We have performed studies of the $K^0_L \rightarrow \pi^+\pi^-\gamma$ direct emission (DE) and inner Bremsstrahlung (IB) vertices, based on data collected by KTeV during the 1996 Fermilab fixed target run. We find $a_1/a_2 = -0.737 \pm 0.034$ GeV$^2$ for the DE form-factor parameter in the $\rho$-propagator parametrization, and report on fits of the form factor to linear and quadratic functions as well. We concurrently measure $\Gamma(K^0_L \rightarrow \pi^+\pi^-\gamma, E^\gamma > 20$ MeV)/$\Gamma(K^0_L \rightarrow \pi^+\pi^-)$ = $(20.8 \pm 0.3) \times 10^{-5}$, and a $K^0_L \rightarrow \pi^+\pi^-\gamma$ DE/(DE + IB) branching ratio of $0.683 \pm 0.011$. We consider two separate form-factor parametrizations. Historically, the $\rho$-propagator form [8]

$$F = \frac{a_1}{(m_\rho^2 - m_K^2) + 2m_K^2E^\gamma} + a_2$$

the first measurement of this form factor from direct fits to the data, using the rare decay mode $K^0_L \rightarrow \pi^+\pi^-\gamma$.

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has been used, where \( a_1/a_2 \) is the quantity of interest. More generally the form factor may be expressed as a Taylor series in \( E^*_\gamma \)

\[
\mathcal{F} = \left(1 + \frac{rE^*_\gamma}{M_K} + \frac{sE^*_\gamma^2}{M_K^2} + \ldots\right),
\]

where the interesting quantities are the slope \( r \) and the quadratic parameter \( s \).

The data presented here were collected by KTeV operating in the E832 configuration [13] during the 1996 Fermilab fixed target run. A proton beam of intensity \(-3 \times 10^{12}\) protons per 19 sec spill incident at an angle of 4.8 mrad on a BeO target was employed to produce two nearly parallel \( K^0 \) beams. In E832, one of these beams was incident on an active \( K^0 \) regenerator. Data collected in the regenerator beam were ignored for this analysis. The E832 detector consisted of a vacuum decay region, a magnetic spectrometer with four drift chambers, photon vetoes, a trigger scintillator bank, a CsI electromagnetic calorimeter, and a muon veto bank.

Signal \( K^0_L \rightarrow \pi^+\pi^-\gamma \) and normalization \( K^0_L \rightarrow \pi^+\pi^- \) candidates were selected from the two-charged-track trigger. Offline, the samples were further purified by requiring the presence of two reconstructed tracks with a good vertex \( x^2 \) within the fiducial aperture of the detector, and no significant activity in the photon veto counters. The two-track vertex was required to be located within the vacuum (non-regenerator) beam. The tracks’ kinematics were required to be inconsistent with \( A^0 \rightarrow p^+\pi^- \) decays. The energy deposited by the charged pions in the calorimeter was required to be less than 0.85\( \times \)the spectrometer momentum in order to eliminate backgrounds from \( K_{e3} \) decays. Candidate events were required to have a decay vertex between 110 and 156 m downstream of the target and a total energy between 20 and 160 GeV.

\( K^0_L \rightarrow \pi^+\pi^-\gamma \) candidates were subject to the additional criterion of requiring that the quantity

\[
P_{\pi\pi}^2 = \frac{(M_K^2 - M_{\pi\pi}^2 - M_{\pi\pi}^2)^2 - 4M_{\pi\pi}^2M_{\pi\pi}'^2 - 4M_{\pi\pi}^2P_{\pi\pi}'^2}{4(P_{\pi\pi}^2 + M_{\pi\pi}^2)}
\]

be negative, i.e., by requiring that the \( \pi^0 \) momentum be imaginary under a \( K^0_L \rightarrow \pi^+\pi^-\pi^0 \) hypothesis. This cut virtually eliminates the \( K^0_L \rightarrow \pi^+\pi^-\pi^0 \) background to the \( K^0_L \rightarrow \pi^+\pi^-\gamma \) event sample. In addition, at least one non-track-associated cluster in the calorimeter was required to possess \( 1.5 \) GeV of energy, and to be at least 3 cm removed from the calorimeter edges. This “photon cluster” had to be at least 20 cm from the nearest track projection to reject background from \( K^0_L \rightarrow \pi^+\pi^- \) events accompanied by pion hadronic showers in the calorimeter. The photon was required to have an energy of at least 20 MeV in the three-body center of momentum.

Figures 2(a) and 2(b) show the vacuum beam \( K^0_{L,S} \rightarrow \pi^+\pi^-\gamma \) data after final cuts. A very clean signal of 8 669 \( K^0_{L,S} \rightarrow \pi^+\pi^-\gamma \) events is achieved, with a background of about 0.5%. Approximately 0.6% of the events in the peak in Fig. 2 are residual \( K^0_S \rightarrow \pi^+\pi^-\gamma \) decays from \( K^0_{L,S} \) generated at the production target. A total of 4 482 706 \( K^0_{L} \rightarrow \pi^+\pi^- \) events were accumulated for the normalization sample with a 0.1% background.

To extract the direct emission form factor, we consider the distribution in \( E_{\gamma}^* \), the photon energy in the center of mass, for the \( p \)-propagator fit result (Table I). Shown also (shaded) are the expected distributions for pure \( E1 \) inner Bremsstrahlung and form-factor-modified \( M1 \) direct emission. The “combined” Monte Carlo plot shown assumes these two are the only contributions to the decay.

\[
K^0_L \rightarrow \pi^+\pi^- \gamma \quad \text{Monte Carlo/data overlay of photon energy distribution in the center of mass, for the } p \text{-propagator fit result (Table I). Shown also (shaded) are the expected distributions for pure } E1 \text{ inner Bremsstrahlung and form-factor-modified } M1 \text{ direct emission. The “combined” Monte Carlo plot shown assumes these two are the only contributions to the decay.}
\]
as the energy shift of the DE spectrum due to the presence of a form factor. We assume that $E_1$ IB and form-factor-modified $M1$ DE are the only significant contributions to the decay. We perform a MINUIT [14] $\chi^2$-minimization fit to combine Monte Carlo DE and IB $E_\gamma^*$ distributions, and extract the relative DE and IB contributions to the data as well as the DE form factor. In the fit, the form-factor parameters ($a_1/a_2$ or $r$ and $s$) and the ratio

$$f = \frac{\Gamma_{DE}}{\Gamma_{DE} + \Gamma_{IB}}$$

(4)

are allowed to float simultaneously. A $\chi^2$ is formed from comparison of the resultant DE + IB summed Monte Carlo histogram with the data, and minimized to obtain the best fit result.

Table I summarizes the numerical fitting results for the three form-factor parametrizations. For the Taylor series parametrization, fits were performed with the quadratic parameter both fixed at zero (floating a single parameter $r_1$) and allowed to vary (floating two parameters $r_2$ and $s_2$). Data Monte Carlo agreement in the $E_\gamma^*$ distribution is shown in Fig. 3, for the $\rho$-propagator form. Figure 4 illustrates the expected effect of the various parametrizations on the pure $M1$ direct emission spectrum.

We see in the data presented in Table I and Fig. 4 clear evidence for a modification to the pure-$M1$ DE spectrum. All fits are good, though the $\rho$-propagator hypothesis stands out slightly: It gives the best $\chi^2$ for a single-parameter fit, and the two-parameter fit results are in good agreement with the values $r_2 = -2.70$ and $s_2 = 3.87$ obtained by Taylor expanding the $\rho$-propagator form. The size of this energy shift, in particular the need to take into account terms of second order in $E_\gamma^*$, is not currently understood within the chiral perturbation theory model for this decay [11].

Additional contributions to the photon energy spectrum are expected from $CP$-violating higher-order multipole contributions to the direct emission amplitude. One possible consequence of these multipole terms is the presence of a charge asymmetry in the $\pi^+$ versus $\pi^-$ Dalitz plot [1]. We exclude asymmetries larger than 2.4% at 90% C.L. with the present data.

One might also expect a contribution to the $E_\gamma^*$ spectrum from $CP$-violating $E1$ DE, which could interfere with the $E1$ IB. We have allowed for such a term, by performing a separate fit in which we assumed an $E1$ DE amplitude constant in $E_\gamma^*$, and searched for the corresponding interference contribution [12] to the $E_\gamma^*$ spectrum. The form-factor parameter and $f$ [see Eq. (4)] are allowed to float simultaneously, while fixing the IB rate at its theoretical value of $7.00 \times 10^{-3}$ [1]. Based on this fit, we set an upper limit of $\Gamma_{IN}/(\Gamma_{DE} + \Gamma_{IB} + \Gamma_{IN}) \leq 0.30$ (90% C.L.) on the contribution to the decay rate of the $CP$-violating interference term. $\Gamma_{IN}/\Gamma_{ALL} = 0.30$ corresponds to a 22% decrease in the $\Gamma_{DE}/\Gamma_{ALL}$ ratio.

Background subtraction under the $E_\gamma^*$ distribution (with shape determined by study of $P_T^2$ sideband) resulted in no statistically significant change in either the form-factor parameter or the $DE/(DE + IB)$ ratio. Accidental activity in the detector was also found to have no statistically significant effect. The form-factor parameters are found to have a slight (1.9%) sensitivity to variations of the lower $E_\gamma^*$ cutoff (nominally 20 MeV) used in the fit. We also assign systematic uncertainties of 1.8% due to detector acceptance, and 1.4% due to the effects of uncertainties in the calorimeter photon energy scale.

Note (Table I) that the measured $DE/(DE + IB)$ ratio is insensitive to the particular choice of form-factor parametrization. Uncertainty in the detector acceptance contributes a 0.7% systematic uncertainty to this ratio, while variations in analysis cuts contribute 0.6%. The systematic uncertainty due to the calorimeter photon energy scale is 0.2%.

We find the absolute $K_L^0 \rightarrow \pi^+ \pi^- \gamma$ branching ratio by normalizing the signal to the $K_L^0 \rightarrow \pi^+ \pi^-$ channel. We determine the ratio of $K_L^0 \rightarrow \pi^+ \pi^- / K_L^0 \rightarrow \pi^+ \pi^- \gamma$ acceptances using a Monte Carlo simulation of the full detector and offline analysis criteria. Based on this, we calculate $\Gamma(K_L^0 \rightarrow \pi^+ \pi^- \gamma)/\Gamma(K_L^0 \rightarrow \pi^+ \pi^-) = (20.8 \pm 0.2 \text{ stat } \pm 0.2 \text{ syst}) \times 10^{-3}$ for $K_L^0 \rightarrow \pi^+ \pi^- \gamma$ events with $E_\gamma^* > 20$ MeV. The systematic uncertainty is due primarily to the effects of $K_L^0$ contamination in the vacuum beam. Assuming no contribution from interference and using the $DE/(DE + IB)$ result from Table I, we obtain the final $\pi^+ \pi^- \gamma$-normalized branching ratios ($14.2 \pm 0.2 \pm 0.2) \times 10^{-3}$ for direct emission and $(6.6 \pm 0.2 \pm 0.2) \times 10^{-3}$ for inner Bremsstrahlung.

Our measured IB branching ratio is consistent with both the QED prediction of $7.00 \times 10^{-3}$ and the most recent experimental result of FNAL E731 [10]; $(7.3 \pm 0.4) \times 10^{-3}$, based on a sample of 3 136 $K_L^0 \rightarrow \pi^+ \pi^- \gamma$ events. The $DE/(DE + IB)$ ratio is also in good agreement with

<table>
<thead>
<tr>
<th>Quantity</th>
<th>$\rho$ propagator</th>
<th>Linear</th>
<th>Quadratic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2$/d.o.f.</td>
<td>38.8/27</td>
<td>43.2/27</td>
<td>37.6/26</td>
</tr>
<tr>
<td>$a_1/a_2$</td>
<td>$(-0.737 \pm 0.026 \pm 0.022)$ GeV$^2$</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$r$</td>
<td>...</td>
<td>$-1.739 \pm 0.062 \pm 0.052$</td>
<td>$-2.93 \pm 0.41 \pm 0.34$</td>
</tr>
<tr>
<td>$s$</td>
<td>...</td>
<td>$3.31 \pm 1.15 \pm 0.96$</td>
<td>...</td>
</tr>
<tr>
<td>$f$</td>
<td>$0.683 \pm 0.009 \pm 0.007$</td>
<td>$0.682 \pm 0.009 \pm 0.007$</td>
<td>$0.684 \pm 0.011 \pm 0.007$</td>
</tr>
</tbody>
</table>
FIG. 4. Ratio (points) of (IB-subtracted) direct-emission data to the expectation for a pure $M1 E^*_\gamma$ spectrum. Vertical scale is arbitrary. Shown for comparison are the best-fit results (Table I) for the $\rho$-propagator (solid), linear (dashed), and quadratic (dotted) form-factor parametrizations. A modification to the pure-$M1$ spectrum is clearly supported by the data.

the E731 result of $0.685 \pm 0.041$. The present form-factor results differ significantly from that reported by E731 ($a_1/a_2 = -1.8 \pm 0.2$ GeV$^2$) but this discrepancy has been understood: The E731 $a_1/a_2$ form factor was inferred [15] from the model of Lin and Valencia [8], on the basis of the measured DE branching ratio, whereas our results are obtained directly by performing fits to the $E^*_\gamma$ distribution. The underlying datasets in the two experiments are consistent with each other, and a reanalysis of the E731 data using our method yields results consistent with ours. The KTeV $K_L^0 \rightarrow \pi^+\pi^-\gamma$ DE form-factor result is also in good agreement with the result $a_1/a_2 = -0.720 \pm 0.029$ GeV$^2$ extracted from the independent $K_L^0 \rightarrow \pi^+\pi^-e^+e^-$ analysis from the KTeV E799 data set [4].

In conclusion, we have made the first direct measurements of the $K_L^0 \rightarrow \pi^+\pi^-\gamma$ direct emission form factor, including $a_1/a_2 = -0.737 \pm 0.034$ GeV$^2$. We find no evidence for new $CP$-violating effects in the photon energy spectrum. Finally, we have made improved measurements of the DE $[(14.2 \pm 0.2 \pm 0.2) \times 10^{-3}]$ and IB $[(6.6 \pm 0.2 \pm 0.2) \times 10^{-3}]$ $K_L^0 \rightarrow \pi^+\pi^-\gamma$ branching ratios, normalized to the $K_L^0 \rightarrow \pi^+\pi^-$ channel.

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