of 24 GeV (though it can be as high as 30 GeV). The beam is debunched and extracted into the primary beamlines, where it is available for experimenters. The cycle time of the AGS under normal operating conditions is 3.4 seconds during which protons are extracted for 1.2 seconds.

### 3.3 The Beam Line

A simple secondary beamline might consist of a target, collimators, a dipole magnet to select the desired momentum and a pair of quadrupole magnets to bring the beam to a focus in the apparatus. The problem with this simple scheme is that it assumes the existence of opaque collimators with sharp edges. Unfortunately, nothing is sharp or opaque to a beam of sub-atomic particles. Putting material in the way of particles to collimate the beam “just makes them [the beam particles] mad.” Tertiary particles are produced which serve as junk to a well defined beam. The predecessor to E865, E777 (also at the AGS) had just such a simple beam line, and was eventually limited by beam related background.

A solution to this problem is to have a beamline with a number of intermediate foci with collimators to clean up the tertiary junk. Since particles produced in the initial collimation and momentum selection are not in the beam phase space, it is unlikely they will go through more than one downstream foci of the beam, especially with doglegs between them, eliminating neutral particles which aren’t swept aside by the beamline magnets.

The E865 beamline is shown in plan view in Figure 8. A schematic of the beamline
showing selected rays is shown in both the bend and non-bend planes in Figure 9. It begins with a copper target 15 cm long and about 4 mm square in transverse cross-section. 15 cm is one interaction length in copper, so about two thirds of the beam should interact in the target. The target is situated on the A beamline of the AGS, with A2 being the E865 beamline.

The first two dipoles, D1 and D2, and the associated collimators define the beam momentum and charge. Between the two dipoles is a long section of magnetic shielding (not shown) to reduce the effect of the near-by dipole magnet on the continuing A1 beam line. The two dipoles deflect the beam by a total of 11.2 degrees from its original angle of 3.5 degrees with respect to the original beam direction. After the two dipoles, a focusing-defocussing pair of quadrupoles, Q1 and Q2, brings the beam to its first focus. Because the beam accepts a range of momenta and because of the bend in the horizontal plane caused by the dipole magnets, this first focus occurs along a line, with different momentum rays focusing at different points. This dispersion allows one to select the range of momenta accepted by placing moveable collimators, CLOP and CHIP, at either end of the focus line, which is tilted with respect to beam. There is a narrow vertical collimator between the two momentum selectors (CQ3, inside Q3). At the focus is a field lens, Q3, a quadrupole magnet that helps to control the chromatic aberrations (different focal lengths for different momenta) of the beam. Since the in-focus rays are close to the axis the quadrupole, they are hardly effected; the further the rays are from focus, the more they are affected by the field lens.

From this point on, the beam is well defined in phase space and momentum. All collimation downstream is designed to remove tertiary particles without touching the beam. After the first focus there is another focusing-defocussing pair, Q4/Q5 and Q6, of which the first element is actually two magnets. Before the beam comes to the next focus it is bent by 3.8 degrees in D3, to remove the line-of-sight path of neutral tertiaries.

There is another field lens at the second focus, Q7, which performs chromatic aberration corrections complimentary to Q3. A small vertical slit (CQ7, inside Q7) at the focus, eliminates as much of the tertiary background as possible without touching the beam. Another dipole follows, D4, which bends the beam by the same amount
Figure 8: A plan view of the A2 beam line at the AGS, showing dipole and quadrupole magnets, the momentum selection collimators and the decay tank.
Figure 9: A schematic representation of the A2 beam line in both bend and non-bend planes, showing beamline elements and selected rays. The bend plane is shown with respect to the on-axis, on-momentum ray (i.e. with the bends removed). Solid lines are on-momentum, dashed are 2% high, dotted are 2% low. Beam magnets are shown schematically with analogy to optical ray-tracing: a convex lens represents a quadrupole which is focussing in the given direction, and concave lens one that is defocussing. A prism represents a dipole in the bend plane; a line represents a dipole in the non-bend plane.
as D3, but in the opposite direction. Another focusing-defocussing quadrupole pair, Q8 and Q9, bring the beam to a focus inside the detector apparatus.

After exiting from the last quadrupole (Q9), the beam enters the evacuated Decay Volume, where all accepted decays are required to occur. The decay volume extends for five meters upstream of the first magnet of the spectrometer system (D5) and continues downstream through the magnet.

While this long beamline reduces the background from teritories, the background from $\pi_{\mu 2}$ ($\pi^+ \rightarrow \mu^+ \nu_\mu$) decays is larger. The muons in these decays are produced with little energy in the pion rest frame and are very close to the beam phase space in the laboratory frame. They tend to get swept into a horizontal band by the beam dipoles. While the long beam line increases this background, it is much lower than the rate from teritories would have been. It is also less pernicious since it consists of muons, one at a time, rather than the multiple particles of various types that can be produced when a beam particle hits a collimator.

The beam has a momentum of 6 GeV with a width of 1–3%. It is designed to deliver $7 \times 10^7$ kaons for every $1.2 \times 10^{13}$ protons per pulse on target. In addition there are about 20 times more pions than kaons and 10 times more protons, for a total beam rate of about $2 \times 10^9$ Hz (actually, $2 \times 10^9$ particles per pulse, with the pulse lasting for 1-2 seconds). One seventh of the kaons decay in our decay volume, giving $1 \times 10^7$ kaon decays per pulse.

Knowing the beam intensity one can estimate the running time of the experiment. With a crude $K_{\pi\mu e}$ acceptance of 10% and PID reconstruction efficiency of 25% indicates that one needs about $4 \times 10^4$ pulses to achieve a single event sensitivity of $1 \times 10^{-10}$. Using the rule of thumb, 1000 pulses per hour, one should be able to reach this sensitivity in about two days. In fact, the beam was run at only about a fifth of the design intensity due to rate problems in the detector and the event reconstruction efficiency is only about 30%. This brings the expected time up to about 35 days. Accelerator and experimental downtime take more time, accounting for the two and a half months of data taking in the 1995 run.
3.4 The Spectrometer System

The spectrometer system determines the kinematics of detected events. It consists of four proportional wire chambers (PWCs) and two dipole magnets (see Figure 6). The PWCs are grouped in pairs on either side of the second spectrometer magnet (D6). By measuring the deflection of the track as it goes through the magnet, momentum is determined. With momentum determined, the tracks are projected upstream through the first magnet to see if they are consistent with coming from a common vertex. If so, the mass and momentum of the decaying particle are determined.

3.4.1 Magnets

The first spectrometer magnet, D5 (the fifth dipole magnet in the beam line), serves to direct positively and negatively charged particles to opposite sides of the detector and to sweep decay products away from the beam region. The beam region is not active, so the acceptance is increased by getting decay products away from the beam. This magnet is included in the spectrometer system because it is downstream of most decay vertices and has a large effect on reconstructing the kinematics of the decay.

The second spectrometer magnet, D6, is the central element in the spectrometer. It provides the $P_T$-kick used to measure the momentum of charged tracks.

Both magnets bend particles in the horizontal plane and thus have primarily a vertical component to their field. However, there is a large longitudinal field component, especially in D6, due to the large gap between the pole tips needed to maximize the acceptance. All three components of the field were measured before any of the other elements of the detector were in place (very little of which is ferromagnetic). This measurement was performed with the ZipTrak field mapping system borrowed from Fermilab. Selected portions of this field map are shown in Figures 10 and 11.

The magnetic field was mapped in different sections. The agreement between neighboring points within the same section was very good. Agreements between the different sections was not so good, due to recalibrations of the hall probes and changes in the electronics reading out the probes. The shape of field matches between patches but the overall normalization of the field strength is uncertain to the order of a few
Figure 10: Plots of the Y and Z components of the magnetic field as a function of the Z position. The plot of the Y component is through the center of both magnets. The plots of the Z components are through the center in X of both magnets, but above the center in Y. The higher of the two is outside the aperture for D5 and a null field is shown for these upstream positions.
Figure 11: Plot of the Y component of the magnetic field at the center (in Z) of D6.
Figure 12: Cross-section of a typical proportional chamber, showing wire anode planes (dotted) and HV cathode planes, along with the exterior gas windows, ground planes, G10 frames (diagonal hatching) and aluminum (P1, P2) or stainless steel (P3, P4) support frames (crossed hatching). Spacing between planes has been exaggerated to show detail.

percent (see Section 5.4.2).

D5 is a 48D48 magnet, a window-frame dipole magnet 48 inches wide and 48 inches thick. The gap between the pole tips is 24 inches. D6 is a 120D36, 120 inches wide and 36 inches thick. Its pole tip gap is 50 inches. Although the Y component of the field grows towards the pole tips (see Figure 11), the integral of BY along Z just through D6 is fairly constant except at the corners. The current in D6 is set so that the field just begins to saturate the iron of the pole tips. Thus, D6 is giving about all the field it can offer. The current in D5 is set so that the integral of the Y component of the magnetic field through both magnets was about zero. Thus each magnet imparts the same transverse momentum (P₁-kick) to particles going through them, but in opposite directions. This P₁-kick is 255 MeV/c.

3.4.2 Proportional Wire Chambers

The four, large-area proportional wire chambers, P1–P4, are used to reconstruct tracks of charged particles and to determine their momentum. Each chamber is a package of four wire anode (sense) planes, five mylar cathode (high voltage, HV) planes, two mylar ground planes and two gas window planes. All this is sandwiched between and supported by two aluminum (P1, P2) or stainless steel (P3, P4) frames. The chamber is illustrated schematically in Figure 12.
The sense planes consist of 20 μm tungsten wire, strung with 60 g of tension, and an interwire spacing of 2 mm. The wires are attached to a G10 (the material used for circuit boards) frame, soldered to the readout artwork at one end, and glued at the other. The four planes have wires in various directions to measure the position in two dimensions of the space point where tracks cross the chamber. The four planes are X, U, V and Y. The U and V wires are rotated by $19.5 = \sin^{-1}(1/3)$ degrees with respect to the X wires. The Y wires, which are strung in the horizontal direction, sag as a result of gravity. To reduce this effect, the Y wires are supported at the middle by a thin strip of kapton which extends vertically from the G10 frame. The wires are strung from one end to the other with the chamber laid flat, then glued to the support and cut. They are read out at each end and act as independent sense wires. These four views allow one to reconstruct a space point even if one of the wires is not present. Three of the planes are required to be present as this gives a much less ambiguous space point than requiring just two wires. At the high rates in these chambers, the combinatorics on two wire space points would be much too high to be useful.

The HV planes are sheets of mylar attached to a similar G10 frames. The mylar is coated on both sides with graphite. The graphite, which has a finite resistance of 100 kΩ per square, is conductive enough to establish the constant voltage necessary to cause the electron avalanche at the sense wires, but resistive enough to limit the charge involved in a complete discharge. The operating voltage for P3 and P4 was about 2600 V, while the voltage for P1 and P2, due to a different gas (see below), was about 3200 V. The spacing between HV and sense planes is 0.5 cm.

The beam line, which goes through the middle of each of the chambers, has a particle flux much too high for the chambers to handle. This beam region of each chamber is deadened by lowering the cathode voltage by 500 V on a small region of each HV plane. Lowering the voltage reduces the multiplication factor of the proportional chamber by several orders of magnitude. Defining these lower voltage regions is easy with the graphite cathodes. Before the graphite is applied to the mylar a mask is placed on the mylar defining separate contiguous regions. Then the graphite is applied and the masks are removed. Each region is electrically isolated from the
others and one can put different voltages on the different regions. Because of the kapton supports for the Y sense wires, the dead region is extended from a region in the center of the chamber with two thin strips going vertically to the top and bottom of the chamber. This also allows one to attach leads to set the voltage on the interior regions. A typical HV plane is presented in Figure 13, showing the dead regions and the thin gaps between regions.

The gas used in P3 and P4 is a standard “magic gas” mixture: 75% argon, 25% isobutane with 0.4% freon-13B1 and a trace of methylal. The gas in P1 and P2 is half freon-14 (CF₄) and half isobutane. The mobility of electrons in this gas is much higher than in the magic gas used in P3 and P4[29]. This allows one to use a shorter gate in these chambers where the rates are higher and reduces problems associated with space charge build-up. This fast gas was not used in P3 and P4 for a number of reasons: the cost of using the gas in the much greater gas volume, lower particle fluxes, and higher operating voltages combined with the larger surface areas leading to larger electrostatic forces between planes.
The spatial resolution for quadruplet space points (SPs with four wires) is 350 μm in X and 520 μm in Y. Combined with the $P_T$-kick of 255 MeV in the magnet gives us a momentum resolution of $\sigma_p/P^2(\text{GeV})^2 = 0.003$ for high momentum tracks. At lower momenta, the resolution worsens due to multiple scattering and is about 0.005 at 1 GeV.

### 3.5 Particle Identification Systems

As stated in the introduction to this chapter, particle identification (PID) systems are the key to searching for $K_{\pi\mu\nu}$. There are three PID systems in the E865 apparatus: two Čerenkov counters, an electromagnetic shower calorimeter and a muon range stack. Each will be discussed separately below.

#### 3.5.1 Čerenkov Counters

There are two independent, atmospheric pressure, threshold Čerenkov counters in the E865 detector. They are placed on either side of the main spectrometer magnet (D6) in order to remove correlated background signals from delta rays. As discussed above, decay products are directed to one side of the apparatus or the other based on their charge, and the requirements placed on the Čerenkov counters are different for the two sides. On the left side of the apparatus, where one expects the electrons from $K_{\pi\mu\nu}$, the emphasis is on having a high threshold so that muons and pions will not have any Čerenkov radiation. That the efficiency for electrons be high is of secondary importance. On the right side of the apparatus, where one expects pions and muons from $K_{\pi\mu\nu}$, the emphasis is on high efficiency to veto all events with positrons. A high threshold is now the secondary concern.

To meet these two differing objective for the respective sides of the apparatus, each Čerenkov counter is split into two halves by a septum at $X = 0$. The left side ($X > 0$) of the counter is filled with H$_2$ gas. H$_2$ gas at atmospheric pressure has a Čerenkov threshold $\gamma_T = 60$. This puts the muon threshold at a momentum of 6.34 GeV, the pion at 8.37 GeV, both well above the spectrum of particles expected
from a kaon decays at 6 GeV. The right side of the counter is filled with either CO₂
(\(\gamma_T = 33.6\)) or CH₄ (\(\gamma_T = 34.9\)). Both of these gasses have muon thresholds below
the upper end of the expected spectrum for \(K_{\mu 4}\) decays, 3.55 GeV for CO₂ and 3.69
GeV for CH₄ (see Figure 55). However, only a small fraction of the spectrum is above
threshold, and few photons are radiated by a particle just above the threshold; there
are other reason to reject these high momentum tracks as well (see Section 5.2.4).

The lower thresholds on the right side imply that a positron on the right will
give more light. The expected number of photons for an ultra-relativistic particle
(the electrons and positrons in this case, with \(\gamma \gg \gamma_T\)) is inversely proportional to
\(\gamma_T^2\). Thus, one expects 3–4 times more photons for a positron on the right than one
does for an electron on the left. Integrating over the response of the mirrors and
the photomultiplier tubes one expects for either counter, their lengths being close to
identical, about 4 photons per positron on the right, but only 1.5 per electron on the
left. This gives, using Poisson statistics, a theoretical efficiency per chamber of over
98% on the right and about 78% on the left.

To maximize the collection efficiency, the Čerenkov photons are focussed onto
photomultiplier tubes (PMTs) using a set of mirrors. The most crucial mirror is
the primary mirror at the downstream end of the gas volume in each counter. The
optimal shape for these mirrors is complex, and thus hard to achieve with lightweight
materials. The solution used in E865 is to make “venetian blind” style primary
mirrors with each vertical slat being a cylindrical section. This allows the mirrored
surfaced to be made of aluminized mylar, supported by a Rohacel (a light, stiff foam)
frame. This system provides acceptable collection efficiency while bringing very little
mass into the spectrometer to downgrade the momentum resolution.

The first counter, C1, is divided into four quadrants; each quadrant is subdivided
into two parts, left and right (see Figure 14). Each part is optimized for viewing by
a single PMT, giving a total of eight PMTs for C1. Since C1 sits partially inside the
large spectrometer magnet (D6) a secondary mirror is used to direct the Čerenkov
photons above or below the active volume, to where the magnetic fields are smaller
and the PMTs can operate with acceptable efficiencies. Each PMT is fitted with a
Winston cone to optimize its collection area.
Figure 14: Right side elevation view of C1, showing primary and secondary mirrors, phototubes, Winston cones and beam zeppelin. The left side is similar, but without the zeppelin.
The second counter, C2, is also divided into quadrants, but the larger collection area requires more subdivision than in C1. Each quadrant is divided into two halves vertically. The central half (closer to the median plane in Y) contributes the bulk of the acceptance and is viewed by four PMTs. The external half is viewed by two PMTs. This gives a total of 24 PMTs for C2. There are no secondary mirrors in C2, but the more intricate mirror geometry requires sending the light from the central halves of each quadrant across the median plane to PMTs on the other side (vertically) of the counter. The outer halves are viewed by PMTs on the same side of the counter (see Figure 15).

Each counter has a hole cut out of the mirrors in the beam region to eliminate beam interactions in the mirror. In addition, rectangular gas balloons, filled with H₂ gas ("zeppelins"), are put in the beam regions to reduce the beam-gas interactions in the heavier gas on the right side of the counter. The zeppelin in C2 stops short of the primary mirror to preserve the optical path for Čerenkov photons crossing the median plane after hitting the primary mirror. Blocking this path leads to unacceptable losses in collection efficiency.

### 3.5.2 The Electromagnetic Calorimeter

An electromagnetic calorimeter is used to measure the energy of electrons and positrons and to detect photons. It is also used to start the tracking process for all charged particles and to trigger the data acquisition system. It consists of 600 square shishka-bob (shashlik), lead-scintillator sandwich modules, each 11.4 cm on a side. These are arranged in a 30 × 20 rectangular array covering the whole acceptance apparatus as defined by charged tracks in the PWCs. A 6 × 3 array of modules is left open to allow the beam to pass through without interacting. The calorimeter is situated just downstream of P4, the last PWC. The arrangement of calorimeter modules is shown in Figure 16. The calorimeter is described in more detail in reference [30].

Each module consists of 1.4 mm thick slices of lead and 4 mm thick slices of plastic scintillator (see Figure 17). Sixty slices of each are interspersed in a stack 32.4 cm deep. This constitutes fifteen radiation lengths of material. 1.4 mm holes are drilled
Figure 15: Right side elevation view of C2, showing mirrors, phototubes, Winston cones and beam zeppelin. The left side is similar, but without the zeppelin. The zeppelin stops short so as not to block photons from the interior mirrors, which cross through the beam region.
Figure 16: Arrangement of modules in the electromagnetic calorimeter. The scintillator slats of the A hodoscope are overlaid (thick lines) to show the correspondence of rows in the calorimeter to A hodoscope slats.
through this stack in a $12 \times 12$ array. Optical fibers doped with a wavelength shifter are strung through the holes. Each fiber goes through two of the holes so that all the fibers end on the same side. These ends are bundled and viewed by a single PMT.

The longitudinal collection of scintillation light leads to very good time resolution for these modules, better than 1 ns. The energy resolution of electrons and positrons is $8.2\% / \sqrt{E(\text{GeV})}$.

Electron and positron PID is made by comparing the energy given by the calorimeter with the momentum given by the spectrometer. This ratio ($E/P$) should be close to unity for electrons and positrons (see Figure 18), but much lower for muons and pions. Muons and pions are minimum ionizing particles in this momentum range and should thus leave signal of about 250 MeV in the calorimeter. Pions can shower hadronically and in this case have larger $E/P$ ratios up to unity and beyond. This is not probable, but it does contribute to PID efficiency for pions. Pions that do shower are not likely to be identified as electrons, however, since they give no light in the Čerenkov counters.

### 3.5.3 The Muon Range Stack

The final PID system is the muon range stack. The muon stack consists of thick plates of steel interspersed with proportional chambers (see Figures 19 and 20). The first eight steel plates are 5 cm thick, each with an XY set of proportional chambers on it upstream side, M1–M8. Thus, the first muon chamber is upstream of any steel. After the eighth plate is the B hodoscope (see Section 3.6). There then follow three steel plates each 10 cm thick, again each with XY muon chambers (M9–M11) on the
Figure 18: A plot of the ratio E/P for typical electrons (solid) and pions (dashed).

upstream side. The last set of muon chambers follows with a 5 cm steel plate. The C Hodoscope (see Section 3.6) is placed at the very end. Each plate has a 48 cm square hole cut out of it for the beam. The muon chambers also have a hole for the beam (see below), as do the hodoscopes.

Muons are required to leave a track in the muon chambers to a depth consistent with their initial momentum. All muons are required to hit the B hodoscope or, equivalently, to leave a track through M8. This imposes a lower limit on the accepted muon momentum, of 0.8 GeV (see Figure 55 on page 121). Muons with a momentum above 1.4 GeV will hit the C hodoscope, but this is not required for $K_{\pi\mu\nu}$ events.

The muon planes each have vertical and horizontal views. Each view consists of a number of modules each containing 32 wires. All the modules are identical, 105 cm long in the wire direction and 42 cm wide. There were originally nine modules in the X direction, but the module which would have been hit by the beam has been removed. Muon tracks that would have hit this module are not required to show an X hit. There are ten modules in the Y direction, five each on the two sides of the
Figure 19: An elevation view of the muon range stack, showing the muon chambers, the B and C hodoscopes, and the ranging steel.
Figure 20: A view, looking downstream, of one plane of muon chambers showing the relative positions of X and Y modules (hidden breaks depicted with dashed line). Note that there is an X module missing and that one Y module has been pulled out partially to leave a hole for the beam.
Figure 21: An end-on view of a muon chamber showing the structure of the chamber and the position and connections of the wires.

$X = 0$ median plane. The module that would have been hit by the beam was pulled back, so that a square hole is left in the array of muon chamber modules. This is possible for the $Y$ view since the beam region is at the end of the module, not at the center as for $X$.

Each module consists of two planes of square individual wire proportional chambers (see Figure 21). The spacing between the wires is 1.3 cm. The chambers are staggered between the two planes to eliminate the possibility of a particle going through the chamber without hitting an active region. Each wire goes through one chamber in each plane. The gas used was the same gas as for P1 and P2 (see Section 3.4.2).

### 3.6 Trigger Hodoscopes

Three arrays of plastic scintillator hodoscopes are used to provide timing information and to trigger the data acquisition system (DAQ).

The A Hodoscope is situated just upstream of the Electromagnetic Calorimeter (see Figure 6). It consists of 30 horizontal scintillator slats in two groups of 15, right and left (see Figures 22 and 16). The scintillator slats are designed to coincide with the half row of Calorimeter modules immediately behind them. Thus they are 11.4 cm high, and long enough to cover the acceptance. The scintillators that would overlap the hole in the calorimeter are shortened so there is no overlap. By using the A Hodoscope in conjunction with the Calorimeter very good timing is obtained for the
Figure 22: A view, looking downstream, of the layout of scintillator segments in the A hodoscope.

hodoscope (see Section 4.2.1). The A hodoscope is also used to discriminate between photons and electrons, since the photons will give no signal in the hodoscope. The coincidence between A Hodoscope and the Calorimeter is the basis of our lowest level trigger (T0, see Section 4.2.1). Not all rows of the calorimeter are covered by the A Hodoscope, these extra rows serve as a fiducial volume to contain showers that occur close to the edge of the acceptance.

The B and C Hodoscope are situated in the muon range stack, B after M8, C after all the last steel plate (see Figure 6). Each consists of 48 vertical scintillator slats, in two groups of 24, up and down (see Figure 23). The width of the inner slats is again 11.4 cm, to match columns of the calorimeter. The outer 3 slats at each side, top and bottom (12 slats in all), are each twice as wide and cover 2 columns of the calorimeter. Both hodoscopes are used to trigger on muons. A coincidence between a clump in the calorimeter and an appropriate B Hodoscope slat satisfies the muon requirement of the T1 trigger (see Section 4.2.2).
Figure 23: A view, looking downstream, of the layout of scintillator segments in the B and C hodoscopes
Chapter 4

Data Acquisition and Selection

Data from the physical detector apparatus must be stored in a useable form and transferred to a storage device for further analysis. In the case of E865, as in other current experiments in high energy physics, the data is converted into digital form for use and storage by computer. Gathering the data, converting it into digital form and transferring it to a permanent storage device are the jobs of the Data Acquisition System (DAQ). The speed at which it can accomplish these tasks is crucial to the experiment, governing the number of events seen in a given amount of time and thus the limit that can be set.

The DAQ consists of several components: electronic devices to readout the different pieces of the detector (Section 4.1), a trigger hierarchy that decides when data should be acquired (Section 4.2), the path for getting collected data to a network of computers for analysis (Section 4.3), a very basic analysis program to further reduce the amount of data stored (the ‘Filter Code’) and an overall controlling process to start and stop data collection, assign run and event numbers, etc., called DAISIE.

4.1 Hardware Readout

There are three kinds of data collected in E865. First are the analog signals whose size represents some physical measurement such (e.g. energy). These are converted to digital form by Analog-to-Digital Converters (ADCs). Second are the timing
Table 1: The types of data generated by each detector subsystem, and the type of electronics used to read it out.

<table>
<thead>
<tr>
<th>Detector Element</th>
<th>Active Part</th>
<th>Data Type</th>
<th>Readout System</th>
</tr>
</thead>
<tbody>
<tr>
<td>PWCs</td>
<td>wires</td>
<td>latch</td>
<td>PCOS 4; CAMAC</td>
</tr>
<tr>
<td>Muon Chambers</td>
<td>wires</td>
<td>latch</td>
<td>Amp/Latch; CAMAC</td>
</tr>
<tr>
<td>Čerenkov Counters</td>
<td>PMT</td>
<td>ADC/TDC</td>
<td>FastBus</td>
</tr>
<tr>
<td>Calorimeter</td>
<td>PMT</td>
<td>ADC/TDC</td>
<td>FastBus</td>
</tr>
<tr>
<td>Hoboscopesc</td>
<td>PMT</td>
<td>TDC</td>
<td>FastBus</td>
</tr>
</tbody>
</table>

differences between signals. These are converted to digital form by Time-to-Digital Converters (TDCs). Third are devices whose state is on or off (e.g. a wire of a PWC). These are called latches (or registers) and often exist in large arrays (e.g. the wire map of a given chamber). The implementation of electronics to handle each of these signals is described below. Table 1 gives the type of data generated by each piece of the detector.

4.1.1 PCOS 4 and the PWCs

The readout of the PWCs is done using the PCOS 4 (Proportional Chamber Operating System, Version 4) System developed specifically for E865 by LeCroy Research Systems[31]. The system consists of cards residing on the chamber and control modules residing in a CAMAC crate close to (but outside of) the experimental area connected to the chamber cards by control and data cables.

The chamber cards contain preamps, discriminators, delays, coincidence units and latches (see Figure 24). Each channel, one of 16 per card, is connected to one wire of the PWC. Signals from the PWC wires are first amplified then discriminated. The discriminator level is programmable via the control module.

To allow time for making a trigger decision, the discriminator output is sent through an electronic, pipe-lined delay line. This delay implemented as a number of identical delay circuits on the card. More than one signal can be in the pipeline at any given time, reducing the dead-time associated with the PCOS readout. The delay of each circuit, and thus of the whole chain is controlled by a bias voltage set via the control modules. After the delay, a latch is set if the signal arrives in coincidence
Figure 24: A block diagram showing some of the inputs, outputs and functional elements of the PCOS 4, on-chamber electronics cards. Two channels out of eight making up a logical card (there are 16 channels per physical card) are shown in detail.

with the gate. The gating signal comes from T1 (see Section 4.2.2). The wire can then be considered hit.

Sixteen wires are connected to each card, and sixteen cards are grouped together into a stream. Each stream is controlled by a master card, one of the sixteen cards in the stream. The master card is connected via a ribbon cable to a control module in the CAMAC crate. The two together control the readout of the latches on all the chamber cards, which is done via a shift register and a serial line connection between the master card and the control module. The control module sends out a clock pulse at a rate of 20 MHz. At each clock pulse, the master card tells each latch to transfer its state to the latch next to it (to the right in terms of bits). The rightmost latch (which actually resides in the master card) is transferred to the control module, and the leftmost latch is reset. The entire stream is read out in 256 clock pulses, or about 12.5 μs. The signal to readout the chambers comes from T2 (see Section 4.2.3); alternatively, if there is no T2 (but there had been a T1), all the latches are reset in parallel, taking only about 650 ns (this signal is called T1AR on Figure 24 for T1 Auto Reset).

Each control module controls up to four streams. All the control modules in a CAMAC crate are controlled by a master PCOS driver, which communicates with them via a private (front-panel) bus. The data from the streams are zero suppressed, and broken up into "logical" cards of eight wires. The two-byte data word consists of
the wire map of the eight wires as the lower byte, and the stream and card number in the upper byte. Each set of four streams defines a logical plane. The stream and card number only take up seven of the eight bits of the upper byte; the last bit being used as “chamber change bit”, set for the first data word in any given logical plane.

4.1.2 The Muon Chambers and Amp/Latch Modules

The muon chambers could in principle have been read out using PCOS 4. However, because E865 is using the same muon chamber modules as in E777, money was saved by reusing the existing electronics.

The muon chamber readout is very similar to PCOS 4. It still consists of a preamp, delay, discriminator and gated latch. The delay, however, comes before discrimination. The only electronics on the chambers are preamplifiers, used to drive 400 ns long physical delay cables. E777 used shorter (200 ns) delays, but the E865 trigger decision takes longer, so two of the delay cables are used in series. Each muon chamber module was connected to one of these “pink” delay cable, 19 cables per plane, 228 cables in all. At the other end, each cable connected to one Amp/Latch module. The Amp/Latch module, as its name suggests, amplifies the delayed signal again, then discriminates it, and latches it if there is a gate (from T1) in coincidence. The data is read out serially into a special control module and transferred to event buffers in CAMAC. The format of the data is the same as for PCOS, except that the chamber change bit now comes on the last data word for a given plane.

4.1.3 FastBus ADCs and TDCs

All the PMT (photomultiplier tube) signals are recorded by FastBus ADC and/or TDC modules, depending on the kind of information (amplitude or time) one is interested in (see Table 1).

For Čerenkov and Calorimeter signals one cares about both amplitude and time, so the output is split. Half the signal goes directly to the ADC, via a physical delay line. (There are 600 channels in the calorimeter, each with its own 50 meter long RG58 cable to serve as a delay line. It makes for quite a bundle of cables!) The other
half of the signal is discriminated with one output of the discriminator going to the TDCs and the other being used for the trigger logic. For the hodoscopes, where one is concerned primarily with timing, the signals go directly to a discriminator (without being split) and then on to the trigger and to the TDCs.

The ADC modules are Lecroy Model 1881[31]. The incoming signal, when in coincidence with a gate from T1 (see Section 4.2.2), charges up a capacitor. At the end of the gate the capacitor is discharged at a constant current, so the amplitude of the incoming signal is proportional to the time taken to read out the capacitor. This time is measured by a TDC (within the same 1881 module) and converted into a count, which is the output of the ADC. The dynamic range of the module is $2^{12}$. This conversion takes 15 μs, and is performed upon a signal from T2 (see Section 4.2.3). Only counts above a given number are saved (zero suppression), and eight event-worth of data can be stored in an on-board buffer. If there is no T2 the ADC can be cleared without being read out in 2 μs.

The TDC modules are Lecroy Model 1872[31]. The TDC operates in common stop mode, meaning that the input signals start the clock running and a common signal stops them. The clock will only start however if the TDC is armed, meaning that a signal is sent to it telling it to get ready to count. The arming signal and the common stop signal are basically the two ends of the gate signal for T1 (see Section 4.2.2) that is sent to the ADCs. The TDC achieves very fine time resolution, by adding one extra layer to the method the ADC uses. A capacitor is charged at a constant current starting with the input and ending with the common stop. The capacitor is then read out at a smaller, constant current and converted to a count. This amounts to a linear expansion of the timing interval. The readout is done one channel at a time (upon receipt of a T2 signal, see Section 4.2.3), taking 3 μs per channel, but only those channels that have a start are readout. It turns out that E865 has about 10% occupancy in its TDC channels, so it takes about 18 μs to read out the 64 channel TDC. Like the ADC, eight events can be stored in an on-board buffer. The TDC can be cleared without reading out in 650 ns.
### Table 2: The readout and reset times of the various hardware readout systems.

<table>
<thead>
<tr>
<th>Readout System</th>
<th>Readout Time</th>
<th>Reset Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCOS 4</td>
<td>12.5 $\mu$s</td>
<td>100 ns</td>
</tr>
<tr>
<td>ADC (LRS 1881)</td>
<td>15 $\mu$s</td>
<td>2 $\mu$s</td>
</tr>
<tr>
<td>TDC (LRS 1872)</td>
<td>18 $\mu$s</td>
<td>650 ns</td>
</tr>
</tbody>
</table>

#### 4.2 The Trigger Hierarchy

It is an unfortunate fact of existence that the process of reading out data as described above in Section 4.1 precludes taking any more data for some amount of time. The time during which the detector cannot take any more data is called deadtime. One would like to minimize the fraction of time that is taken by deadtime since the accelerator is running, at considerable expense, whether the detector is ready for data or not.

Some operations with data intrinsically take longer than others. The analog pulse from a PMT or PWC wire implies a short deadtime while voltages are restored. This time is measured in nanoseconds, and dictates the time resolution of the experiment more than contributing to the overall deadtime. Resetting PCOS 4 latches (see Section 4.1.1) or ADC/TDC modules (see Section 4.1.3) takes longer, up to 2 $\mu$s for the ADC. Actually reading out the data takes longer yet, about 15 $\mu$s.

To keep the deadtime low, one would like to readout, or better yet arm, the DAQ hardware only when an interesting event has taken place. However, to know whether something interesting has really happened one must readout the system and analyze the data. To get around this Catch-22, a trigger hierarchy is implemented, using quickly accessible signals at the low levels, and using less readily available information at the higher levels where much or most of the uninteresting events have been already removed. There are four levels in the trigger hierarchy, called T0–T3, described individually below and summarized in Tables 3 and 4.
<table>
<thead>
<tr>
<th>( \mu e )</th>
<th>ELER LM</th>
<th>ELER HM</th>
<th>( \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T0</td>
<td>SB</td>
<td>SB</td>
<td>SB</td>
</tr>
<tr>
<td>T1</td>
<td>CLand \cdot CRor \cdot B</td>
<td>CLand \cdot CRand</td>
<td>PS (10)</td>
</tr>
<tr>
<td>T2</td>
<td>PWCLIK</td>
<td>PWCLIK</td>
<td>PWCLIK</td>
</tr>
<tr>
<td>T3</td>
<td>PWCLIK</td>
<td>PWCLIK</td>
<td>PWCLIK</td>
</tr>
</tbody>
</table>

SB 3 A\cdot SH (1 and 2 per side) passing either Satish box matrix
CLand C1L \cdot C2L
CRand C1R \cdot C2R
CRor C1R \oplus C2R
C1L Signal above threshold in the left side of C1.
C1R Signal above threshold in the right side of C1.
C2L Signal above threshold in the left side of C2.
C2R Signal above threshold in the right side of C2.
B Signal above threshold in the B hodoscope.
PS Prescale factor.
LM 2 A\cdot SH \cdot CK, one either side, passing Low Mass matrix.
PWCLIK Require good signal from hardware PWCLIK box.

Table 3: Summary of the trigger requirements for various streams used in this analysis.

<table>
<thead>
<tr>
<th>Rate</th>
<th>Deadtime</th>
<th>Total Deadtime</th>
</tr>
</thead>
<tbody>
<tr>
<td>T0</td>
<td>2M</td>
<td>N/A</td>
</tr>
<tr>
<td>T1</td>
<td>2K</td>
<td>650 ns</td>
</tr>
<tr>
<td>T2</td>
<td>1200</td>
<td>18 ( \mu s )</td>
</tr>
<tr>
<td>T3</td>
<td>900</td>
<td>100 ( \mu s )</td>
</tr>
</tbody>
</table>

Table 4: Approximate trigger rates and deadtimes incurred during normal running.
4.2.1 T0

The first layer of the trigger decides when something interesting has happened in the detector. At the most basic level this means that charged particles have gone through the apparatus leaving detectable ionization trails. However, most of the particles going through the apparatus are not interesting, so the trigger needs to be more specific. E865 is looking for a decay of a kaon into three charged particles. Thus, the trigger should require three charged particles in coincidence. Also, conservation of momentum indicates that there should be at least one particle on each side of the detector (left/right) and a pattern to the vertical arrangement of the three tracks.

The most prompt signals available to make this decision are the outputs of the discriminators for PMT signals. The calorimeter modules and A hodoscope slats together form T0. These two are used together because the calorimeter is sensitive to photons as well as charged particles, while the A hodoscope, with its long scintillator slats, does not by itself have very good time resolution. Using the coincidence of an A hodoscope slat with one of the calorimeter modules in the row behind it, one can correct for the time-of-flight in the scintillator and get very good time resolution. This adjustment is performed by a home-built module called “Larry’s box”, after its designer Larry Leipuner. The signal from the A hodoscope is delayed by a fixed amount (about 1 ns) sequentially for each calorimeter module signal in that row. Coincidences between the signals are checked after the appropriate delay in the chain for each calorimeter module (see Figure 25, page 52). Any coincidence is thus correctly adjusted in time. This coincidence is called A·SH.

T0 requires three A·SHs, with at least one on each side of the detector (recall that the A hodoscope is divided into two halves, left and right, see Figure 22). As mentioned above, conservation of momentum places a constraint on the set of rows that can be hit. For example, one can’t have three rows all in the upper half. This constraint is implemented as a lookup table. The A·SH on one side selects a number of combinations for the two rows on the other side. See Figure 26 for two examples. Only those allowed combinations can act as a trigger. If there is more than one hit on the selecting side, the union of their regions is taken.

The allowed patterns were determined by Monte Carlo calculation, and reflect only
the conservation of momentum in regards to kaon decays at 6 GeV. The patterns are implemented as a home-built lookup table module called a “Satish Box”, after its designer Satish Dhawan. The sets of A·SHs on one side act as an address of a table whose value at that address is the allowed pattern of hits for the other side. There are two parallel boxes for T0 so a hit on either side can open the window for a set of two hits on the other. If the inputs from the selected side match the pattern, the Satish Box outputs a NIM signal. The OR of the signals from the two Satish boxes is T0.

4.2.2 T1

The next layer of the trigger, T1, adds information from the Čerenkov counters and the muon hodoscopes (B and C). It is in fact a collection of three different triggers which relate to the three different event streams to be used in this analysis.

That portion of T0 consisting of actual kaon decays is dominated by the decay\[ K^+ \rightarrow \pi^+\pi^+\pi^- (K_{3\pi}) \]. A fraction of these are kept as a normalization sample. A prescale unit was used to give a T1 for every 20,000 (or later, 30,000) T0 coming in, independent of any other activity in the detector. This is the TAU stream. It serves as the minimum bias sample, and the source of the \( K_{3\pi} \) normalization.

The next most common kaon decay included in T0 is \[ K^+ \rightarrow \pi^+\pi^0, \pi^0 \rightarrow e^+e^-\gamma \n\]
Figure 26: A plot of the Satish Box selection matrix. A hit on one side opens up a window for a certain combination of hits on the other side. There is one plot for each A counter on the single side. There are 15 A counters on each side, so the limits on the plots are 1–15 in both directions. Only ordered pairs of hits on the double side are considered, so only the upper diagonal half of the plot is filled. The allowed regions for two singles equally distant from the center (e.g., 1 and 15, 2 and 14, etc.) are roughly mirror images about the $x + y = 15$ line, and single number 8 is symmetric about this line.
(K_Dal). K_Dal is distinguished by having both an electron and a positron on the two sides of the detector, so T1 requires at least one PMT in each half (left/right) of both Čerenkov detectors to have a signal above threshold (CLand · CRand). This limits our acceptance for the decay slightly, since neither the electron nor the positron are allowed to switch sides. This is the ELER (electron on the left, electron on the right) stream.

Finally, $K^+ \rightarrow \pi^+ \mu^+ e^-$ has an electron on the left, no positron on the right and a muon. Thus we require light on the left in both Čerenkov detectors (CLand), no light on the right in either Čerenkov detector (CRor) and a B hodoscope signal aligned with one of the clumps in the calorimeter. This is the MuE stream.

The correlations between calorimeter modules and B hodoscope slats is made using columns of the calorimeter and a Satish Box (see Section 4.2.1) lookup table. Larry's Box, which gives the time corrected A hodoscope signal for T0, has an output for every calorimeter module that is active and in coincidence with the A signal. These signals, from only those calorimeter modules involved in an A·SH, are rearranged into columns by “Swiss Mix” modules (built by our Swiss collaborators). The outputs of the four Swiss Mix modules (left/right, up/down) are completely analogous to the output of the Larry's Boxes with rows of calorimeter modules replaced by columns. There is one output from the Swiss Mix, for any column that has a module contributing to an A·SH. The output of the Swiss Mix then acts as the selector in a Satish Box, the other inputs being the discriminated B hodoscope signals. Any B hodoscope signal passing through this Satish Box contributes to the MuE stream T1 trigger. (The B hodoscope signals that are passed then serve as selectors for C hodoscope hits as well. This is only used in the MuMu trigger, which is not used in this thesis, and was not implemented in the 1995 run.)

4.2.3 T2

T2 was trivial for the TAU and MuE streams, and served primarily to separate the ELER stream into low mass (LM) and high mass (HM) components. Low and high mass refer to the invariant mass of the electron-positron pair. In the $\pi^0 \rightarrow e^+ e^- \gamma$
decay, the electron-positron pair come from a virtual photon, and are expected to have a low invariant mass, peaked close to zero and bounded above by the $\pi^0$ mass. In the decay $K^+ \rightarrow \pi^+ e^+ e^-$ ($K_{ee}$), the electron-positron pair are expected to have a high invariant mass, with much of the spectrum above the $\pi^0$ mass. This trigger, therefore serves as a way of separating the common $K_{Dal}$ decay from the more interesting and rarer $K_{ee}$ decay. The ELER HM stream is not used in this analysis, but the ELER LM stream is.

The LM/HM distinction is made by looking at the vertical separation of the electron-positron tracks. The PWCs have not been read out yet, however, so a more indirect way of determining the vertical separation of the tracks is used. It relies on the correlation between phototubes in the Čerenkov counters and A·SH elements for electrons. A lookup table, stored in hardware (LRS 2365[31]), relates which Čerenkov PMTs go with which A·SH. The coincidence of the two is called A·SH·CK. The vertical separation of the electron-positron tracks is thus translated into the separation between the two A hodoscope slats, by tagging those slats that are correlated with light in the Čerenkov counters.

Those electron-positron pairs hitting A slats at the same height (on the different sides) or one apart are considered low mass events. The rest are high mass. Another hardware lookup table (LRS 2372[31]) contains the matrix defining the combinations for low mass events.

The ELER LM stream is then prescaled by a factor of 10 before being passed to T2.

4.2.4 T3

T2 initiates the readout of the PWCs. Thus, after T2 (12.5 $\mu$s after), PWC information is available to the system. This information is used to eliminate events unlikely to give us useful information due to large track multiplicities. This is done via the PWC Likelihood Function. If the likelihood is below a certain value, the event is eliminated.

The PWC Likelihood is calculated by summing the logarithms of the probabilities
of the hit multiplicities in each logical PWC plane. Each physical PWC plane (16 total; 4 × 4) is divided into two logical planes (left and right halves for X and Y view, but not halves for the U and V views). The multiplicity distributions for good events are determined by taking data without the PWC Likelihood requirement in place, selecting a sample of good $K_{\pi\pi}$ and $K_{Dal}$ events, and finding their multiplicity distributions. These multiplicity distributions then determine the probability of finding a given number of hits in a given logical chamber. To reduce the number of channels considered, the number of data words (see section 4.1.1) is used rather than the number of wires. The logarithm of this probability is summed over all the planes. This is the PWC Likelihood, which has a certain distribution for real events, like $K_{\pi\pi}$ and $K_{Dal}$. A cut point is set at the tenth percentile of this distribution.

The point of the PWC Likelihood cut is to eliminate events with many tracks, "junky" events. Since the branching ratio for $K_{\pi\mu}$ is so low, one expects that most of the triggers for it will be from events with many junk tracks. The cut also eliminates events where many of the planes do not have enough hits to have been the result of a kaon decay.

The PWC Likelihood function is calculated in hardware for each event. Each plane is looked at in turn. The number of data entries (one for each active logical card) for the plane is counted, and this number (along with the number of the logical plane) serves as an address to a location in memory holding the value of the log of the probability of having that multiplicity in the plane. This value is transferred to another register where it is added to a running sum. After going through all the logical planes, the running sum is compared to the cut value, and if the sum exceeds the cut, the T3 signal is generated.

4.3 The Data Path

The T3 Trigger (see Section 4.2.4) is the signal to the DAQ system that there is interesting data in the detector readout hardware. This data must now be gathered together in one place, put into a data structure and sent off for further analysis and storage. The collection of data and redistribution of it as an event is handled by a
specialized computer called the CERN Host Interface (CHI).

The first job of the CHI is to gather all the data describing the event into its own memory buffer. This is accomplished in two steps. First, the latched data residing in CAMAC crates, such as the wire data and trigger latches, are transferred to FastBus memory modules, Lecroy Module 1892[31]. This transfer is actually performed by the Smart Crate Controllers (SCC) on the CAMAC crates, but is initiated by the CHI upon receipt of T3. This transfer, over flat ribbon cables, takes 600 ns/word. When the transfer is complete the SCC notifies the CHI.

After the transfer to FastBus is complete, the CHI can signal the apparatus to begin taking data again, because all the FastBus modules have on-board buffering of data. The buffers store up to eight events, only seven which are ever used by E865. A dedicated piece of hardware, the “Up-Down Counter”, keeps track of the number of events stored in the FastBus buffers, adding one for every new event, subtracting one every time the CHI finishes copying an event. When the Up-Down Counter reads seven events, no data can be taken.

When the CHI resets the apparatus, the apparatus has been dead from the time the T1 signal had first been formed (see Section 4.2.2 and Table 4). Not having to wait for the CHI to copy the event to its own data buffer cuts down significantly on the dead time.

The CHI transfers all the FastBus data into its own memory. This transfer through the FastBus backplane, takes about 1 μs. The CHI has 4 MB of memory on-board, enough to contain more than an entire spill’s worth of data, up to about 2500 events (the typical event is about 1.5 kB). All the data is put into the event data structure, where it is assigned a unique identifier consisting of a run and event number. The CHI continues to fill up its memory buffer with events until it receives an end-of-spill signal (EOS).

On receiving the EOS interrupt, the CHI changes from data gathering mode to data distribution mode. The data will be analyzed before being written to tape, since writing data to tape takes much longer than moving it from place to place in memory buffers. There are two DEC Alpha workstations doing this analysis, both connected, via a Turbo Channel interface, to a VME Crate. (VME stands for Versa Module
Eurocard, though perhaps a better mnemonic is Virtual Memory Extension.) The VME crate acts as shared memory for the CHI and the two Alphas. The CHI copies an event into the shared memory where either of the Alphas can access it. When the workstations are working on the data, they place a software (semaphor) lock, on the data, to inform the CHI not to touch that area of the shared memory. When the workstation is done analyzing the event, the semaphor lock is removed, and the CHI can copy more data into the shared memory.

The two workstations do analysis on events before they are written to tape. This analysis was intended to be a simplified version of the full analysis done off line, but time limitations didn’t allow any track or vertex reconstruction. The code that is in use, called the “Filter Code”, makes sure the event has enough information to be an interesting event. That is to say that there is enough energy recorded by the calorimeter, in enough clumps; that there are enough PWC card entries to make enough space points; and that light was seen in the Čerenkov counters (if there was any expected).

The final step in the data path is from the Alpha workstations to magnetic tape. This is handled by the overall operating system, DAISIE. DAISIE runs on yet another computer (in this case an old VAX 3100) with a tape drive attached. The event is copied from the Alphas to the VAX 3100 via Ethernet at up 500 kB/sec. This is, in fact, the bottleneck of the whole DAQ system, limiting it to about 800 events per spill. The tape drive is attached to DAISIE via the SCSI bus. DAISIE sends a few events per spill to another VAX for run monitoring. The complete data path is summarized in Figure 27.
Figure 27: A block diagram of the E865 DAQ system.
Chapter 5

Data Analysis and Reduction

There are two main tasks involved in analyzing an event: Determining the kinematics of the event (Section 5.1) and determining the types of particles involved (Section 5.2). These tasks require knowledge of the positions and responses of all the detector elements. Determining the parameters corresponding to alignments and calibrations of the detector elements involves much of the total data analysis effort (Section 5.3). As with the trigger, data analysis can be performed with more or less care, more care taking more time. Thus data reduction proceeds in several passes, more care being taken each time (Section 5.4).

5.1 Event Building

Determining the kinematics of an event means finding the tracks left by charged particles, determining the momentum of these tracks and projecting the tracks back through the separator magnet (D5) to a vertex (if it exists). Only those events with a well defined vertex will be further considered. Once the vertex is found, the kaon momentum can be reconstructed and projected back up the beamline to the target.

Finding the tracks involved in the decay is not always an easy task due to the long gate needed to operate the PWCs, the combinatorics of having 16 wire planes, and background tracks. The task is made simpler by restricting attention to the regions of the chambers where one expects hits and finding space points in each
chamber before linking together the points from separate chambers into tracks. The calorimeter provides a well timed, localized place to begin the search for tracks and it is used to limit the search area of each chamber. Thus, active calorimeter modules are the starting point in determining event kinematics.

5.1.1 Clumps

A clump is a set of contiguous, active calorimeter modules. In general, this can lead to complicated topologies for the active modules making up the clump. In E865, however, a clump is a square of nine modules, with the center module having the most energy. This simplification is due to the size of the calorimeter modules, which are roughly the size of expected electromagnetic showers of few GeV particles. (This fact is used explicitly when looking for photons: 90% of the energy in a photon clump is required to be contained in a $2 \times 2$ box.)

The search for clumps begins with in-time A hodoscope slats. For each in-time slat, the corresponding row of calorimeter modules is scanned for in-time modules with energy above the seed threshold (75 MeV). If there is more than one in-time module in the row, the largest reading is considered first. Knowing the position of this module the timing of the A hodoscope module is corrected and rechecked for being in-time. (This is the job Larry's Box, section 4.2.1, does in the fast trigger logic.) If the calorimeter and A hodoscope modules are in time, a clump has been found and the energy in the 8 surrounding modules is added to the seed energy. The energy weighted time and position are found, and shape parameters are calculated (the fraction of energy in the center module, $2 \times 2$ square and $4 \times 4$ square; these are only used for photons). The modules constituting the clump ($3 \times 3$ square) are marked so they may not act as seeds for new clumps.

After all tracks have been found and fit (see Sections 5.1.2 through Sections 5.1.5), the calorimeter is searched for clumps not associated with a track and with no active A counter covering them. These are photon candidates. The timing and position of the clump are determined by energy weighted averages as for charged clumps. The time of the photon candidate is required to be within 2 ns of the T0 time determined
from the three charged clumps making up the event and their A counters. To reduce backgrounds from neutrons 90% of the energy in the shower is required to lie in a $2 \times 2$ square of modules, and the total energy is required to be above 400 MeV. Photons are used to veto $K_{\pi\mu\epsilon}$ candidates and to allow the full reconstruction of $K_{Dal}$ events.

### 5.1.2 Space Points

Tracks from kaon decays in the beam are kinematically restricted to range of momenta. This range varies from decay to decay, but not so much as to erase the correlation between the position of a clump in the calorimeter and the position of hits in each of the four proportional chambers. This is obvious for P4, being just upstream of the calorimeter; it is not obvious for P1 which has the spectrometer magnet intervening. Nevertheless, there is a correlation, which is used to define the region of each chamber where tracks associated with a given clump are allowed to pass. Only wires passing through such an allowed region are considered when looking for tracks.

Each chamber has four planes of sense wires, oriented vertically (X view), horizontally (Y view) and two at $\pm 19.5^\circ$ (U and V views). Neglecting inefficiencies, each track should register in all four planes. The first goal of the space point finding algorithm, is to look for sets of four wires (or clusters, two or three contiguous wires) consistent with being hits on the same track.

U and V wires together give a measurement of both an X coordinate and a Y coordinate.

\[
\begin{pmatrix}
V \\
U
\end{pmatrix} = 
\begin{pmatrix}
\cos \theta & \sin \theta \\
\cos \theta & -\sin \theta
\end{pmatrix} 
\begin{pmatrix}
X \\
Y
\end{pmatrix}
\]

\[X_{UV} = \frac{V + U}{2\cos \theta}\]  \hspace{1cm} (18)

\[Y_{UV} = \frac{V - U}{2\sin \theta}\]  \hspace{1cm} (19)

Thus, for every UV pair one can calculate the residuals $\Delta X$ and $\Delta Y$,

\[\Delta X = X - X_{UV}\]  \hspace{1cm} (20)

\[\Delta Y = Y - Y_{UV}\]  \hspace{1cm} (21)
for all the X and Y candidate wires, and look for those within 15 wires of the UV crossing. If two such wires are found, the set of four wires is considered to be a space point candidate.

To determine whether the four wires are actually consistent with having come from one track, and to determine the best X and Y coordinates for the quadruplet, a $\chi^2$ analysis is performed. If $(x, y)$ is the position of a space point and $(X, V, U, Y)$ are the positions (along the wire axis) of the four wires constituting it, then

$$
\chi^2 = \frac{(X - x)^2}{\sigma^2} + \frac{(Y - y)^2}{\sigma^2} + \frac{(V - x \cos \theta - y \sin \theta)^2}{\sigma^2} + \frac{(U - x \cos \theta + y \sin \theta)^2}{\sigma^2}
$$

(22)

where $\sigma = 0.58 \text{ mm} = 2/\sqrt{12} \text{ mm}$, is the RMS resolution of one wire. Minimizing the $\chi^2$ gives

$$
x = \frac{X + (V + U) \cos \theta}{1 + 2 \cos^2 \theta}
\quad (23)
$$

$$
y = \frac{Y + (V - U) \cos \theta}{1 + 2 \sin^2 \theta}
\quad (24)
$$

The $\chi^2$ can then be rewritten in terms of the two residuals $\Delta X$ and $\Delta Y$, which have already been calculated:

$$
\chi^2 = \frac{2 \cos^2 \theta}{1 + 2 \cos^2 \theta} \frac{\Delta X^2}{\sigma^2} + \frac{2 \sin^2 \theta}{1 + 2 \sin^2 \theta} \frac{\Delta Y^2}{\sigma^2}
\quad (25)
$$

$\chi^2$ less than 30 are accepted in the first pass.

This analysis doesn’t take into account the angle of the track, which is equivalent to assuming that all the measuring planes were at the same Z position. The fact that they aren’t introduces errors into the $\chi^2$ so that one cannot take it literally, yet. This is the reason for the loose $\chi^2$ cut (which is still much tighter than the 15 wire window implied by the $\Delta X$ and $\Delta Y$ cuts). To correct for this effect, one needs to know the angle that the track makes with respect to the planes and the spacing between them. This information isn’t known until after the tracks are formed which requires knowledge of the space points. The loose, uncorrected cut on the $\chi^2$ is used to select the space points before finding tracks. After having found tracks, the space point $\chi^2$
Figure 28: Two plots showing the $\chi^2$ distribution for quadruplet and triplet space points. Both distributions are qualitatively what one would expect for $\chi^2$ distributions for the given degrees of freedom (two and one, respectively), with quantitative differences coming from backgrounds and the non-gaussian nature of the uncertainties.

is recalculated using the track angle to project all wires to the same $Z$ plane. At this point a $\chi^2$ cut of 7 per degree of freedom (2 degrees of freedom in a quadruplet space point) is applied. Tracks which loose space points due to this cut may be discarded if they no longer have three good space points. An example of the corrected $\chi^2$ distribution is given in Figure 28. The track angle correction is only performed in the last data reduction pass (see Section 5.4.3).

Having found all quadruplet space points, one looks for sets of three wires forming triplet space points. The analysis is similar to the one above, but the exact form of the $\chi^2$ and best fit coordinates are different. Wires may belong to more than one space point, but different space points can only share one wire.

Doublet space points are not considered before the track finding phase of the analysis. However, track momentum resolution is significantly reduced in three space point tracks. Much of this degradation can be recovered if one looks for doublet space points in the missing chamber. This search begins after the momentum fitting phase of the analysis (see Section 5.1.4), which gives a prediction for the position
of the track in the missing chamber. A very small window is considered (to reduce complications from combinatorics; the window is 1 cm on each edge) for two wire combinations. These wires make a space point and the wires are added to the track's list of wires. If more than one wire in a given view is found, the one closest to the predicted position is used. With extra wires in place the track is refit. This doublet space point search is also done only at the last reduction pass.

Because the angle between the X and UV wires is $19.5^\circ$, the X resolution of the chambers is better than the Y resolution. From Equation 25 one sees that the resolutions are

$$
\sigma_x^2 = \frac{1}{1 + 2 \cos^2 \theta} \sigma^2
$$

$$
\sigma_x = 0.60 \cdot 0.58 \text{ mm}
$$

$$
= 0.35 \text{ mm}
$$

(26)

$$
\sigma_y^2 = \frac{1}{1 + 2 \sin^2 \theta} \sigma^2
$$

$$
\sigma_y = 0.90 \cdot 0.58 \text{ mm}
$$

$$
= 0.52 \text{ mm}
$$

(27)

One can also find the resolution of the residuals $\Delta X$ and $\Delta Y$:

$$
\sigma_{\Delta X}^2 = \frac{1 + 2 \cos^2 \theta}{2 \cos^2 \theta} \sigma^2
$$

$$
\sigma_{\Delta X} = 1.25 \cdot 0.58 \text{ mm}
$$

$$
= 0.72 \text{ mm}
$$

(28)

$$
\sigma_{\Delta Y}^2 = \frac{1 + 2 \sin^2 \theta}{2 \sin^2 \theta} \sigma^2
$$

$$
\sigma_{\Delta Y} = 2.35 \cdot 0.58 \text{ mm}
$$

$$
= 1.36 \text{ mm}
$$

(29)

This is pictured in Figure 29 (where the units are in terms of the wire spacing, 2 mm, so the RMS is half the resolution given above).

### 5.1.3 Finding Tracks

It is the task of the track finding algorithm to group space points from different chambers into a track. This is done by testing the hypothesis that the four points
lie along a straight path with one kink in the midplane of the spectrometer magnet (D6). Tracks are also found using three space points by testing the hypothesis that the middle point lies on the line connecting the other two space points in the YZ projection. The four point track hypothesis is parametrized by the following $\chi^2$:

$$\chi^2 = \frac{(x^\text{us} - x^\text{ds})^2}{\sigma_x^2} + \frac{(y^\text{us} - y^\text{ds})^2}{\sigma_y^2} + \frac{(Q_y^\text{us} - Q_y^\text{ds})^2}{\sigma_{Q_y}^2} + \frac{2(y^\text{us} - y^\text{ds})(Q_y^\text{us} - Q_y^\text{ds})}{\sigma_y Q_y}$$

(30)

where $x$ and $y$ are the position of the track at D6 from the upstream (us; P1 and P2) or downstream (ds; P3 and P4) projection and $Q_y$ is the slope of the track in that projection.

The search begins with a clump from the calorimeter. Only those space points lying within the windows projected upstream from the clump are considered. First, combinations of four space points are considered. If the $\chi^2$ defined above is below the cut value (30), the track is added to the list, and its constituent space points are removed from consideration. There is also a cut on the difference between the projected track position at the calorimeter and the centroid of the clump. When all four point tracks for a given clump have been found, three point tracks are considered. All the tracks for a given clump are sorted according to their $\chi^2$. In the first two data
reduction passes, only one track (the one with the best $\chi^2$) is considered per clump. In the final pass, three tracks are allowed for each clump. This allows one to choose the best track based on how well it combines with other tracks to form a vertex (see Section 5.1.5). Allowing multiple possibilities for the track attached to a particular clump improves the mass resolution, but takes extra time as each of the tracks has to be fit to find the momentum. Thus, it is only done at the last stage of the data reduction. A given space point may belong to only one track in a given clump, but may belong to different tracks in different clumps at this stage in the analysis. The momentum fitting procedure, which uses the individual wires rather than space points (and the wires may not then be shared), will resolve any ambiguities between tracks.

5.1.4 Momentum Fitting

Five parameters are needed to fully describe a track in the E865 Detector: the X and Y positions of the track at P1; the tangents (slopes) of the track, projected onto the XZ and YZ planes, also at P1; and the rigidity of the track in the spectrometer (defined as the ratio of the charge to the momentum of the track, which is proportional to the bend angle). Determining these five parameters is the heart of the whole analysis.

The momentum is fit by minimizing a $\chi^2$ parametrized by the five parameters above. This $\chi^2$ measures the difference between the measured wire positions in the chambers and the expected wire positions of the track given by the parameters. The expected position at each wire plane is found by propagating the track from the center of P1 using either a hard-edge approximation for the magnetic field (straight trajectories between the magnets, circular trajectories inside the magnets) or the measured field map.

Let $x_i$ represent the measured wire positions, $\sigma_i$ the uncertainty in that measurement, $\alpha_\mu$ the five track parameters and $f_i(\alpha_\mu)$ the expected wire positions. Then the $\chi^2$ is written

$$\chi^2 = \sum_i \frac{(x_i - f_i(\alpha_\mu))^2}{\sigma_i^2}$$

(31)

The fitted parameters, $\alpha_\mu$, are obtained by $\chi^2$ minimization. Since the $f_i(\alpha_\mu)$ aren't known analytically, a first-order Taylor expansion of $\alpha$ about a first guess $\alpha_0$ (taken
from the track finding procedure) is made to linearize the system of equations

$$\hat{\alpha} = \alpha_0 - \left( \frac{\partial^2 \chi^2}{\partial \alpha^2} \right)^{-1}_{\alpha_0} \cdot \frac{\partial \chi^2}{\partial \alpha} \bigg|_{\alpha_0}$$  \hspace{1cm} (32)

This allows a semi-analytical solution. Defining

$$r_i = x_i - f_i$$ \hspace{1cm} (33)

$$f_{i,\mu} = \frac{\partial f_i}{\partial \alpha_\mu}$$ \hspace{1cm} (34)

$$y_\mu = -\frac{1}{2} \frac{\partial \chi^2}{\partial \alpha_\mu}$$

$$= \sum_i r_i f_{i,\mu} / \sigma^2$$ \hspace{1cm} (35)

$$C^{-1}_{\mu\nu} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial \alpha_\mu \partial \alpha_\nu}$$

$$= \sum_i f_{i,\mu} f_{i,\nu} / \sigma^2$$ \hspace{1cm} (36)

$$\delta \alpha_\mu = \left( \frac{\partial^2 \chi^2}{\partial \alpha^2} \right)^{-1}_{\alpha_0} \cdot \frac{\partial \chi^2}{\partial \alpha} \bigg|_{\alpha_0}$$

$$= C_{\mu\nu} y_\nu$$ \hspace{1cm} (37)

Equation 32 becomes (dropping the 0 subscript)

$$\hat{\alpha}_\mu = \alpha_\mu + \delta \alpha_\mu$$ \hspace{1cm} (38)

and the new residuals for equation 31 are

$$\hat{r}_i = r_i + f_{i,\mu} \delta \alpha_\mu$$ \hspace{1cm} (39)

The only things to be found numerically are $f_i$ and $f_{i,\mu}$. This requires making six trial tracks, one for the initial guess ($f_i$), and five for the derivatives with respect to each of the parameters ($f_{i,\mu}$). The $f_{i,\mu}$ are found by taking the difference between the position of the initial guess at a given chamber and the position of the trial with the indicated parameter changed by a small amount, then dividing by the magnitude of the change in the parameter. Finding these six trial tracks for each prospective track
(and there can be several prospective tracks for each clump) is the single most CPU intensive task in the whole analysis, and the most important. With the trial tracks in hand, one calculates \( \hat{\alpha}_i \) to find the best track parameters, and uses \( \hat{r}_i \) to calculate the new \( \chi^2 \) of this best fit track.

The new \( \chi^2 \) has to pass a quality cut (\( \chi^2 / \text{dof} < 23 \), in the final pass) for further consideration of the track. This very large cut value is due shortcomings in the knowledge of the magnetic field and individual wire positions. The mean of the \( \chi^2 \) distribution is about 2.5 with a much larger tail than one would expect (see Figure 30). This shows up in the space point \( \chi^2 \) as well, where the number of space points decreases more slowly with \( \chi^2 \) than one would expect. This uncertainty does not, however, show up as a gross error or broadening of the reconstructed kaon mass or momentum distributions.

In the above analysis, the measurement errors at each plane are assumed to be uncorrelated. This is not the case because scattering at a given point introduces correlations in all measurements downstream of that point. The formalism of the procedure changes only in the introduction of the correlation matrix \( V_{ij} \) replacing the

---

**Figure 30:** Plots of the space point and momentum fitting \( \chi^2 \)'s as seen in \( K_{\pi\pi} \) data (histogram) compared with the expectation determined by MC (error bars).
Thus, Equation 31 becomes
\[
\chi^2 = \sum_{ij} r_i V_{ij}^{-1} r_j \tag{40}
\]

Everything proceeds as before, and we find
\[
y_\mu = \sum_{ij} r_i V_{ij}^{-1} f_{j,\mu} \tag{41}
\]
\[
G_{\mu\nu}^{-1} = \sum_{ij} f_{i,\mu} V_{ij}^{-1} f_{j,\nu} \tag{42}
\]

Thus, the matrix \( V_{ij} \) also to be calculated
\[
V_{ij} = \langle \delta w_i, \delta w_j \rangle \tag{43}
\]

where \( \delta w_i \) is the deviation of a measurement from the expected position due to multiple scattering and the finite wire spacing. The calculation of \( V_{ij} \) is detailed in Appendix A.

Since this fit is so time intensive, only the hard edge approximation is used in the first pass of data reduction. The second pass uses the field map, but with 20 cm steps when calculating the trajectories. The third pass uses the full 5 cm resolution of the field map.

The momentum resolution as a function of momentum is shown in Figure 31. The resolution is determined by how well one can measure the bend angle, which neglecting multiple scattering, is independent of momentum. The relative error is proportional to the momentum, since the higher momentum tracks bend less.

\[
\sigma_\mu = \text{const} \tag{44}
\]
\[
\frac{\sigma_\mu}{C} = \frac{\sigma_P}{P} \propto P \tag{45}
\]

Thus,
\[
\frac{\sigma_P}{P^2} = \text{const} \tag{46}
\]

Figure 31 shows a resolution of about 0.3\% \( \times P \) (GeV) for four point tracks. This goes up to 0.7\% \( \times P \) (GeV) if one point is missing. The later is dominated by the case in which the space point in P4 is missing. Those tracks missing P1 or P2 are nearly as
good as the four space point case, those missing P3 come in at 0.6% × P (GeV) and those missing P4 only achieve 1.0% × P (GeV). This is the motivation for using two wire space points in three point tracks (see Section 5.1.2). The extra wires, especially in P4, improve the resolution markedly.

Figure 31 also shows that scattering limits the resolution at low momenta. Multiple scattering invalidates the uncertainty analysis above, so σP/P² is no longer constant.

5.1.5 Finding the Vertex

The distinguishing feature of kaon decays is the common vertex of the tracks of the decay products. The ability to find this vertex is crucial to the ability to separate kaon decays from coincidental backgrounds. The size of the reconstructed vertex is used as a measure of the quality of an event.

A rough vertex is found by looking for the position of closest approach of the tracks in just the Y-view. This is done by minimizing the distance perpendicular to the Z axis of each track to a common point. The analysis is similar to that presented below (where the actual distance between a track and a point is considered). There is no bend in the Y plane, so propagating the track upstream is trivial, and having only two degrees of freedom makes the matrix inversion simple.

Using this guess at the vertex position, tracks are propagated upstream through D5, and their positions and angles are found at this Z. The true best-fit vertex is then found according to the following algorithm.

Parametrize the tracks as follows

\[ x(z) = x_0 + (z - z_0)Q_x \]
\[ y(z) = y_0 + (z - z_0)Q_y, \]

where Q_x and Q_y are the tangents of the angles of the track projected onto the XZ and YZ planes (i.e. \( Q_x = \frac{\partial x}{\partial z}, Q_y = \frac{\partial y}{\partial z} \)). Then the distance squared between a point, \((x_1, y_1, z_1)\) and this track is

\[ S^2 = (x(z) - x_1)^2 + (y(z) - y_1)^2 + (z - z_1)^2, \]
Figure 31: Plots showing the momentum resolution of the fitting routine for four point tracks and for three point tracks. In the absence of multiple scattering, $\sigma_P/P^2$ should be constant. The three point track momentum resolution is dominated by the case in which P4 is missing; missing P1 and P2 give resolutions on par with the four space point case.
and the Z coordinate along the track for the position of closest approach is given by

\[
\frac{1}{2} \frac{\partial S^2}{\partial z} = 0
\]

\[
= (x(z) - x_1)Q_x + (y(z) - y_1)Q_y + (z - z_1)
\]

\[
= (x_0 - x_1)Q_x + (y_0 - y_1)Q_y - z_0(Q_x^2 + Q_y^2) + (1 + Q_x^2 + Q_y^2) - z_1
\]

\[
z = \frac{z_0(Q_x^2 + Q_y^2) + z_1 + (x_0 - x_1)Q_x + (y_0 - y_1)Q_y}{1 + Q_x^2 + Q_y^2}
\]

(50)

Now consider how \( S^2 \) changes as one moves the reference point

\[
\frac{1}{2} \frac{\partial S^2}{\partial x_1} = (x - x_1)(\frac{\partial x}{\partial x_1} - 1) + (y - y_1)\frac{\partial y}{\partial x_1} + (z - z_1)\frac{\partial z}{\partial x_1}
\]

(52)

The partial derivatives are determined using the chain rule and Equation 51,

\[
\frac{\partial x}{\partial x_1} = \frac{\partial x}{\partial z} \frac{\partial z}{\partial x_1}
\]

\[
= Q_x \cdot \frac{Q_x}{1 + Q_x^2 + Q_y^2}
\]

(53)

Thus

\[
\frac{1}{2} \frac{\partial S^2}{\partial x_1} = -(x - x_1) + [(x - x_1)Q_x + (y - y_1)Q_y + (z - z_1)] \frac{Q_x}{1 + Q_x^2 + Q_y^2}
\]

(55)

Likewise

\[
\frac{1}{2} \frac{\partial S^2}{\partial y_1} = -(y - y_1) + [(x - x_1)Q_x + (y - y_1)Q_y + (z - z_1)] \frac{Q_y}{1 + Q_x^2 + Q_y^2}
\]

(56)

\[
\frac{1}{2} \frac{\partial S^2}{\partial z_1} = -(z - z_1) + [(x - x_1)Q_x + (y - y_1)Q_y + (z - z_1)] \frac{1}{1 + Q_x^2 + Q_y^2}
\]

(57)

The second derivatives also follow

\[
\frac{1}{2} \frac{\partial^2 S^2}{\partial x_1^2} = -\frac{\partial x}{\partial x_1} + 1 + \left[\frac{\partial x}{\partial x_1} - Q_x + \frac{\partial y}{\partial y_1} - Q_y + \frac{\partial z}{\partial z_1}\right] \frac{Q_x}{D}
\]

\[
= -\frac{Q_x^2}{D} + 1 - \frac{Q_x^2}{D} + \left[Q_x^2 + Q_y^2 + 1\right] \frac{Q_x^2}{D^2}
\]

\[
= 1 - \frac{Q_x^2}{D}
\]

(58)

\[
\frac{1}{2} \frac{\partial^2 S^2}{\partial y_1^2} = 1 - \frac{Q_y^2}{D}
\]

(59)
\[
\frac{1}{2} \frac{\partial^2 S^2}{\partial z_1^2} = 1 - \frac{1}{D} \\
\frac{1}{2} \frac{\partial^2 S^2}{\partial x_1 \partial y_1} = -\frac{Q_x Q_y}{D} \\
\frac{1}{2} \frac{\partial^2 S^2}{\partial x_1 \partial z_1} = -\frac{Q_x}{D} \\
\frac{1}{2} \frac{\partial^2 S^2}{\partial y_1 \partial z_1} = -\frac{Q_y}{D}\]

where \(D = 1 + Q_x^2 + Q_y^2\).

With more than one track, one considers the sum of the distances of the tracks to the reference point. The derivatives are just as before, except that one has to sum over all the tracks \((x_0, y_0, Q_x \text{ and } Q_y \text{ all change with the track, } x_1, y_1 \text{ and } z_1 \text{ don't})\). Thus

\[
S^2 = \sum_i S_i^2
\]

\[
\frac{\partial S^2}{\partial x_1} = \sum_i \frac{\partial S_i^2}{\partial x_1},
\]

and so forth. To find the minimum value of \(S^2\), one expands in a multivariate Taylor expansion about some initial guess \(x_0\) (where \(x\) refers to the vector of three components and \(x_i\) is one of those components):

\[
S^2(x) = S^2(x_0) + \frac{\partial S^2}{\partial x_i} \bigg|_{x=x_0} (x-x_0)_i + \frac{1}{2} \frac{\partial^2 S^2}{\partial x_i \partial x_j} \bigg|_{x=x_0} (x-x_0)_i (x-x_0)_j
\]

\[
= S^2(x_0) + g_i (x-x_0)_i + \frac{1}{2} V^{-1}_{ij} (x-x_0)_i (x-x_0)_j
\]

Setting the derivative to zero, one finds

\[
\frac{\partial S^2}{\partial x_i} = 0 = g_i + V^{-1}_{ij} (x-x_0)_j \\
(x-x_0)_j = -V_{ij} \cdot g_i
\]

One would like to use \(S\) as measure of the vertex quality, cutting on it to distinguish between good and bad events. Unfortunately, the quality of the vertex depends upon how far upstream the vertex took place. This is a result of the fact that the track is measured in the spectrometer; the further upstream one projects the track
from the spectrometer, the larger the uncertainty in the position of the track. Thus, 
decays occurring in the upstream part of the decay volume have a worse $S$ distribution 
than those occurring downstream. One can get around this limitation by normalizing 
the mean of the $S$ distribution at any vertex position to a linear parameterization 
of what is seen in a very clean sample of $K_{\pi\pi}$ events. The renormalized $S$ is called 
$S'$. $S'$ has a flat distribution in the vertex position, allowing one to cut on it (see 
Figure 32).

The above algorithm is only accurate for those regions where the tracks can be 
approximated as straight lines. For vertices in the downstream end of the decay volume 
and into D5, this is not the case. Even for these events, if the initial guess, due to just 
the $Y$ slopes of the tracks, is good enough, then the linear approximation still holds. 
To get better vertex resolution in this region, all vertices downstream of $Z = -70$ cm 
are run through the analysis again, using the vertex in hand as the initial guess of 
the vertex position for the next pass.

At the upstream end of the decay volume, only events with $V_Z > -490$ cm are 
considered.

Since multiple tracks are allowed per clump (in the last data reduction pass), 
all the different track possibilities are used to search for vertices. The number of 
vertices found then depends on the number of possible combinations of tracks (with, 
of course, only one track at a time per clump). Not all combinations give a viable 
vertex, however. The vertices that are found are ranked according to an Event $\chi^2$, 
which weighs equally the quality of the particular tracks and the quality of the vertex. 
It is formed by taking the average $\chi^2$ of the three tracks, normalized so the mean of 
the individual momentum fits $\chi^2$ were unity and adding it to the normalized $S^2$ of 
the vertex, again normalized such that the mean of the $S^2$ distribution is unity (and 
flat with respect to the vertex position). The set of tracks with the best Event $\chi^2$ is 
chosen.
Figure 32: Plots showing the distribution of $S$ and the $<S>$ vs $V_z$ for a large sample of $K_{ee}$ events. The plots in the right-hand column have been normalized so that the $V_z$ dependence of $<S>$ has been removed. This normalized $S$ is called $S'$. 
Figure 33: A typical $K_{\pi\pi\pi}$ event event display, showing the projection of the reconstructed kaon upstream through the beamline to the target. The target is on the left and the decay vertex is visible on the right.

5.1.6 Hitting the Target

For the decays of primary interest in this thesis, $K_{\pi\mu}$ and $K_{\pi\pi\pi}$, the three charged particle tracks carry all the kinematic information of the event. This means one can reconstruct the parent kaon momentum and project it back upstream, through the beamline to the target (see Figure 33). By requiring the parent kaon to hit the target and have the right momentum, one discriminates against decays with missing particles, such as $K_{Dal}$, and events from the coincidence of a kaon decay with a beam muon or another kaon decay.

The uncertainty in momentum resolution leads to a blurring of the target image.
using the reconstructed kaon decays. Thus the reconstructed target position of the kaon is not used. Rather, one uses the probability that a good event (reconstructed $K_{\pi\pi}$ events) had a given kaon momentum and position in phase space at the target. A likelihood function is formed using three distributions: $Q_x$ vs $X$, $Q_y$ vs $Y$, and $X$ vs $P_K$ (see Figure 34). $X$, $Q_x$, $Y$ and $Q_y$ are all found by propagating the reconstructed kaon upstream through the beamline to the $Z$ position of the target. The number entries in each bin of the plots, divided by the total number on the plot is taken as the probability that a good event will have that pair of coordinates. The logarithms of the three probabilities are added together to give the Target Likelihood Function (see Figure 35).

Decays such as $K_{Dal}$ with missing momentum, in the form of an undetected or unanalyzed photon, will not have a good reconstructed kaon, and as such will get pairs of target parameters in bins unpopulated by $K_{\pi\pi}$ events. The contribution to the likelihood function in this case is a default value of $-50$, where typical cuts on the Target Likelihood are of order $-20$. These cases are thus cut automatically.

When the photon (see Section 5.1.1) in $K_{Dal}$ decays is detected and its momentum is added to the reconstructed kaon, one gets a viable projection back to the target. However, the additional uncertainties smear out the Target Likelihood distribution somewhat (see Figure 36).

5.2 Particle Identification (PID)

Identifying the proper particle type is crucial to the success of the experiment. With the beam consisting entirely of pions, protons and kaons, the only decay products to be concerned about are pions, electrons and muons. Pions are taken as the default, and muons and electrons are identified by means of the particle identification (PID) systems. The Čerenkov counters and the calorimeter are used to separate electrons from pion. Muons are identified by means of the muon range stack.
Figure 34: Scatter plots of the distributions used to form the Target Likelihood Function. The plots are all considered at the Z position of the target. The full sample of $K_{\pi\pi}$ events is used to make these plots.
Figure 35: A plot of the Target Likelihood distribution for the full sample of $K_{\pi\pi}$ events used in this thesis.

### 5.2.1 Light on the Track

Čerenkov light from a track can illuminate only a few of the many photomultiplier tubes (PMTs) in either of the Čerenkov counters. Matching PMTs to the track is done at the plane of the primary mirror in each counter. The track is projected to the primary mirror using the best fit track (see Section 5.1.4). Each PMT views a specific region of the primary. If the track intersects the primary mirror plane within this region, light from that photomultiplier can contribute to light on the track.

Timing is important as well. After a slewing correction, signals with TDCs within 5 ns of T0 are accepted. One finds events in the unbiased $K_{\text{Dal}}$ sample which have reasonable ADC values along the electron or positron track, but an early TDC. The assumption in this case is that some uncorrelated signal triggered the TDC, but that the ADC represents the light due to the electron or positron. To increase the electron PID efficiency, these events are accepted as well. One doesn’t find late TDCs, as expected, so large positive times ($t > 5$ ns) are excluded.

The Čerenkov counter performance can be tested with a sample of $K_{\text{Dal}}$ events taken without any PID triggers. This sample forms a small subset of the TAU stream taken for normalization purposes. This sample is prescaled at the level of $\sim 1/23000$. 
Table 5: The photoelectron yields and efficiencies of the four Čerenkov counters for electrons and positrons. The two counters on the right are also evaluated under the condition that the track go only through good regions of both counters.

The events are found kinematically by looking for a photon, then finding a combination of one positive and one negative track that gives the $\pi_0$ mass when combined with the photon. The reconstructed kaon, *with the photon added*, is required to be consistent with hitting the target (but with a more lenient cut on the Target Likelihood) and the invariant mass of the $\pi_0$ with the remaining charged track to be consistent with the kaon mass. Figure 36 shows the reconstructed pion and kaon resolution and the Target Likelihood distribution.

The average photoelectron yields and efficiencies of the four counters are given in Table 5. The right side inefficiencies are partly due to cracks in the coverage of the primary mirrors and to partly defects in the primary mirror itself. To limit the effect of this on $K_{\pi\mu e}$ backgrounds, pions in $K_{\pi\mu e}$ candidate events are required to not go through any questionable parts of the counters. Placing this fiducial volume requirement on the positrons in the $K_{Dal}$ sample improves the average yield and efficiency slightly.

### 5.2.2 $E/P$

Since each track is associated with a calorimeter clump from the beginning (the clump seeds the search for the space points), it is trivial to compare the energy deposited in the calorimeter to the momentum found by the track fit. One expects that electrons and positrons will deposit all their energy in the calorimeter giving $E/P$ ratios of close to unity. Muons are minimum ionizing particles and as such have small $E/P$ ratios.
Figure 36: Plots showing the reconstruction of $K_{Dal}$ events from the $K_{ππ}$ data sample (no PID requirements). The first two plots show the reconstructed masses of the $π_0$ and the $K^+$. The third plot shows the Target Likelihood distribution after the photon momentum is added to the system.
Figure 37: A plot of the distribution of $E/P$ for pions. The dashed histogram shows the case where muons from $\pi_{\mu 2}$ decays are removed.

Pions are sometimes minimum ionizing particles, but also have hadronic interactions in the calorimeter. The calorimeter is not compensated for hadronic showers, and much of the visible energy of the shower leaks out of the back end of the calorimeter. The result is that the pions show a minimum ionizing peak at low $E/P$ and a broad shower peak with a maximum of about 0.5. See Figure 18 on page 38 for examples of electron and pion distributions in the calorimeter, taken from the unbiased $K_{Dal}$ sample (see Section 5.2.1). A higher statistics sample of pions, from the $K_{\pi\pi\pi}$ sample, with and without $\pi_{\mu 2}$ decays, is show in Figure 37.

Electrons are required to have $E/P$ within 0.2 of unity. Pions and muons are required to have $E/P < 0.85$, or if there is light in one of the Čerenkov counters, $E/P < 0.6$. Tables 6 and 7 show the efficiencies of these cuts for particles of various momenta. Figure 38 shows the resolution and linearity of the calorimeter response for electrons.

5.2.3 Muon Range

Muons are distinguished by the fact that they are too heavy to undergo bremsstrahlung radiation, and thus make a shower in the calorimeter, yet they do not interact
Table 6: Percentage of electrons with $E/P$ within a given range of unity. Taken from the unbiased $K_{Dal}$ sample.

<table>
<thead>
<tr>
<th>$0.9 &lt; P &lt; 1.1$</th>
<th>$0.7 &lt; E/P &lt; 1.3$</th>
<th>$0.8 &lt; E/P &lt; 1.2$</th>
<th>$0.9 &lt; E/P &lt; 1.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>89</td>
<td>94</td>
<td></td>
</tr>
<tr>
<td>81</td>
<td>90</td>
<td>94</td>
<td></td>
</tr>
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<td>91</td>
<td>94</td>
<td></td>
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<tr>
<td>75</td>
<td>92</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>74</td>
<td>91</td>
<td>94</td>
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</tr>
<tr>
<td>70</td>
<td>85</td>
<td>94</td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Percentage of pions with $E/P$ greater than a given value. Taken from the unbiased $K_{\pi\pi}$ normalization sample.

<table>
<thead>
<tr>
<th>$0.9 &lt; P &lt; 1.1$</th>
<th>$E/P &gt; 0.6$</th>
<th>$E/P &gt; 0.8$</th>
<th>$E/P &gt; 0.85$</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.2</td>
<td>19.7</td>
<td>16.2</td>
<td>14.0</td>
</tr>
<tr>
<td>10.7</td>
<td>7.0</td>
<td>5.1</td>
<td>4.1</td>
</tr>
<tr>
<td>8.3</td>
<td>5.2</td>
<td>3.7</td>
<td>3.0</td>
</tr>
<tr>
<td>10.1</td>
<td>2.5</td>
<td>2.2</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Figure 38: The plot on the left shows the resolution of $E/P$ for electrons and positrons versus the momentum of that particle. Also shown is the $0.082/E^{1/2}$ line expected of the calorimeter. The difference is due to the resolution of $P$. The plot on the right shows the linearity of the response for particles of different momenta.
hadronically like pions. This makes muons very penetrating. To check the muon hypothesis for a given track, one looks for a track in the muon range stack and activity in the B and C hodoscope arrays. A muon of given energy has an expected range in the steel of the range stack. Any muon track is required to hit the B hodoscope, after 40 cm of steel. This places a lower limit of about 0.8 GeV on our acceptance for muons. Muons above this momentum are required to reach their expected range, or to exit the range stack out the back if they are energetic enough.

Tracks are projected from P4 into the range stack, and a three standard deviation window about this projection in each plane is searched for hits. The window takes into account the multiple scattering in the steel of the stack. Horizontal and vertical planes are considered separately. The number of planes seen is required to be at least half of the number expected from the range to be considered a muon track. The X-view is complicated slightly by the one missing segment in each plane (see Figure 20), but the expected number of hits takes this missing segment into account. B and C hodoscopes are assigned by forming a $\chi^2$ from the residual between the center of the hodoscope slat and the projected position of the track at that plane, all divided by the expected error from multiple scattering and the finite size of the hodoscope slats. This $\chi^2$ is required to be less than 12 in the B hodoscope for all muon candidates.

It is difficult to measure the efficiency of muon-finding algorithm since a clean sample of muons is unavailable. $K_{\mu3}$ decays could be used if one knew the initial kaon momentum, but I have been unable to isolate this signal well enough using just the central beam momentum and a kaon proceeding along the Z axis. I have instead estimated the efficiency from GEANT monte carlo, putting in the efficiency of each plane by hand. The efficiency for detecting muons by this method is 80% (see Section 6.2).

### 5.2.4 PID Requirements

The PID requirements vary with the level of the analysis. The first pass of the data reduction analysis has no PID requirements at all since one is mostly concerned with getting rid of those events without viable vertices, and the pass was started before
Electron

- C1 PE  > 0
- C2 PE  > 0
- $E/P$  > 0.5

Muon

- B Hodoscope  $\chi^2 < 12$
- # of Muon Hits  # Expected  > 0.5
- $E_{Cal}$  < 0.45 GeV

Pion

- Two out of three:
  - C1 PE  < 0.5
  - C2 PE  < 0.5
  - $|E/P - 1|$  > 0.2

Table 8: PID Requirements for the various particle types in Pass 2.

full understanding of the PID system responses was obtained. The second pass has loose cuts on the PID indicators while the final analysis has tighter cuts. These cuts are listed in Tables 8 and 9. A PID likelihood analysis was considered, but found to offer no real advantage over the cut based analysis presented here.

The cut on momentum for muons is attempt to reduce background from beam muons from $\pi_{\mu2}$ decays (see Figure 39). This also has the desirable effect of eliminating muons above the Čerenkov threshold on the right. The pion cut on NMY (number of Y plane muon chamber hits) is to eliminate muons whose transverse momentum, plus scattering in the first planes of the muon range stack carry them out the top or bottom of the stack before hitting the B hodoscope.

5.3 Alignment and Calibration

Before one can use the information from the proportional chambers, calorimeter or Čerenkov counters, one must know their response to tracks and their exact position in the detector. For the proportional chambers, position is the most important item;
**Electron**

On Left Side

Negative Charge

C1 PE  > 0.3
C2 PE  > 0.3
|E/P − 1| < 0.2

**Muon**

Positive Charge

B Hodoscope $\chi^2 < 12$

# of Muon Hits > 0.5

NMY ≥ 6

$E_{Cal} < 0.45$ GeV

$P_{Tot} < 3.4$ GeV

C1 PE  < 1.2
C2 PE  < 1.2

E/P < 0.85

Two out of three:

C1 PE  < 0.4
C2 PE  < 0.4
E/P < 0.6

**Pion**

On Right Side

Positive Charge

Fiducial Volume Cut for Čerenkov efficiency

C1 PE  < 1.2
C2 PE  < 1.2

E/P < 0.85

Two out of three:

C1 PE  < 0.4
C2 PE  < 0.4

E/P < 0.6

If |$Y_B$| > 0.8:

NMY < 4

Table 9: $K_{\pi\mu\epsilon}$ PID Requirements for the various particle types in the final analysis. NMY is the number of Y plane muon chamber hits associated with the track.
Figure 39: A plot of the momentum spectrum of muon tracks in candidate $K_{\pi\mu e}$ events after Pass 2. The peak above $P = 3.4$ GeV is due to beam muons combining with a coincidental kaon decay to give a $K_{\pi\mu e}$ signature. To reduce this background only those events with muon momentum below 3.4 GeV are accepted.

for the calorimeter and Čerenkov counters, the gain and pedestal of the PMTs are paramount.

5.3.1 Spectrometer

The alignment of the proportional wire chambers (PWCs) uses data from muon runs, where the analysis magnets and beamline are turned off and the detector is triggered on a single A·SH. These triggers come from single muons created near the target that penetrate all the material between the target and the detector. The alignment proceeds in two steps. First, the space point residuals in each chamber are checked for consistency against rotations and uniform wire spacing of the planes within that chamber. This can be done with no knowledge of hits in the other chambers or of the angle of the track. However, since the planes within one chamber are at different $Z$ positions, there is a small effect from the angle of the track. This effect must be corrected to get consistency between the planes within one chamber. Fortunately, the accuracy to which the track must be known is not that great: chamber positions
given by survey are sufficient.

There are are two residuals, $\Delta X$ and $\Delta Y$ which can be plotted with respect to the $X$ and $Y$ coordinates, giving four plots to determine the positions of the four wire planes. In fact, one does linear fits to the residual versus position plots, giving two parameters per plot, eight parameters total. On the other hand, one has 10 parameters to determine, four offsets, three plane angles and three plane wire spacings. The overall angle and wire spacing aren’t observable. This underdetermination is circumvented by assuming the nominal wire spacing until one has found the best combination of angles, then trying to better that solution by using only small changes to the wire spacings.

To find how small changes in the individual plane angles affect the residual distributions, parametrize the wire planes $(XVUY)$ in terms of the actual position $(x, y)$ as follows

\[
X = x \cos \theta_X + y \sin \theta_X \quad (69)
\]
\[
V = x \cos \theta_V + y \sin \theta_V \quad (70)
\]
\[
U = x \cos \theta_U - y \sin \theta_U \quad (71)
\]
\[
Y = -x \sin \theta_Y - y \cos \theta_Y \quad (72)
\]

where $\theta_V \approx \theta_U$ and $\theta_X, \theta_Y \ll 1$. Using equation 17, one can calculate the effect of small changes in angles on the residuals

\[
d(\Delta X) = (-x \sin \theta_X + y \cos \theta_X) d\theta_X - \frac{1}{2 \cos \theta} [(-x \sin \theta_V + y \cos \theta_V) d\theta_V + (-x \sin \theta_U - y \cos \theta_U) d\theta_U]
\]
\[
\approx y d\theta_X - \frac{(-x \tan \theta + y) d\theta_V + (-x \tan \theta - y) d\theta_U}{2} \quad (73)
\]
\[
d(\Delta Y) \approx -x d\theta_Y - \frac{(-x + y \cot \theta) d\theta_V + (x - y \cot \theta) d\theta_U}{2}, \quad (74)
\]

where $\theta$ is the nominal angle of the $VU$ planes. Now looking at the slopes with respect to $X$ and $Y$ we have

\[
\frac{d}{dy} d(\Delta X) = d\theta_X - \frac{1}{2} (d\theta_V - d\theta_U) \quad (75)
\]
Using the observed slopes in each of the four plots, one tries to determine the angles that give the smallest slopes in all four plots. Unfortunately, the matrix defined by the equations above is singular, so one cannot just invert the matrix to find the best solution of plane angle settings. The matrix is singular as a result of the fact that four lines in a plane will generally not have a common vertex. However, one knows in this case that the four wires in a true space point must come from a common vertex, given the finite spatial resolution implied by the wire spacing, so there is a solution in this case. The singularity of the matrix then leads to a non-unique solution due to the invariance of the residuals to a global rotation of all four planes in the chamber.

The solution is found by searching through a range of possible plane rotation angles, using the equations above to find the slopes of the fits to the residual distributions in each case. The slopes from the four plots are combined into a sort of unnormalized $\chi^2$ (i.e., the slopes are squared and added together). The set of angles with the smallest $\chi^2$ is chosen. To remove the ambiguity in the solution, the rotations are required to give no overall rotation to the plane, and if further specification is necessary, the set of angles with the smallest deviation from the nominal angles is chosen.

This solution can be improved by letting the wire spacings of the chamber vary. This effect is expected to be much smaller in practice, than that due to rotations of the planes. The analysis is quite similar to that above. The result is wire chambers aligned to within 100 $\mu$rad as measured by the slopes on the residual plots (see Figure 40). The wire offsets are much more straightforward, and the offsets on the residual plots are aligned to about 1/20th of a wire spacing.

The overall positions of the chambers are given first by survey and then adjusted in the second part of the alignment procedure. The muon tracks establish straight lines through the four chambers. We use the positions of hits in P1 and P4 to predict
Figure 40: Two plots showing a scatter plot of the residual $\Delta X$ versus $X$ for P1. The second plot shows the mean of that distribution with respect to $X$ fit to a line. Both plots are shown after the alignment is complete.

the hit positions in P2 and P3. The residuals in these chambers are then used to determine the overall rotations and offsets of the chambers. Any systematic slope of residuals on a $\Delta X(\Delta Y)$ vs $X(Y)$ plot is attributed to an offset in the Z direction that needs to be corrected. At this stage of the alignment analysis, all four planes of each chamber must be moved together to maintain the internal consistency of the chambers determined in the first stage.

This alignment program leaves the possibility of twists and offsets between P1 and P4. There are no real handles in the data to estimate these misalignments. The positions of P1 and P4 are somewhat constrained by having the kaon project back to the target, but the uncertainties of the beamline magnet settings are much larger than the uncertainty in the chamber positions. The size of the calorimeter modules is also too large to provide much of a constraint on the overall positions. A two body charged decay of a neutral particle would allow one to look for a twist offset between chambers; unfortunately, there are not enough of these in charged kaon and pion decays to allow us to do this alignment. These ambiguities (which are limited by survey accuracy) affect the resolution only at very high order.
5.3.2 Calorimeter

Calibration of the calorimeter consists of determining the pedestal and gain for each of the 600 PMTs. Pedestals, ADC signal levels with no input to the PMT, are determined in by looking at the ADC response with a random gate. The peak representing no light in the module is taken as the pedestal.

To determine the relative gains of each tube and to monitor changes in the gain over time, one looks at signals from cosmic ray muons. These muons are required to go vertically through the calorimeter, hitting modules in both the top and bottom rows and leaving a track through all the rows in between. The muons are assumed to be minimum ionizing and no correction is made to account for those tracks with lower energy, except to require that the track pass vertically through the entire calorimeter. One can fit the hit modules to a line and use this to determine how much energy is left in each module. This gives the relative gain of each module. Cosmic ray data was taken between spills whenever E865 was taking data.

Knowing the relative gains, one can determine the overall gain with the Dalitz pair electrons. Knowing the energy of the electron or positron, one looks to see the ADC values associated with modules in the clump. The relative energy in each module is known from the cosmic ray calibration. Thus, one compares the ADC in any module with the fraction of the energy it should have seen using the known momentum of the track. This gives the absolute gain of all the modules.

5.3.3 Čerenkov Counters

Calibration of the PMTs in the Čerenkov Counters is similar to those in the Calorimeter. The pedestals are taken from the ADC values when there is no activity in the PMT. The gains are determined by looking for the single photoelectron peak in $K_{Dal}$ events when there is an electron and a positron in the detector.
Table 10: Statistics for the MuE stream in the various passes. NTUPLE refers to the analysis of Pass 3 data to make the NTUPLE used in the rest of the analysis (i.e., there are only 11163 events in the final NTUPLE).

5.4 Data Reduction Passes

The reduction of E865 data proceeded in a series of passes. Each pass is more restrictive than the last, and because of the culling done in earlier passes, takes less time. Statistics for the various passes are given in Table 10.

5.4.1 Pass 1

Pass 1 is the simplest, longest and most effective of the data passes. The basic requirement is the existence of a vertex. In fact, most of the events on tape at this point do not have vertices, even for events with a TAU trigger, so this is a very good way to remove useless events. Unfortunately, determining the presence of a good vertex is time consuming, even in the hard-edge field approximation. The complete analysis took about six months using three dedicated DEC Alpha workstations.

Pass 1 looks for at least three clumps in the calorimeter, connecting to at least three momentum fit tracks. Only the best track from each clump is kept at this stage. These tracks have to form a vertex with $S$, the RMS distance of closest approach (see Section 5.1.5), less than 10 cm. The tracks are fit using the hard-edge field approximation. No PID is performed since this pass was begun before the PID responses were well known.

The data from this pass are split into a number of streams: TAU, 3TRK and 5TRK. The TAU stream consists of those events with a pre-scaled $\tau$ trigger (see Section 4.2.2). The 3TRK stream contains events with a MuE or ELER trigger. The
3TRK stream, as its name suggests contains events with five tracks in the vertex. It is not used for this thesis.

### 5.4.2 Pass 2

The magnetic field map is first used in the Pass 2 analysis, with a step size of 20 cm. The field strength of the field map is taken as measured, though it was later found that it should be scaled up by about 2% (see Section 3.4.1) to get the correct reconstructed kaon mass in $K_{\pi\pi}$ decays; this correction is instituted for Pass 3. Again a vertex with $S < 10$ cm was required, allowing only one track per clump. Pass 2 requires, in addition, that the reconstructed kaon momentum be within 1 GeV of the nominal 6 GeV beam momentum (this is, in fact, only required of the MuE stream, see below). The analysis of the 3TRK stream took approximately two weeks using the same three workstations.

Particle identification (PID) is introduced with this pass, using the cuts listed in Table 8. Unambiguous PID determination is not required during this pass, just that at least one combination of particle types among the tracks in the vertex satisfies the requirements of the stream.

The TAU stream from Pass 1 is not reanalyzed in this pass. The 3TRK stream is split into three streams: MuE, EEPS, and HIMS. The MuE stream requires one of each particle type (electron, muon and pion) among the three tracks with no track in the vertex not supplying one of the required types. The electron must have a negative charge, while the other two tracks must be positively charged. The EEPS and HIMS streams require two electrons of different charges (an electron and a positron) and a positive pion. The difference between the two streams is the HIMS trigger latch, requiring that the calorimeter rows of the two electron clumps be at least two apart for the HIMS stream. The EEPS stream takes one in ten of all the good $\pi ee$ events. This is the source of the $K_{\text{Dal}}$ normalization sample (which is not used as a normalization in this thesis, but is used to determine PID efficiencies and the $K_{\text{Dal}}$ background rate). There is an additional prescale of a factor of ten in T2 for this channel (see Section 4.2.3), so the sample is actually prescaled by a factor of 100.
5.4.3 Pass 3

A step size of 5 cm is used in Pass 3, along with the correctly scaled magnetic field map. (The map is scaled to give the correct reconstructed kaon mass in $K_{\pi\pi\pi}$ decays.) In this pass one wants the best possible resolution of tracks and vertices, having winnowed down the sample to a manageable size in the previous two passes. With good resolution on the vertex, one also has confidence in the ability to project the kaon back to the target and cut on how well it hits. This is the biggest reduction factor for this pass. The analysis of the MuE stream took two to three days on one DEC Alpha workstation.

To achieve the best resolution for tracks and vertices a number of additional analysis steps are introduced in this pass. All space points are corrected for the known angle of the track. Those space points with a $\chi^2$ per degree of freedom greater than seven are eliminated. Tracks require at least three space points, so if the eliminated space points reduce the number on the track to less than three, the track is eliminated as well. On the other hand, if the track has exactly three good space points, the fourth chamber is searched for doublet space points.

These two measures increase one's confidence in the $\chi^2$ for the momentum fit, which is required to be less than 23 per degree of freedom. This is much larger than one would expect, and represents shortcomings in the knowledge of wire positions and in the field map. The mean of the $\chi^2$ per degree of freedom is about 2.5, making the cut of 23 seem more reasonable, but still quite large.

For any clump, the best three tracks are kept. All different combinations of the tracks from various clumps are considered to give the best vertex. The best vertex (see Section 5.1.5) weighs the quality of the tracks and the $S^2$ of vertex equally. Both $S$ and the normalized $S'$ are also required to be less than ten.

Finally, the Target Likelihood, when the kaon is projected back to the target, is required to be greater than -30 (see Section 5.1.6). Events passing these requirements are written into a disk resident NTUPLE for further analysis. This analysis is done for both the TAU stream and the MuE stream. A similar analysis, with no Target Likelihood requirement is performed for the EEPS stream. The MuE and TAU stream cuts for Pass 3 are listed in Table 11.
The final step in the analysis is done using the NTUPLEs created in Pass 3. The purpose of this final analysis is to eliminate as many of the backgrounds as possible without overly limiting the signal. The biggest source of background is from $K_{\text{Dal}}$ events, and there are two effective ways of dealing with this background, the $M_{ee}$ cut and the Target Likelihood cut.

The electron and positron in $K_{\text{Dal}}$ decays come from a virtual photon, so their invariant mass distribution is peaked at very low masses and is bounded by the $\pi^0$ mass. The invariant distribution mass of the electron together with the pion or the muon (when declared to be a positron) in $K_{\pi\mu\text{e}}$ decays, on the other hand, is mostly above the $\pi^0$ mass. This quantity is called $M_{ee}$. Much of the Dalitz background can be eliminated by cutting on $M_{ee}$, while $K_{\pi\mu\text{e}}$ is minimally cut (see Section 6.2). The cut used in this analysis is 0.05 GeV.

The other handle on the Dalitz background is to require a good target likelihood. Few Dalitz decays have a good Target Likelihood since they are missing the momentum of the photon. A Target Likelihood of greater than -19 was required in the final analysis, though again higher cuts are considered. The $K_{\pi\pi\pi}$ sample, with its better mass resolution, has a slightly better Target Likelihood distribution than expected from $K_{\pi\mu\text{e}}$ decays. The Target Likelihood value at the same percentile as -19 for $K_{\pi\mu\text{e}}$ (determined from Monte Carlo) is -18.8.

Another important factor, which hasn’t been stressed until now is the timing of the signals. The timing signals of the A hodoscope slats, calorimeter modules and

\begin{align*}
\chi^2_{sp}/\text{DoF} &< 7 \\
\chi^2_{\text{Fit}}/\text{DoF} &< 23 \\
5.55 \text{ GeV} &< P_K < 6.35 \text{ GeV} \\
\text{Target Likelihood} &> -30
\end{align*}

Table 11: Kinematic Cuts for MuE and TAU streams in Pass 3.

### 5.5 Final Cuts

The electron and positron in $K_{\text{Dal}}$ decays come from a virtual photon, so their invariant mass distribution is peaked at very low masses and is bounded by the $\pi^0$ mass. The invariant distribution mass of the electron together with the pion or the muon (when declared to be a positron) in $K_{\pi\mu\text{e}}$ decays, on the other hand, is mostly above the $\pi^0$ mass. This quantity is called $M_{ee}$. Much of the Dalitz background can be eliminated by cutting on $M_{ee}$, while $K_{\pi\mu\text{e}}$ is minimally cut (see Section 6.2). The cut used in this analysis is 0.05 GeV.

The other handle on the Dalitz background is to require a good target likelihood. Few Dalitz decays have a good Target Likelihood since they are missing the momentum of the photon. A Target Likelihood of greater than -19 was required in the final analysis, though again higher cuts are considered. The $K_{\pi\pi\pi}$ sample, with its better mass resolution, has a slightly better Target Likelihood distribution than expected from $K_{\pi\mu\text{e}}$ decays. The Target Likelihood value at the same percentile as -19 for $K_{\pi\mu\text{e}}$ (determined from Monte Carlo) is -18.8.

Another important factor, which hasn’t been stressed until now is the timing of the signals. The timing signals of the A hodoscope slats, calorimeter modules and
Čerenkov counters have all been adjusted for slewing and spatial position. By making tight timing cuts, one reduces backgrounds from coincidental events. Čerenkov light is considered within a 5 ns window about the nominal time of the event, as well as light with early or overflowed TDCs (see Section 5.2).

The timing of the A hodoscopes and shower modules making up the event is combined into an RMS value to gauge the degree to which the trigger elements were in-time. The square of this six-element RMS time, is required to be less than 1.5 ns².

A couple other cuts are designed to reduce the background from $K_{\text{Dal}}$ decays: no in-time photons and no extra (fourth) track which can form a good Dalitz pair with the electron (meaning the fourth track comes close to the vertex, looks like an electron and has a low $M_{ee}$). To reduce backgrounds from spurious tracks, formed when the chambers are noisy and many space points are reconstructed, the number of possible tracks, calculated combinatorically from the number of space points in each chamber in the windows opened by a particular clump, is required to be less than 1000 for each track. This large cut value reflects the combinatorics of space points in four chambers: $\sqrt[4]{1000} = 5.6$, so one doesn’t need an overwhelming number of space points in each chamber to get very large numbers of possible combinations.

All events with two muons are eliminated, as well as any event with ambiguous particle identification. The particle identification cuts are those listed in Table 9. The final cuts are listed in Table 12, along with a summary of all the other cuts.

The same cuts apply to the TAU stream except for the PID requirements (no PID at all in the TAU stream) and the $M_{ee}$ cut.

### 5.6 Analysis Results

A grand total of 113 $K_{\pi\mu\epsilon}$ candidate events passed all the cuts for the 1995 run. A scatter plot of $S'$ versus $M_{\pi\mu\epsilon}$ is shown in Figure 41. An equivalent plot for a portion of the $K_{\pi\pi\pi}$ sample is shown in Figure 42. The reconstructed mass for the full $K_{\pi\pi\pi}$ sample, with full cuts and $S' < 3$ is shown in Figure 43.

On the basis of Monte Carlo studies the $S'$ distributions for $K_{\pi\mu\epsilon}$ and $K_{\pi\pi\pi}$ events are expected to be similar. 96% of the $K_{\pi\pi\pi}$ events have an $S' < 3$. The width
Figure 41: $S'$ vs Kaon Mass for the final $K_{\pi\mu\nu}$ sample. The signal region is indicated by the small box. There are no events in the signal region. The mass range of the signal region is determined from Monte Carlo and the measured resolution in $K_{\pi\pi\pi}$ decays.
Figure 42: $S'$ vs Kaon Mass for part of the final $K_{\pi\pi}$ normalization sample. The signal region is indicated by the small box. The mass range of the signal region contains 95% of the events.
Figure 43: The reconstructed kaon mass for the final $K_{\pi\pi}$ normalization sample (with $S' < 3$). The signal region is indicated by the shaded part of the histogram and only these events are included in the number of events and in the gaussian fit to shape of the peak.
Table 12: The Final Cuts on the $K_{\pi\mu\nu}$ candidates.

of the reconstructed kaon mass of $K_{\pi\mu\nu}$ is expected to be 60% larger than that of
$K_{\pi\pi\pi}$ due to larger amount of kinetic energy available in the $K_{\pi\mu\nu}$ decay. 95% of
the $K_{\pi\pi\pi}$ sample fell within 7.5 MeV of the central kaon mass. The corresponding
width for $K_{\pi\mu\nu}$ is 12 MeV. This sets our signal region for $K_{\pi\mu\nu}$ decays to $S' < 3$ and
$481.6$ MeV $< M_{\pi\mu\nu} < 505.6$ MeV. No events are observed. The origin of the outlying
events on the signal plot will be discussed in Section 7.2.

239306 $K_{\pi\pi\pi}$ events are observed in the signal region having passed all cuts. The
TAU stream is prescaled by a factor of 20,00 or 30,000, depending on the run number.
Multiplying the number of events in each run by the prescale for that run one finds a
total of $5.43(1) \times 10^9$ $K_{\pi\pi\pi}$ decays in the normalization sample (see Figure 44), where
the uncertainty comes from the uncertainty in the setting of the Target Likelihood
for $K_{\pi\pi\pi}$ decays (see Figure 57 on page 125, Figure 58 on page 126, and Section 6.2).

$\chi^2_{Rp}/\text{DoF}$ $< 7$
$\chi^2_{PFit}/\text{DoF}$ $< 23$
$V_Z > -490$ cm
$|P_K - 5.95$ GeV$| < 0.4$ GeV
Target Likelihood $> -19$ (-18.8 for $K_{\pi\pi\pi}$)
$M_{\pi\nu}(\pi$ or $\mu)$ $> 0.05$ GeV (not $K_{\pi\pi\pi}$)
$T^2_{\text{RMS}} < 1.5$ ns$^2$
$N_{\text{TrackComb}} < 1000$
No photons
No extra Dalitz pair tracks
Unambiguous PID (not $K_{\pi\pi\pi}$)
Figure 44: Plots of the number of $K_{\pi\pi}$ events passing all analysis cuts per run number, the prescale factor for each run number, and the cumulative number of $K_{\pi\pi}$ events, prescale factor removed, seen as function of run number and day of the run.
Chapter 6

Normalization and Acceptance Calculations

With no events in the signal region, one can set an upper limit on the branching ratio of $K^+ \rightarrow \pi^+ \mu^+ e^- (K_{\pi\mu\epsilon})$. To do this one needs to know the number of kaon decays in the Decay Volume that one is sensitive to, factoring in the acceptance of the detector and the reconstruction efficiencies. One way of calculating this number is to look at another decay mode whose branching ratio is well known and whose reconstruction efficiencies are similar to $K_{\pi\mu\epsilon}$. There are two good candidates: $K^+ \rightarrow \pi^+ \pi^+ \pi^- (K_{\pi\pi\pi})$ and $K^+ \rightarrow \pi^+ \pi^0$, $\pi^0 \rightarrow e^+ e^- \gamma (K_{\text{Dal}})$. $K_{\pi\pi\pi}$ has the advantage that all the decay products are charged particles, as in $K_{\pi\mu\epsilon}$ allowing us to reconstruct a kaon mass and project back to the target. It also has the advantage of having a large branching ratio (limiting backgrounds in the normalization sample) and a large acceptance. The drawback is that no PID efficiencies come into the analysis of this mode, so these efficiencies must be determined by other means for $K_{\pi\mu\epsilon}$, introducing possible uncertainties. The $K_{\text{Dal}}$ mode has complimentary features. It has an electron on the left like $K_{\pi\mu\epsilon}$ but the presence of the unanalyzed photon removes the possibility of reconstructing a kaon mass or projecting back to the target. If one includes the photon in the analysis, one must also understand the biases introduced in finding the photon and how it effects the reconstructed mass and target resolutions. The $K_{\pi\pi\pi}$ normalization mode is used in this thesis. Another analysis of this data uses the $K_{\text{Dal}}$
mode[32].

The 90% confidence level (CL) upper limit of the branching ratio for $K_{\pi\mu\nu}$ denoted $\text{BR}(K_{\pi\mu\nu})$, is written in terms of $\text{BR}(K_{\pi\pi\pi})$, the branching ratio for $K_{\pi\pi\pi}$:

$$\text{BR}(K_{\pi\mu\nu}) < \text{BR}(K_{\pi\pi\pi}) \cdot \frac{N(K_{\pi\mu\nu})}{N(K_{\pi\pi\pi})} \cdot \frac{\text{Acc}(K_{\pi\pi\pi})}{\text{Acc}(K_{\pi\mu\nu})} \cdot C$$  \hspace{1cm} (79)

where $\text{Acc}()$ is the acceptance of the E865 detector for the given decay modes, $N(K_{\pi\pi\pi})$ is the number of $K_{\pi\pi\pi}$ events seen after removing the prescale factor, $N(K_{\pi\mu\nu})$ is the 90% CL upper limit on the number events expected consistent with seeing none (which is 2.3 using standard Poisson statistics), and $C$ is a correction factor taking into account the different efficiencies that come into play in the two modes.

The calculation of the acceptances relies primarily on Monte Carlo (MC) calculation as detailed in Section 6.1. The correction factors are determined from data where possible, and with MC where not. The details of the correction factor calculation are given in Section 6.2.

6.1 The Acceptance

The acceptance of the detector for a given decay mode is defined as that fraction of decays that satisfy these conditions:

1. The decay vertex is in the decay volume.

2. The decay tracks lie completely within a fiducial volume defined roughly by the edges of all the detector elements. This excludes the dead regions of the chambers, hodoscopes and calorimeter. It does not exclude a fiducial volume cut applied to the $K_{\pi\mu\nu}$ candidates to remove inefficient regions of the Čerenkov counters.

3. The topology of the three charge tracks satisfies the T0 matrix requirements (see Section 4.2.1). In addition, $K_{\pi\mu\nu}$ is required to have the electron on the right and the pion on the left. No such side requirement is imposed on $K_{\pi\pi\pi}$ (either in the acceptance calculation or the normalization sample analysis).
When computing $K_{\text{Dal}}$ acceptances (for computing expected backgrounds) the side requirements are the same as for $K_{\pi\mu e}$ with the additional requirement that the positron be on the right. If the photon acceptance is included, there is no fiducial requirement on the photon track, only the requirement that the photon clump in the calorimeter not be adjacent to an active A hodoscope slat.

The acceptance is calculated by the Monte Carlo (MC) method. A sequence of pseudorandom numbers is used to generate the momenta and directions of the products of kaon decays subject to kinematic constraints of the conservation of momentum and energy. The decay products traverse a simulation of the E865 detector to determine the fraction of the decays that will be accepted. Physical effects such as energy loss, multiple scattering, delta ray production, etc., are included. Both event generation and detector simulation are accomplished with the GEANT package from CERN [33]. In addition, GEANT allows one to model the response of the detector elements and to create simulated data to test analysis code and check reconstruction efficiencies. Being able to compare MC data with physical data from the detector gives one confidence in the acceptance calculation where one has only the MC calculation to rely on. It also allows one to estimate some trigger level efficiencies that would be difficult to obtain otherwise.

### 6.1.1 The KAY Monte Carlo Program

The GEANT simulation of the E865 detector is called KAY [34]. KAY includes a very detailed description of the detector: the individual wires in the PWCs, the detailed structure of the calorimeter modules (lead and scintillator sheets, wave shifting fibers, foil around the edges; less detail on the PMT and the gathered wave shifting fibers) and cell structure of the muon chambers. The measured field map is used to propagate the particles through the detector.

Kaons can be generated at the target and transported through the beamline to the detector. This is an inefficient and CPU intensive way to proceed, however, and KAY can also generate kaons at a given point along the beam line from a table containing a sampling of the beam phase space. This table was generated from data, a clean
sample of reconstructed $K_{\pi\pi}$ events, so that uncertainties in the modeling of the beam line would not effect the acceptance calculation. It is used for the acceptance calculation and all the MC/data comparisons shown below.

The T0 topology and side requirements are difficult to apply during MC event generation. These are applied later in the analysis of the MC generated data. A special section of the data generated by KAY has the track position information used to generate the detector response. This information is used to apply T0 to the event without introducing reconstruction efficiencies.

### 6.1.2 Comparison of Data and MC Distributions

How well the MC simulates the detector can be checked by comparing distributions of physical data to the same distribution in MC. One is especially interested in event reconstruction variables such as vertex position, kaon momentum and Target Likelihood; the momentum distribution of the three daughter particles and the distributions of hits in the four chambers (see Figures 45 through 49). The acceptance is especially sensitive to the distribution of vertex positions along the beamline.

To get the MC and data distributions to match, one must introduce several position dependent reconstruction efficiencies. These may be thought of as part of the acceptance, but are separated here into efficiencies since they depend on running conditions in a way in which one doesn’t usually expect of an acceptance. These efficiencies are summarized at the top of Table 13, on page 124.

The first of these efficiencies, $\epsilon_{SP}$, is a space point reconstruction efficiency. The MC data is generated without any wire efficiencies. Instead the observed space point reconstruction efficiency for physical data is used to cut some of the MC data while it is being analyzed. There is a large position dependence in the space point reconstruction efficiency due to a large swath of P3 being 40-50% efficient. The rest of the chambers are mostly above 95% efficient.

In fact, one also has to introduce a correlation between missing P3 and missing one of the other chambers. The three space point track reconstruction algorithm is much less efficient when P3 is missing than for any of the other chambers. Without this
Figure 45: A comparison of the reconstructed vertex position in data (histogram) and MC (error bars) for $K_{\pi\pi}$ decays.
Figure 46: A comparison of the reconstructed kaon momentum and the Target Likelihood distribution between data (histogram) and MC (error bars) for $K_{\pi\pi}$ decays.
Figure 47: A comparison of the pion momentum distributions in data (histogram) and MC (error bars) for $K_{\pi\pi}$ decays.
Figure 48: A comparison of PWC hit distributions in P1 and P2 between data (histogram) and MC (error bars) for $K_{\pi\pi\pi}$ decays.
Figure 49: A comparison of PWC hit distributions in P3 and P4 between data (histogram) and MC (error bars) for $K_{\pi\pi\pi}$ decays.
correlation, the MC does not correctly match the knee in the PWC X hit distribution (at $X = -75$ cm in P3, $X = -80$ cm in P4, see Figure 49). This is $\varepsilon_{SP}$. Another correlation resulting in an overall track reconstruction efficiency is due to activity in the chamber from the halo of the beam. This is manifest as a dip in the track reconstruction efficiency, $\varepsilon_{\text{Trk}}$, symmetric about the dead region in X (at P4) and uniform in Y.

A final track reconstruction efficiency, $\varepsilon_{A}$, is due to a bad A hodoscope slat. It limited to the center slat on the left hand side.

With these efficiencies in place the agreement between MC and data is excellent. There is a slight discrepancy in the Z distribution of the vertex due to the Z resolution of the vertex and the fact that MC events were generated only for $Z > -490$ cm. Some of upstream vertices are reconstructed to have a Z position $V_Z < -490$ cm and hence cut.

The analysis introduces a 20 MeV shift in the reconstructed kaon momentum, determined by MC comparing the input and reconstructed kaon momentum. This shift was removed from the table of kaon momenta used to generate the MC events.

Since the beam phase space look-up table used in the MC was generated from $K_{\pi\pi}$ events, one would like to see how well the vertex and momentum distributions match in Dalitz decays (see Figures 50 and 51). The agreement is again excellent, though the low acceptance of the mode leads to low MC statistics for comparison. The good agreement between data and MC is achieved only after including the three most common Dalitz decays: $K^+ \rightarrow \pi^+\pi^0$, $K^+ \rightarrow \pi^0\mu^+\nu_\mu$ and $K^+ \rightarrow \pi^+\pi^0\pi^0$, all followed by $\pi^0 \rightarrow e^+e^-\gamma$.

For the purpose of comparison, I have also included a plot of the MC predicted Z vertex distribution for $K_{\pi\mu\ell}$ decays in Figure 52. It is peaked much further forward than the distribution for $K_{\pi\pi}$ decays (see Figure 45) because the acceptance is limited by the outer apertures rather than the inner dead region apertures. However, it is not as forward peaked as the distribution for $K_{\text{Dal}}$ decays (see Figure 50) because it has less kinetic energy available for the decay products and because it doesn’t have the low momentum Dalitz decay products.
Figure 50: A comparison of the reconstructed vertex position in data (histogram) and MC (error bars) for $K_{\mathrm{Dal}}$ decays.
Figure 51: A comparison of the pion momentum distributions in data (histogram) and MC (error bars) for $K_{\text{Dal}}$ decays.
Figure 52: A plot of the distribution of accepted Z vertex position for $K_{\pi\mu\epsilon}$ decays.

### 6.1.3 Acceptances

With some assurance of the validity of the MC simulation of the detector, one looks at the acceptance. In the $K_{\pi\pi}$ mode, for 600000 events generated, 150981 were accepted, giving a acceptance

$$\text{Acc}(K_{\pi\pi}) = 0.252$$

In $K_{\pi\mu\epsilon}$, for 200000 events generated, 18950 were accepted, giving an acceptance

$$\text{Acc}(K_{\pi\mu\epsilon}) = 0.0948(5)$$

where the uncertainty shown is solely from statistics. Systematic uncertainties are discussed below.

In $K_{Dal}$ (used in background calculations), for 400000 events generated, 12977 were accepted, giving an acceptance

$$\text{Acc}(K_{Dal}) = 0.0324(3)$$

If one looks for a photon clump as well, in the $K_{Dal}$ MC sample, one finds at least one photon 41.8(6)% of the time. This gives the total acceptance for $K_{Dal}$ decays including the photon of

$$\text{Acc}(K_{Dal} + \gamma) = 0.0135(2)$$
This last figure has more systematic uncertainties than the other acceptances since it requires reconstruction of the photon clump (it doesn’t get the photon information from the MC block). However, the probability of finding a spurious photon clump is small, as demonstrated by the fact that one finds two photon clumps in the MC $K_{\text{Dal}}$ sample 0.5% of the time.

In calculating the background to $K_{\pi\mu\nu}$, one needs to know the acceptance for $K_{\text{Dal}}$ combined with $\pi_\mu2$. In this mode, for 400,000 events generated, 990 were accepted, giving an acceptance of

$$\text{Acc}(K_{\text{Dal}}, \pi_\mu2) = 2.47(8) \times 10^{-3}$$

(84)

Similarly, one needs to know the acceptance for $K_{\mu3}$. In this mode, for 200,000 events generated, 8084 were accepted, giving an acceptance of

$$\text{Acc}(K_{\mu3}) = 0.0404(4)$$

(85)

The MC data used to compute the acceptance (the generated positions in the detector used to calculate the detector response, which is also stored as a special section of the data format for MC events) is only known to integer centimeters for the acceptance calculation. To estimate systematic uncertainty in the acceptances due to this coarse position information, the MC data is analyzed three times with different dead region sizes: once with large dead regions that insure that all dead region tracks are excluded, once with small dead regions that insure that all tracks outside the dead regions are included and once with dead regions as close to the nominally correct value as possible. By looking at the relative difference in acceptance between the three runs one gets a handle on the systematic uncertainty in the acceptance calculation.

The acceptances for $K_{\pi\pi\pi}$ decays range from 23.9% to 25.8% for the large and small dead regions, respectively. The acceptances for $K_{\pi\mu\nu}$ ranged from 8.95% to 9.70%, for the same. The systematic uncertainty for the two is correlated, so it is better to find the uncertainty in the ratio $\text{Acc}(K_{\pi\pi\pi})/\text{Acc}(K_{\pi\mu\nu})$. For the large dead region case this is 2.67(2) (where the given uncertainty is from statistics); for the small dead region case, 2.66(2). The dead regions used in the acceptance calculation gives 2.66(2). These are all consistent with each other, showing that most of the
systematic errors in the acceptance calculation cancel in the ratio of acceptances. Thus, the ratio
\[
\frac{\text{Acc}(K_{\pi\pi})}{\text{Acc}(K_{\pi\mu\epsilon})} = 2.66 \pm 0.02 \text{ (stat)} \pm 0.01 \text{ (sys)}
\] (86)
will be used to calculate the branching ratio limit.

### 6.2 Efficiencies and Normalization Corrections

The Normalization Correction Factor in Equation 79 is made up of all the reconstruction efficiencies that effect \(K_{\pi\mu\epsilon}\) and \(K_{\pi\pi}\) differently. Explicitly we have

\[
C = \frac{\epsilon_{SP}^{\pi\pi} \epsilon_{SPc}^{\pi\pi} \epsilon_{Trk}^{\pi\pi} \epsilon_{A}^{\pi\pi}}{\epsilon_{SP}^{\mu\epsilon} \epsilon_{SPc}^{\mu\epsilon} \epsilon_{Trk}^{\mu\epsilon} \epsilon_{A}^{\mu\epsilon}} \frac{1}{\epsilon_e \epsilon_{\mu} \epsilon_V \epsilon_P \epsilon_{M_{ee}} \epsilon_{2\mu} \epsilon_{C}}
\] (87)

where \(\epsilon_e\) and \(\epsilon_\mu\) are the electron and muon PID efficiencies, \(\epsilon_V\) is the pion efficiency for passing the positron veto, \(\epsilon_P\) is the efficiency of the 3.4 GeV cut on pion and muon momenta, \(\epsilon_{M_{ee}}\) is the efficiency of the \(M_{ee}\) cut including either muon or pion with the electron, \(\epsilon_{2\mu}\) is the efficiency for \(K_{\pi\mu\epsilon}\) decays not having two muon identifies tracks, and \(\epsilon_C\) is the efficiency of the Čerenkov fiducial volume cut for pions. The efficiencies in the first factor are discussed above in Section 6.1.2. All of the other efficiencies effecting \(K_{\pi\pi}\) reconstruction are adjusted to be the same as those for \(K_{\pi\mu\epsilon}\), so no correction factors appear.

The electron PID efficiency is deduced from an unbiased sample of \(K_{\text{Dal}}\) events culled from the TAU data stream. The events were selected with kinematic cuts, so the PID efficiencies don't come into play (see Section 5.2.1). This allows one to impose the electron PID requirements after selecting the sample to determine the efficiency of the cut. Applying the electron cuts listed in Table 9 we find an electron PID efficiency of 50% ± 2%. This includes both the effects of the electron trigger and the analysis PID requirements. The 4% relative uncertainty in the electron PID efficiency is due to the low statistics in the unbiased \(K_{\text{Dal}}\) sample. The systematic uncertainty is much smaller based on how this efficiency varies with the strictness of the kinematic cuts used to select the sample. The distributions used in determining the electron PID efficiency are shown in Figure 53.
Figure 53: Plots of the three distributions going into the electron PID analysis along with the relevant cuts. The sample in each case is the electron distribution in an unbiased $K_{Dal}$ sample.
The muon PID efficiency is the most troublesome factor in the whole normalization. The problem is obtaining a clean sample of unbiased muons in the right momentum range. Muons are common in kaon decays but hard to separate kinematically from pions. Muons result from $\pi_{\mu 2}$ decays in $K_{\pi\pi\pi}$, but the momentum is not well known and there is the possibility of contamination from beam muons. Three pronged kaon decays with muons include $K_{\mu 3}$ and $K_{\mu 4}$ decays, in both cases with a neutrino to foul the kinematic reconstruction. In the end, one is forced to rely on MC simulation using the measured muon plane efficiencies. One has some faith in this calculation because the inefficiency is expected to be due primarily to multiple scattering in the iron (which is well modelled by GEANT) and the experimentally measured single plane efficiencies. Using the first four muon cuts in Table 9 on page 87, one finds a muon PID efficiency of 80.4%. The MC generated distributions are shown in Figure 54.

To reduce the background from $K_{Dal}$ decays, pions and muons are required to pass a positron veto. This consists of the cuts on the number of Čerenkov photoelectrons and on the value of $E/P$ listed in Table 9 on page 87. To keep the efficiency high, two sets of cuts are used. All three loose cuts are required, but only two out of three of the tight cuts are. To estimate the losses from this cut, one looks at the pions from $K_{\pi\pi\pi}$ events in data, applying the positron veto (which isn’t applied in the normalization analysis) to pion tracks on the right. Despite the two tiered cut scheme, only 80.7% of the pions pass the veto.

The momentum spectrum of muons in $K_{\pi\mu e}$ decays (from MC) is shown in Figure 55 with the value of the cut indicated. Only 95.8% of $K_{\pi\mu e}$ decays (with a flat phase space decay matrix) have the muon with momentum below 3.4 GeV.

$M_{ee}$ denotes the invariant mass of an electron-positron pair. To reduce the background from $K_{Dal}$ decays, one requires $M_{ee} < 50$ MeV in $K_{\pi\mu e}$ candidate events with the pion or the muon taken as a positron. The $M_{ee}$ spectrum for phase-space $K_{\pi\mu e}$ decays is shown in Figure 56. The efficiency of this cut is 92.6%. One can control the level of $K_{Dal}$ background in the $K_{\pi\mu e}$ candidate sample by raising or lowering this cut (see Section 7.2).

Although a $\pi_{\mu 2}$ decay of the pion in a $K_{\pi\mu e}$ event might be allowed, it introduces
Figure 54: Plots of the four distributions going into the muon PID analysis along with the relevant cuts. The sample in each case is the muon distribution in a MC generated $K_{\pi\mu\epsilon}$ sample.
an uncertainty in the mass reconstruction: which positive particle with a muon PID tag is the real muon, and which came from the pion. To remove this uncertainty, one requires unambiguous PID determination of all tracks. Specifically, one requires exactly one muon tag, which eliminates events where the pion undergoes $\pi_\mu^2$ decay. One can also get a muon tag for a pion with a coincidental muon from the beam. To determine the number of pions from each source, one compares the likelihood of getting a muon in $K_{\pi\pi}$ decays between MC and data. The MC contains just the probability of decay, without the background of coincidental muons. The difference between the two is then accountable to the background, which one adds to the MC prediction for $\pi_\mu^2$ decay in $K_{\pi\mu}$ pions. However, the MC and data muon tag rates in $K_{\pi\pi}$ events are consistent with one another to 0.2% (the MC figure is slightly larger), so the coincidental background must be below this. The efficiency is just one minus the muon decay tag probability from MC. (The muon decay tag probability per track is different in $K_{\pi\mu}$ and $K_{\pi\pi}$ decays, being 5.0% and 7.7%, respectively.)

Because of cracks in the coverage of the Čerenkov counters, there are regions on the right side of the detector where the positron veto efficiency is not very good. Events with muons or pions in these regions are excluded. This region is determined using data from the MuE stream, with an electron on the left and a positron candidate
Figure 56: Plots of the $M_{ee}$ spectra in MC $K_{e\mu\nu}$ events with the pion or the muon taken as a positron. The cut at 50 MeV is indicated.
by $E/P$ on the right with low $M_{ee}$, i.e. a Dalitz pair. The third track is required to be a muon. This sample is composed roughly equally of $K_{Dal}$ (followed by $\pi_{\mu 2}$) and $K_{\mu 3}$ (with $\pi_{ee}$). The positron track is projected to the middle of D6, where the probability of missing C1 or C2 is tabulated. If the probability is greater than 1% for a given bin (8 x 7 cm), it is not used as part of the fiducial volume. The efficiency of this fiducial volume cut was 96.4%.

One final cut eliminates $K_{\pi\pi\pi}$ events from the final $S'$ versus $M_{\pi\mu e}$ plot, by cutting on the reconstructed mass of the three tracks all taken as pions ($M_{\pi\pi\pi}$). This is required to be over six standard deviations away from the kaon mass. It eliminates very few $K_{\pi\mu e}$ decays, but must be take into account for calculating the expected number of $K_{\pi\pi\pi}$ background events on the plot.

Systematic uncertainties haven’t been discussed in any of these factors. These are all less than the 4% relative uncertainty in the electron PID efficiency, which dominates the uncertainty in the correction factor as a whole.

The various correction factors and other reconstruction efficiencies are summarized in Table 13. Assuming the factors to be independent, one finds

$$
C = \frac{0.413 \pm 0.005}{0.444 \pm 0.005} \frac{1}{0.264 \pm 0.011} = 3.52 \pm 0.14
$$

Also included in Table 13 are some of the reconstruction efficiencies which effect $K_{\pi\mu e}$ and $K_{\pi\pi\pi}$ decays equally. In the case of the Target Likelihood cut, this is because the cut was explicitly set in each case to be at the same percentile of the data. To do this one plots the normalized, integrated MC Target Likelihood distributions for the two different decays, picks the desired cut level for $K_{\pi\mu e}$, and reads the appropriate cut for $K_{\pi\pi\pi}$ off the plot (see Figure 57). It was crucial here to have the actual beam as reconstructed from the small sample of $K_{\pi\pi\pi}$ events, as this gives a Target Likelihood distribution close to what one actually sees in the data. The uncertainty in the cut value leads to an uncertainty in the size of the $K_{\pi\pi\pi}$ normalization sample (see Figure 58). The Target Likelihood distribution from MC data generated with a beamline model is much less peaked; one isn’t sure how appropriate it is to compare $K_{\pi\pi\pi}$ and $K_{\pi\mu e}$ Target Likelihoods with this distribution.
<table>
<thead>
<tr>
<th>Cut</th>
<th>$\epsilon_{\pi\pi}$</th>
<th>$\epsilon_{\pi\mu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space Points ($\epsilon_{SP}$)</td>
<td>0.83(1)</td>
<td>0.81(1)</td>
</tr>
<tr>
<td>SP Correlations ($\epsilon_{SPc}$)</td>
<td>0.728(5)</td>
<td>0.751(5)</td>
</tr>
<tr>
<td>Tracks ($\epsilon_{Trk}$)</td>
<td>0.708(5)</td>
<td>0.736(5)</td>
</tr>
<tr>
<td>A Hodoscope ($\epsilon_{A}$)</td>
<td>0.966(2)</td>
<td>0.992(2)</td>
</tr>
<tr>
<td>Event Reconstruction (total)</td>
<td>0.413(5)</td>
<td>0.444(5)</td>
</tr>
<tr>
<td>$T_{RMS}^2 &lt; 1.5$ ns$^2$</td>
<td>0.810(1)</td>
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</tr>
<tr>
<td>No in-time photons</td>
<td>0.912(1)</td>
<td>(0.912)</td>
</tr>
<tr>
<td>Target Likelihood</td>
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<td>(0.930)</td>
</tr>
<tr>
<td>Track Combinations</td>
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<td>(0.962)</td>
</tr>
<tr>
<td>No extra dalitz-like tracks</td>
<td>0.992</td>
<td>(0.992)</td>
</tr>
<tr>
<td>Electron PID ($\epsilon_e$)</td>
<td></td>
<td>0.50(2)</td>
</tr>
<tr>
<td>Muon PID ($\epsilon_\mu$)</td>
<td>0.804(5)</td>
<td></td>
</tr>
<tr>
<td>Positron Veto ($\epsilon_V$)</td>
<td>0.807(2)</td>
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<tr>
<td>$M_{ee} &gt; 0.05$ GeV ($\epsilon_{M_{ee}}$)</td>
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</tr>
<tr>
<td>No second muon (decay)</td>
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<td></td>
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<tr>
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</tr>
<tr>
<td>$\epsilon_C$ (Fiducial)</td>
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<td></td>
</tr>
<tr>
<td>$</td>
<td>M_{\pi\pi} - 0.4936</td>
<td>&gt; 0.015$ GeV</td>
</tr>
<tr>
<td>$K_{\pi\mu}$ relative efficiency</td>
<td></td>
<td>0.264(11)</td>
</tr>
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</table>

Table 13: Reconstruction efficiencies for $K_{\pi\pi}$ and $K_{\pi\mu}$. $K_{\pi\mu}$ efficiencies in parentheses are defined to be the same as for $K_{\pi\pi}$. 
Figure 57: Comparison of the fraction of the decays (from MC) with a Target Likelihood value below the given value for $K_{\pi\pi}$ (solid) and $K_{\pi\mu\nu}$ (dashed) decays. The error bars at each point are binomial and correlated with the errors at the neighboring points. The cut values for each decay are indicated.
Figure 58: The number of $K_{\pi\pi}$ normalization events as a function of the Target Likelihood cut. The cut used, and its uncertainties and how they contribute to the uncertainty of the size of the normalization sample are indicated by the lines.
Figure 59: A comparison of the $S'$ distribution for MC $K_{\pi\pi}$ (histogram) and $K_{\pi\mu\nu}$ (error bars) decays.

For all the other reconstruction cuts, except the size of mass window, one expects the efficiencies to be the same for the two decays (see Figure 59).
Chapter 7

Conclusion

7.1 The Final Result

All the pieces are now in hand to calculate an upper limit on the branching ratio of $K^+ \rightarrow \pi^+\mu^+e^-$ ($K_{\pi\mu\epsilon}$). The 90% confidence level limit was given in terms of the number of observed $K^+ \rightarrow \pi^+\pi^+\pi^-$ ($K_{\pi\pi\pi}$) events as in equation 79, repeated here for convenience.

$$\text{BR}(K_{\pi\mu\epsilon}) < \text{BR}(K_{\pi\pi\pi}) \cdot \frac{N(K_{\pi\mu\epsilon})}{N(K_{\pi\pi\pi})} \cdot \frac{\text{Acc}(K_{\pi\pi\pi})}{\text{Acc}(K_{\pi\mu\epsilon})} \cdot C \quad (89)$$

The factors on the right hand side are:

$$\text{BR}(K_{\pi\pi\pi}) = 0.0559 \pm 0.0005 \quad (90)$$
$$N(K_{\pi\mu\epsilon}) = 2.3 \quad (91)$$
$$N(K_{\pi\pi\pi}) = (5.43 \pm 0.01) \times 10^9 \quad (92)$$
$$\frac{\text{Acc}(K_{\pi\pi\pi})}{\text{Acc}(K_{\pi\mu\epsilon})} = 2.66 \pm 0.02 \quad (93)$$
$$C = 3.52 \pm 0.14 \quad (94)$$

This gives an 90% confidence level upper limit on the branching ratio for $K_{\pi\mu\epsilon}$ of

$$\text{BR}(K_{\pi\mu\epsilon}) < (2.22 \pm 0.09) \times 10^{-10} \quad (95)$$
7.2 Expected Backgrounds

Even though no $K_{\pi\mu e}$ candidate events are found in the signal region, one would like to understand the origin of the other events shown in Figure 41 (page 98), and the levels of expected backgrounds.

One expects events with $S' < 3$ to come from actual single kaon decays, giving the good vertex reconstruction, but with misidentified particles. Events with $M_{\pi\mu e} < 0.44$ GeV are expected to come from $K_{\pi\pi\pi}$ decays, while those with $M_{\pi\mu e} > 0.44$ GeV are expected to be $K_{D\pi}$ decays.

7.2.1 $\pi^-$ as $e^-$ Misidentification Probability

The size of the background due to $K_{\pi\pi\pi}$ decays hinges on the probability of identifying a negative pion as an electron. To estimate this probability, one analyzes the $K_{\pi\pi\pi}$ normalization sample with the cuts used for the $K_{\pi\mu e}$ analysis, without the electron or muon requirements. Specifically this includes having the negative pion on the left, and a positron veto for the positive pions. The pions on the right are also required to be in the Čerenkov efficient fiducial volume. These cuts are not normally applied to the $K_{\pi\pi\pi}$ sample, but one wants to understand how these events contribute background to the $K_{\pi\mu e}$ sample. With this sample of $K_{\pi\pi\pi}$ events, one finds the probability that the negative particle look like an electron in each of the three electron indicators:

- Photoelectrons in C1: $C1_{\text{PE}} > 0.3$
- Photoelectrons in C2: $C2_{\text{PE}} > 0.3$
- $E/P$ Ratio: $0.8 < E/P < 1.2$

The results are given in Table 14.

Table 14 also gives the probability of each indicator signalling an electron given that one of the other indicators also indicated an electron. This gives an indication of correlations between the different indicators. There are no correlations for real pions, so any observed correlations must come from actual electrons, either electrons that coincide with pions within the time resolution of the various indicators, or electrons
from one decay that combine with another decay to fake a $K_{\pi\pi}$ event. The observed correlation is small, and only marginally significant. One expects 7 events with C1 and C2 indicating an electron, one sees 12; 14 events with C1 and $E/P$, one sees 16; and 88 events with C2 and $E/P$, one sees 95.

Since there is no significant correlation of the Čerenkov indicators with $E/P$, one expects that the correlation between the C1 and C2 indicators is due to extra electrons, outside the timing resolution of the calorimeter and A counters (1 ns), but within the timing window for the Čerenkov counters (10 ns). One can estimate the probability of an electron going through the detector in any 10 ns window. 30% of kaon decays have a $\pi^0$. Of these 1% undergo Dalitz decays, giving an electron for every $3 \times 10^{-3}$ kaon decays. Estimating the rate of kaon decays in our decay region to be $5 \times 10^6$ Hz (half the design rate), gives $1.5 \times 10^4$ electrons per second, or a probability of $1.5 \times 10^{-4}$ in a 10 ns window. This gives about 20 electrons in the sample of 120,000, however, only a fraction of these will come close enough to the pion for its light to contribute. This factor should be of order four, since C2 is divided into four quadrants on each side of the detector. Thus one expect about five out-of-time electrons in our sample, which is approximately the size of the correlation one sees.

<table>
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<th></th>
<th>None</th>
<th>C1PE &gt; 0.3</th>
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<th>0.8 &lt; $E/P$ &lt; 1.2</th>
</tr>
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<tbody>
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<td>2384</td>
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</tr>
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<td>16</td>
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<tr>
<td>C1PE &gt; 0.3</td>
<td>0.0030(2)</td>
<td>0.0050(14)</td>
<td>0.0036(9)</td>
<td></td>
</tr>
<tr>
<td>C2PE &gt; 0.3</td>
<td>0.0195(4)</td>
<td>0.032(9)</td>
<td>0.021(2)</td>
<td></td>
</tr>
<tr>
<td>0.8 &lt; $E/P$ &lt; 1.2</td>
<td>0.0367(5)</td>
<td>0.043(11)</td>
<td>0.040(4)</td>
<td></td>
</tr>
</tbody>
</table>

Table 14: Table of $\pi^-$ as $e^-$ misidentification. The upper part has the actual numbers in each sample, while the lower part has the frequencies and is obtained by dividing each number in the upper part by the number in the first row and the same column. Each row and column is labelled with the requirements used to obtain that sample.
Taking the uncorrelated probabilities (from the first column of the second part of Table 14), we find a \( \pi^- \) as \( e^- \) misidentification probability of

\[
P(\pi^- \text{ as } e^-) = (2.1 \pm 0.1) \times 10^{-6}
\]

(96)

However, one really should take the C1/C2 correlation into account. Using the uncorrelated probability of having a C2 indicator, and the correlated probabilities, given C2, of seeing C1 or \( E/P \) (from the third column) one finds

\[
P(\pi^- \text{ as } e^-) = (4 \pm 1) \times 10^{-6}
\]

(97)

### 7.2.2 The \( K_{\pi\pi} \) Background

Using the \( \pi^- \) as \( e^- \) misidentification probability found in the previous section one proceeds to find the expected number of background events from \( K_{\pi\pi} \) decays. Analyzing the \( K_{\pi\pi} \) normalization sample, with the cuts used in the \( K_{\pi\mu} \) sample, again without the electron PID requirement, but with the muon PID requirement, one obtains the \( M_{\pi\mu} \) distribution (the invariant mass distribution with the particles taken as a pion, muon and electron) given in Figure 60. In the lower of the two histograms, the \(|M_{\pi\pi} - M_K| > 0.015 \text{ GeV} \) cut has been applied (as it is in the \( K_{\pi\mu} \) analysis). In this case, there are 241 events with \( M_{\pi\mu} > 0.4 \text{ GeV} \), 2 events with \(|M_{\pi\mu} - M_K| < 0.012\). Multiplying these by the average prescale factor, 22780 (see Section 5.5), one finds \( 5.5 \times 10^6 \) events with \( M_{\pi\mu} > 0.4 \text{ GeV} \), \( 4 \times 10^4 \) events in the signal region. Putting in the \( \pi^- \) as \( e^- \) misidentification probability of \( 4 \times 10^{-6} \), one finds

\[
\text{BG}(K_{\pi\pi}, M_{\pi\mu} > 0.4) = 22 \pm 6
\]

(98)

\[
\text{BG}(K_{\pi\pi}, \text{signal}) = 0.16 \pm 0.11
\]

(99)

There are 15 events in the data sample with \( M_{\pi\mu} < 0.45 \text{ GeV} \), where 93\% of the \( K_{\pi\pi} \) events lie. There is one event on the \( K_{\pi\mu} \) signal plot with \( M_{\pi\mu} = 0.475 \text{ GeV} \); one expects about 0.3 events from \( K_{\pi\pi} \) with \( M_{\pi\mu} > 0.475 \text{ GeV} \).
Figure 60: A plot of $M_{\pi\mu\ell}$ for $K_{\pi\pi}$ events with $S' < 3$ and one muon. All $K_{\pi\mu\ell}$ cuts are applied except electron PID. The solid histogram includes the cut $M_{\pi\pi} > 0.5083$ GeV, the dashed histogram does not. The $K_{\pi\mu\ell}$ signal region is indicated by the pair of lines.

### 7.2.3 The Positron Veto Miss Probability

The background due to $K_{Dal}$ decays depends on the probability that a positron on the right will miss being vetoed as such. To estimate this probability one uses the unbiased $K_{Dal}$ sample discussed in Section 5.2.2. The distributions of the three PID indicators (see Figure 61) for the positrons in this sample are used to find the probability that a given positron will miss the veto. Because of the two tiered cut scheme, one must be careful to use the correct conditional probabilities in calculating the veto miss probability.

The probability that a given positron will miss the veto is the probability that the positron will miss the loose cuts,

$$P_L = P_{L1} P_{L2} P_{L3} = 0.00120 \pm 0.00003 \quad (100)$$

times the sum of the probabilities that the positron will miss two of the tight cuts but not the third, given the positron missed the loose cuts,

$$P_T(P_L) = P_{T1}(P_L) \cdot P_{T2}(P_L) \cdot (1 - P_{T3}(P_L)) + \text{perm} = 0.167 \pm 0.010 \quad (101)$$
Figure 61: Plots of the three distributions going into the positron veto. The sample in each case is the positron distribution from the unbiased $K_{Dal}$ sample.
Table 15: Table of probabilities for a given positron to fall below the two level of positron veto cuts. The sample is the unbiased $K_{\text{Dal}}$ sample. The actual distributions are shown in Figure 61.

Thus,

$$P(e^+ \text{ as } \pi^+) = P_L \cdot P_T(P_L) = (2.00 \pm 0.13) \times 10^{-4}$$

(102)

The values for the various factors are given in Table 15.

### 7.2.4 The Dalitz Background

To estimate the background due to $K_{\text{Dal}}$ and $K_{\mu3}$ decays, one finds the fraction of Dalitz events from the ELER stream that pass the $S'$, Target Likelihood, and $M_{ee}$ cuts of the $K_{\pi\mu\epsilon}$ analysis and whose pion also has a muon tag. The $M_{\pi\mu\epsilon}$ distribution of this sample is shown in Figure 62. The 324 events with $0.4 \text{ GeV} < M_{\pi\mu\epsilon} < 0.6 \text{ GeV}$ represent $(2.9 \pm 0.3) \times 10^{-3}$ of all Dalitz events with $S' < 3$; the 68 events in the signal region, $(5 \pm 2) \times 10^{-4}$.

The number of $K_{\text{Dal}}$ decays is estimated by looking at the number of $K_{\pi\pi\pi}$ events in the normalization sample.

$$N(K_{\text{Dal}}, \pi_{\mu2}) = N(K_{\pi\pi\pi}) \cdot \frac{\text{BR}(K_{\pi2})\text{BR}(\pi_{ee\gamma})}{\text{BR}(K_{\pi\pi\pi})} \cdot \frac{\text{Acc}(K_{\text{Dal}}, \pi_{\mu2})}{\text{Acc}(K_{\pi\pi\pi})} \cdot C_{\text{Dal}}$$

(103)

where $C_{\text{Dal}} = \epsilon_e \epsilon_\mu \epsilon_\nu \epsilon_P = 0.29 \pm 0.01$ ($\epsilon_P = 0.90$ for $K_{\text{Dal}}$ decays, the other efficiencies are the same as for $K_{\pi\mu\epsilon}$). Thus

$$N(K_{\text{Dal}}, \pi_{\mu2}) = (7.0 \pm 0.3) \times 10^5$$

(104)

Likewise, the number of $K_{\mu3}$ Dalitz decays is

$$N(K_{\mu3}, \pi_{ee\gamma}) = N(K_{\pi\pi\pi}) \cdot \frac{\text{BR}(K_{\mu3})\text{BR}(\pi_{ee\gamma})}{\text{BR}(K_{\pi\pi\pi})} \cdot \frac{\text{Acc}(K_{\mu3})}{\text{Acc}(K_{\pi\pi\pi})} \cdot C_{\text{Dal}}$$

(105)

$$= (1.72 \pm 0.07) \times 10^6$$

(106)
Figure 62: A plot of $M_{\pi\mu\epsilon}$ for Dalitz events with $S' < 3$, $TLIK > -19$, $M_{cc} > 0.05$ GeV and one muon. All $K_{\pi\mu\epsilon}$ cuts are applied except positron veto. The $K_{\pi\mu\epsilon}$ signal region is indicated by hatching.

One can neglect Dalitz decays of $K^+ \to \pi^+\pi^0\pi^0$ because the branching ration is an order of magnitude lower than $K_{Dal}$ and there is no initial muon present as in $K_{\mu3}$. Combining the two figures above gives the number of Dalitz decays that can contribute to $K_{\pi\mu\epsilon}$ background.

$$N(Dalitz) = (2.42 \pm 0.08) \times 10^6$$ (107)

With this number of Dalitz decays one expects, applying the kinematic reduction factors together with the missed veto probability probability of $2.00 \times 10^{-4}$

$$BG(Dalitz, 0.4 < M_{\pi\mu\epsilon} < 0.6) = 1.4 \pm 0.1$$ (108)

$$BG(Dalitz, \text{signal}) = 0.24 \pm 0.10$$ (109)

To check the positron veto miss probability, one can look at the same calculation with the $M_{cc}$ cut removed. Figure 63 shows the sample of $K_{\pi\mu\epsilon}$ data with $S' < 2$, $TLIK < -19$ and $M_{\pi\mu\epsilon} > 0.45$ GeV. Of the 14 events on the plot, 10 are consistent with coming from Dalitz decays. The kinematic reduction factor in this case is $2.7\% \pm 0.1\%$. One expects 13 background events in this case, agreeing with the 10 seen.
Figure 63: A plot of $M_{ee}$ vs $M_{\pi\mu\epsilon}$ for $K_{\pi\mu\epsilon}$ candidates with $S' < 2$. The $M_{ee} > 0.05$ GeV cut has been removed.

There are five background events in the range $0.45 \text{ GeV} < M_{\pi\mu\epsilon} < 0.6 \text{ GeV}$ with $S' < 3$, though only one is consistent with being a Dalitz decay. There are no events in the signal region. The total expected background from both $K_{\pi\pi\pi}$ and $K_{\text{Dal}}$ is $0.40 \pm 0.15$. The probability of expecting 0.4 events and finding none is 67%.

The event with $M_{\pi\mu\epsilon} = 0.475 \text{ GeV}$ (see Figure 64) is consistent with being either a $K_{\pi\pi\pi}$ or $K_{\text{Dal}}$ decay. It is slightly more likely to be $K_{\text{Dal}}$ by the mass distribution, but PID and the upstream location of the vertex indicate $K_{\pi\pi\pi}$ as being more likely.

### 7.2.5 Other Backgrounds

The $K_{\pi\mu\epsilon}$ events with $S' > 3$ can be partially attributed to overlapping decays. A real kaon decay combines with a particle from another kaon decay or with a beam muon to form a three pronged event. The bad $S'$ is due to one of the tracks not being part of the real vertex.

This background has not been estimated accurately, but one can try to do an order-of-magnitude estimation using $K_{\text{Dal}}$ decays where the positron is not accepted combining with a beam muon (from $\pi_{\mu2}$) to fake a $K_{\pi\mu\epsilon}$ event. There are roughly $10^4$ $K_{\text{Dal}}$ decays per spill in the Decay Volume. Of these, perhaps 10% will have the
electron and pion accepted and the positron not (the full $K_{\text{Dal}}$ acceptance is 3%),
for $10^3$ accepted events per spill. One expects roughly $10^7$ beam muons per spill
(decaying in a 10 m region upstream of the detector with a decay length of 200 m).
Thus, one expects 50 coincidences per spill, or $10^7$ coincidences for the run.

This number of coincidences is reduced by a number of factors: the muon momentum
being less than 3.4 GeV ($10^{-2}$), the muon coming close to the $K_{\text{Dal}}$ vertex
($10^{-1}$), the momentum combining to project back to the target ($10^{-2}$). These factors
bring the expected background to 100 about the same as is seen.

A number of other background modes from single kaon decays are listed in Table 16,
together with their expected contribution.

I have included event displays of the three closest events to the signal region
(see Figures 64 through 66). The displays shows plan and elevation views of the
detector. PWC space points and muon wires are indicated by short crossing lines on
the respective chambers. The Čerenkov PMTs are indicated by the uniformly sized

<table>
<thead>
<tr>
<th>Mode</th>
<th>PID Assumption</th>
<th>Number of Decays</th>
<th>PID prob</th>
<th>Kinematic Reduction</th>
<th>BG Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{\text{Dal}}$</td>
<td>$\pi^+ as \mu^+$</td>
<td>$1 \times 10^7$</td>
<td>$1 \times 10^{-6}$</td>
<td>$5 \times 10^{-4}$</td>
<td>$5 \times 10^{-3}$</td>
</tr>
<tr>
<td>$K_{\text{Dal}}$</td>
<td>$e^+ as \pi^+$</td>
<td>$1 \times 10^7$</td>
<td>$5 \times 10^{-5}$</td>
<td>$5 \times 10^{-5}$</td>
<td>$2 \times 10^{-2}$</td>
</tr>
<tr>
<td>$K_{\text{Dal}}, \pi_\mu^2$</td>
<td>$e^+ as \pi^+$</td>
<td>$7 \times 10^5$</td>
<td>$2 \times 10^{-4}$</td>
<td>$5 \times 10^{-4}$</td>
<td>$7 \times 10^{-2}$</td>
</tr>
<tr>
<td>$K_{\mu^3, \pi_\mu^2}$</td>
<td>$e^+ as \pi^+$</td>
<td>$2 \times 10^6$</td>
<td>$2 \times 10^{-4}$</td>
<td>$5 \times 10^{-4}$</td>
<td>$2 \times 10^{-1}$</td>
</tr>
<tr>
<td>$K_{\pi^\pi}$</td>
<td>$\pi^+ as \mu^+$</td>
<td>$6 \times 10^8$</td>
<td>$2 \times 10^{-8}$</td>
<td>$1 \times 10^{-5}$</td>
<td>$1 \times 10^{-3}$</td>
</tr>
<tr>
<td>$K_{\pi^\pi}, \pi_\mu^2$</td>
<td>$\pi^- as e^-$</td>
<td>$1 \times 10^9$</td>
<td>$4 \times 10^{-6}$</td>
<td>$5 \times 10^{-5}$</td>
<td>$2 \times 10^{-1}$</td>
</tr>
<tr>
<td>$K_{\pi^\pi}, \pi_\mu^2$</td>
<td>$\mu^- as e^-$</td>
<td>$5 \times 10^8$</td>
<td>$5 \times 10^{-9}$</td>
<td>$5 \times 10^{-5}$</td>
<td>$1 \times 10^{-4}$</td>
</tr>
<tr>
<td>$K_{\pi^\pi}, \pi_\mu^2$</td>
<td>$\mu^- as e^-$</td>
<td>$1 \times 10^8$</td>
<td>$1 \times 10^{-6}$</td>
<td>$5 \times 10^{-5}$</td>
<td>$5 \times 10^{-3}$</td>
</tr>
<tr>
<td>$K_{\pi^\pi}, \pi_\mu^2$</td>
<td>$\pi^+ as \mu^+$</td>
<td>$2 \times 10^3$</td>
<td>$1 \times 10^{-5}$</td>
<td>$1 \times 10^{-4}$</td>
<td>$2 \times 10^{-6}$</td>
</tr>
<tr>
<td>$K_{\pi^\pi}, \pi_e^2$</td>
<td>$\pi^- as e^- in \tilde{C}1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_{\pi^\pi}, \pi_e^2$</td>
<td>$\pi^+ as \mu^+$</td>
<td>$1 \times 10^3$</td>
<td>$5 \times 10^{-3}$</td>
<td>$1 \times 10^{-4}$</td>
<td>$5 \times 10^{-4}$</td>
</tr>
<tr>
<td>$K_{\pi^\pi}, \pi_e^2$</td>
<td>$\pi^- as e^- in \tilde{C}1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_{\pi^\pi}, \pi_e^2$</td>
<td>None</td>
<td>$1 \times 10^3$</td>
<td>1</td>
<td>$1 \times 10^{-4}$</td>
<td>$1 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Table 16: Order of magnitude estimation of the number of background events due to
various decay modes assuming the particle misidentification probabilities given above.
Figure 64: The event just to the left of the signal region in Figure 41, with $M_{\pi\mu\nu} = 0.476$ GeV and $S' = 1.28$.

boxes on the plan view; going from left to right, the boxes indicate lower to higher portions of the primary mirrors. The size of any PMT signal ADC is indicated by a superimposed box, and in-time TDCs are indicated by a cross. The sum of all the calorimeter modules in a given row or column is indicated by the size of the bar on the calorimeter and if any are in-time, a cross is again present. In-time A, B and C hodoscope slats are indicated as well, with the A hodoscopes on the elevation view (and left side slats are shown downstream of the right side slats) and B and C on the plan view (with upper and lower halves as in the Čerenkovs).

The first event (1851 83079, Figure 64) can be from either $K_{\pi\pi}$ or $K_{\text{Dal}}$. The $K_{\text{Dal}}$ explanation is slightly favored by the mass distributions, but the far upstream vertex and the lack of any positron PID indications along the dashed track (which is
Figure 65: The event just to the right of the signal region in Figure 41, with $M_{\pi\mu\nu} = 0.514$ GeV and $S' = 2.37$. 
Figure 66: The event just off the upper left corner of the signal region in Figure 41, with $M_{\pi\mu e} = 0.516$ GeV and $S' = 3.09$. 
the only track that could combine with the electron to form a Dalitz pair).

The second event (1808 1785941, Figure 65) is most likely a combination of a Dalitz decay with a beam muon or another kaon decay. The $S'$ of 2.37 is above the bulk (95th percentile) of what one would expect for real kaon decays. One might say that the dashed and dotted tracks could form a Dalitz pair, especially with the hints of some Čerenkov activity along the dashed track. However, the dashed track is identified as a muon, which is obvious on inspection, and it is very unlikely for a muon to coincide so closely with another particle (less than $10^{-4}$ probability).

The third event (2063 2372808, Figure 66), despite its bad $S'$ of 3.09, is probably a Dalitz decay. The solid and dotted tracks form the Dalitz pair and the muon PID comes from the dashed track. There is some activity in the Čerenkovs for the positron (solid) track, but not enough for a positive identification to remove the event.

### 7.3 Concluding Remarks

While the upper limit on the $K_{\pi\mu e}$ branching ratio presented in this thesis does not surpass the previously published value, it represents only a fraction of the data currently on tape. The 1996 run of should be able to lower the limit by at least a factor of four. In addition the large reconstruction efficiencies due to P3 will be largely absent. Another analysis[32] of the same data using the $K_{\text{Dal}}$ normalization found a slightly smaller, but consistent, limit of $2.1 \times 10^{-10}$.

A third run of E865 searching for $K_{\pi\mu e}$ is also planned for 1998, with an improved DAQ system and pixel hodoscope in the beam. The beam hodoscope should allow the isolation of $K_{\mu 3}$ decays and a more accurate determination of the muon efficiency.

Combining this result with the previously published result[1] of $2.1 \times 10^{-10}$ gives

$$\text{BR}(K_{\pi\mu e}) < 1.08 \times 10^{-10} \quad (110)$$

This allows one to set new limits on Horizontal Gauge bosons and leptoquarks. Using Equation 11 one finds

$$M_H \frac{g}{g_X} > 46 \text{ TeV} \quad (111)$$
Likewise, for leptoquarks, using Equation 13 one finds

\[ M_{LQ} \frac{g}{g_{LQ}} > 155 \text{ TeV} \] (112)

Recently, the H1[35] and ZEUS[36] collaborations have reported an anomaly in high-\(Q^2\) \(e^+p\) collisions at the HERA collider. More events were seen than expected, 24 with an expectation of 13.4(10) for \(Q^2 > 15000\) GeV\(^2\), 6 with an expectation of 1.52(18) for \(Q^2 > 25000\) GeV\(^2\). One explanation[37] of the extra events is the creation of an S-channel resonance, a leptoquark with mass of order 200 GeV. The size of the coupling constant is constrained to be about 0.05, about the same size as the weak coupling constant.

Such a leptoquark should contribute to \(K_{\pi\mu e}\) if it couples equally to all generations. Using the limit given above in Equation 112 with a mass, \(M_{LQ} = 200\) GeV, one finds

\[ \frac{g_{LQ}}{g} < 1.2 \times 10^{-3} \] (113)

This coupling is much too small to explain the HERA anomaly.
Appendix A

The Multiple Scattering Correlation Matrix

This appendix goes in detail through the calculation of the multiple scattering correlation matrix used to find the best fit tracks to a series of wires in the spectrometer.

The entity to be calculated is

\[ V_{ij} = \langle \delta w_i, \delta w_j \rangle \]

where \( \delta w_i \) is the deviation of a wire measurement from the position of the track with no multiple scattering. \( \delta w_i \) includes the effects of both multiple scattering deflections and wire spacing.

There four proportional chambers, each containing four measurement planes. The track measurements thus break down into four groups of four, with the multiple scattering occurring between the groups. The correlations between different types of planes will be related to a simpler correlation between just one type of plane between the different chambers. The X plane is used to set forth these simplified correlations.

An approximation is used that restricts multiple scattering to only a few point scatterers and a few extended regions. The point scatterers are

- \( P2 \) (subscript 2), which includes the upstream windows of C1;

- The mirrors in C1 (subscript C) which includes the downstream windows;
• P3 (subscript 3), which includes the upstream windows of C2.

(P1 and P4 are not included because they are at the ends of the tracks and play no role in changing how that track is fit. The mirror and downstream windows of C2 are left out as well, being close enough to P4 that the same argument applies.) The extended regions are

• P1 to P2 (subscript 12), medium: Air;
• P2 to C1 downstream window (subscript 2C) medium: H₂, CH₄, CO₂;
• C1 downstream window to P3 (subscript C3) medium: He;
• P3 to P4 (subscript 34), medium: H₂, CH₄, CO₂.

(Note that two subscripts denotes an extended region whereas one subscript denotes a point scatterer.)

The effect of multiple scattering at any point is the sum of individual effects upstream of that point. Thus with the scattering elements defined above

\[
\begin{align*}
\delta x_1 & = 0 \\
\delta x_2 & = \delta_{12} \\
\delta x_3 & = \delta_{12} + \phi_{12}(z_3 - z_2) + \\
& \quad \delta_{2C} + \phi_{2C}(z_3 - z_C) + \\
& \quad \delta_{C3} + \\
& \quad \phi_2(z_3 - z_2) + \phi_C(z_3 - z_C) \\
\delta x_4 & = \delta_{12} + \phi_{12}(z_4 - z_2) + \\
& \quad \delta_{2C} + \phi_{2C}(z_4 - z_C) + \\
& \quad \delta_{C3} + \phi_{C3}(z_4 - z_3) + \\
& \quad \delta_{34} + \\
& \quad \phi_2(z_4 - z_2) + \phi_C(z_4 - z_C) + \\
& \quad \phi_3(z_4 - z_3)
\end{align*}
\]
where \( \phi_{ij}, \delta_{ij} \) are the deflection angle and shift in position caused by multiple scattering in an extended medium, and \( \phi_i \) is the deflection angle caused by multiple scattering at a point. Fortunately, those elements with different subscripts are uncorrelated, so when the expectation value of the products is taken, most of the terms drop out. Thus

\[
< \delta x_1 \delta x_1 > = 0 \tag{119}
\]

\[
< \delta x_2 \delta x_2 > = < \delta x_{12}^2 > \tag{120}
\]

\[
< \delta x_2 \delta x_3 > = < \delta x_{12}^2 > + < \delta x_{12} \phi_{12} > (z_3 - z_2) \tag{121}
\]

\[
< \delta x_2 \delta x_4 > = < \delta x_{12}^2 > + < \delta x_{12} \phi_{12} > (z_4 - z_2) \tag{122}
\]

\[
< \delta x_3 \delta x_3 > = < \delta x_{12}^2 > + 2 < \delta x_{12} \phi_{12} > (z_3 - z_2) + < \phi_{12}^2 > (z_3 - z_2)^2 + \nonumber \\
< \delta x_{2c}^2 > + 2 < \delta x_{2C} \phi_{2C} > (z_3 - z_C) + < \phi_{2C}^2 > (z_3 - z_C)^2 + \nonumber \\
< \delta x_{2C}^2 > + \nonumber \\
< \phi_x^2 > (z_3 - z_2)^2 + < \phi_C^2 > (z_3 - z_C)^2 \tag{123}
\]

\[
< \delta x_3 \delta x_4 > = < \delta x_{12}^2 > + < \delta x_{12} \phi_{12} > [(z_3 - z_2) + (z_4 - z_2)] + \nonumber \\
< \phi_{12}^2 > (z_3 - z_2)(z_4 - z_2) + \nonumber \\
< \delta x_{2C}^2 > + < \delta x_{2C} \phi_{2C} > ((z_3 - z_C) + (z_4 - z_C)) + \nonumber \\
< \phi_{2C}^2 > (z_3 - z_C)(z_4 - z_C) + \nonumber \\
< \delta x_{2C}^2 > + < \delta x_{2C} \phi_{2C} > (z_4 - z_3) + \nonumber \\
< \phi_x^2 > (z_3 - z_2)(z_4 - z_2) + < \phi_C^2 > (z_3 - z_C)(z_4 - z_C) \tag{124}
\]

\[
< \delta x_4 \delta x_4 > = < \delta x_{12}^2 > + 2 < \delta x_{12} \phi_{12} > (z_4 - z_2) + < \phi_{12}^2 > (z_4 - z_2)^2 + \nonumber \\
< \delta x_{2C}^2 > + 2 < \delta x_{2C} \phi_{2C} > (z_4 - z_C) + < \phi_{2C}^2 > (z_4 - z_C)^2 + \nonumber \\
< \delta x_{2C}^2 > + 2 < \delta x_{2C} \phi_{2C} > (z_4 - z_3) + < \phi_{2C}^2 > (z_4 - z_3)^2 + \nonumber \\
< \delta x_{2C}^2 > + \nonumber \\
< \phi_x^2 > (z_4 - z_2)^2 + < \phi_C^2 > (z_4 - z_C)^2 + \nonumber \\
< \phi_x^2 > (z_4 - z_3)^3 \tag{125}
\]

Multiple scattering occurs in two directions, but these processes are identical and
the amount in each direction is independent. Thus

\[ \langle \delta y_i \delta y_j \rangle = \langle \delta x_i \delta x_j \rangle \]  
(126)

\[ \langle \delta x_i \delta y_j \rangle = 0 \]  
(127)

To find the correlations in the oblique views we note that

\[ \delta u_i = \delta x_i \cos \theta - \delta y_i \sin \theta \]  
(128)

\[ \delta v_i = \delta x_i \cos \theta + \delta y_i \sin \theta \]  
(129)

Thus, plugging in equations 126 and 127 we get

\[ \langle \delta u_i \delta u_j \rangle = \langle \delta x_i \delta x_j \rangle \]  
(130)

\[ \langle \delta v_i \delta v_j \rangle = \langle \delta x_i \delta x_j \rangle \]  
(131)

\[ \langle \delta x_i \delta u_j \rangle = \langle \delta x_i \delta x_j \rangle \cos \theta_{ui} \]  
(132)

\[ \langle \delta x_i \delta v_j \rangle = \langle \delta x_i \delta x_j \rangle \cos \theta_{vi} \]  
(133)

\[ \langle \delta y_i \delta u_j \rangle = -\langle \delta x_i \delta x_j \rangle \sin \theta_{ui} \]  
(134)

\[ \langle \delta y_i \delta v_j \rangle = \langle \delta x_i \delta x_j \rangle \sin \theta_{vi} \]  
(135)

\[ \langle \delta u_i \delta v_j \rangle = \langle \delta x_i \delta x_j \rangle (\cos \theta_{ui} \cos \theta_{ui} - \sin \theta_{ui} \sin \theta_{ui}) \]  
(136)

Finally, the contribution of multiple scattering for continuous scattering is given by

\[ \langle \phi_{ij}^2 \rangle = \left( \frac{0.0136 \text{ GeV}}{\beta c p} \right)^2 \frac{\delta_{ij}}{L_{ij}^R} \left[ 1 + 0.038 \ln \left( \frac{s_{ij}}{L_{ij}^R} \right) \right]^2 \]  
(137)

\[ \langle \delta_{ij} \phi_{ij} \rangle = \frac{1}{2} (z_j - z_i) \langle \phi_{ij}^2 \rangle \]  
(138)

\[ \langle \delta_{ij}^2 \rangle = \frac{1}{3} (z_j - z_i)^2 \langle \phi_{ij}^2 \rangle \]  
(139)

where \( s_{ij} = (z_j - z_i) \sec \theta_{ir} \) is the path length in the medium, and \( L_{ij}^R \) is the radiation length of that medium. For point scattering this can be replaced by

\[ \langle \phi_{ij}^2 \rangle = \left( \frac{0.0136 \text{ GeV}}{\beta c p} \right)^2 l_i \left[ 1 + 0.038 \ln (l_i) \right]^2 \]  
(140)
where \( l_i \) is the number of radiation lengths seen as the particle traverses the point. For the case \( \theta_{\text{track}} = 0 \), these values are

\[
\begin{align*}
  l_2 &= 0.00235 \\
  l_C &= 0.00280 \\
  l_3 &= 0.00257
\end{align*}
\]

(141) \hspace{1cm} (142) \hspace{1cm} (143)

For reference, the values for the radiation lengths used in the continuous media were

\[
\begin{align*}
  L_{12}^R &= 30420 \text{ (Air)} \\
  L_{2C}^R &= 680889 \text{ (H}_2\text{)} \\
  &= 64850 \text{ (CH}_4\text{)} \\
  &= 18310 \text{ (CO}_2\text{)} \\
  L_{C3}^R &= 692655 \text{ (He)} \\
  L_{34}^R &= 680889 \text{ (H}_2\text{)} \\
  &= 64850 \text{ (CH}_4\text{)} \\
  &= 18310 \text{ (CO}_2\text{)}
\end{align*}
\]

(144) \hspace{1cm} (145) \hspace{1cm} (146) \hspace{1cm} (147) \hspace{1cm} (148) \hspace{1cm} (149) \hspace{1cm} (150) \hspace{1cm} (151)

Finally we must add the measurement errors due to the finite wire spacing. These are identical for each plane and all independent. Thus

\[
\begin{align*}
  V_{ij}^{\text{MS}} &= \langle \delta x_i \delta x_j \rangle \\
  V &= V^{\text{MS}} + I\sigma^2
\end{align*}
\]

(152) \hspace{1cm} (153)
Bibliography


[34] Stefan Pislak. *KAY: E865 Monte-Carlo*. Zurich, Switzerland, 1996.

