Maple: An Introduction

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March 12, 1992

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1 Introduction

This introductory article contains basic information for the new user of Maple who wants to spend only a couple of hours learning Maple. This document covers how to use Maple as a calculator, recovering from errors, Maple's notation for arithmetic, mathematical constants and functions, how to get help, how to solve equations, do linear algebra, and to define and plot mathematical functions in two and three dimensions. For each section there are several examples. At the end there are some exercises which you can try to solve. Note, we have not attempted to show any of Maple's programming facilities in this article.

2 Getting Started

You should be able to start Maple by typing maple. [Note: On the physics workstations, in Xwindows, type xmaple. See the next paragraph.] The Maple logo should appear almost immediately. After the Maple prompt > has appeared, Maple is ready to receive your input commands. You should see

MAPLE V
Copyright (c) 1981-1990 by the University of Waterloo.
All rights reserved. MAPLE is a registered trademark of
Waterloo Maple Software.
Type ? for help.
Note, on workstations running Xwindows, you can run a version of Maple which has a more sophisticated user interface called xmaple. [674 students, this means you. See “Using the X-Windows Interface...” below.] Just type xmaple instead of maple. After a few seconds the Maple session window should appear which contains the Maple logo followed by the Maple prompt. You can now enter a command.

**ATTENTION**: every Maple command must end with a semicolon ;

This is a grammatical requirement of the Maple language. The semicolon tells Maple that this is the end of the input. Type now \texttt{4/6;} followed by a return. Maple answers with

$2/3$

To refer to the latest result, one uses the double quote character "". Try it now

$> ";\$

$2/3$

You can use the value of "" in an expression as if it were a variable, for example

$> 2*"+1;\$

$7/3$

In order to save results, one assigns them to a variable, e.g.

$> R := ";\$

$R := 7/3$

If you want to interrupt a calculation push the interrupt key once. This is <Ctrl> C under Unix. When the prompt character appears, Maple is ready for further input. To exit Maple, you can type quit. Note, under xmaple you can click on the Interrupt button to interrupt Maple and on the Quit button to exit Maple.

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Added for 674:

**Using the X-Windows Interface in xmaple**

The fact that you are using Maple in a windows environment makes it easier to do a number of editing operations. The slidebar on the left of the screen can be used to move up and down through your window, to review previous lines or to hide them from view to make available new space. To use the slidebar, move the cursor to the top edge of the grey region of the slide bar, click the middle button, and drag the top edge of the slide bar up and down. The material in the window will move in the opposite direction.

You can do line editing with the mouse. Move the cursor to any point and click the left mouse button. The xmaple position indicator “~” will appear at that point, and you can add or delete text.
You can save much typing using the x-windows interface. Suppose you want to avoid retyping a line, or part of a line, that occurred previously. Set the cursor at the beginning of the material you want to repeat; push the left mouse button, and drag the cursor to the right, until you have covered all the material you want to “lift,” then release the button. Next, move the cursor to the point at which you want to deposit the text, and click the left mouse button to make the **xmaple** position indicator “-” appear. Last, click the middle button and the text will be deposited at that point.

3 Arithmetic and Maple Notation

The basic arithmetic operators and constants known to Maple are

<table>
<thead>
<tr>
<th>Maple notation</th>
<th>Meaning</th>
<th>Mathematical notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>x+y</td>
<td>addition</td>
<td>+</td>
</tr>
<tr>
<td>-x and x−y</td>
<td>negation and subtraction</td>
<td>−x and x − y</td>
</tr>
<tr>
<td>x*y</td>
<td>multiplication</td>
<td>xy</td>
</tr>
<tr>
<td>x/y</td>
<td>division</td>
<td>x/y</td>
</tr>
<tr>
<td>x^y or x**y</td>
<td>exponentiation</td>
<td>x^y</td>
</tr>
<tr>
<td>k!</td>
<td>factorial</td>
<td>k!</td>
</tr>
<tr>
<td>I or sqrt(-1)</td>
<td>complex unit</td>
<td>i or √−1</td>
</tr>
<tr>
<td>Pi</td>
<td></td>
<td>π</td>
</tr>
<tr>
<td>infinity</td>
<td></td>
<td>∞</td>
</tr>
<tr>
<td>abs(x)</td>
<td>absolute value</td>
<td></td>
</tr>
<tr>
<td>sqrt(x) or x^(1/2)</td>
<td>square root</td>
<td>√x</td>
</tr>
<tr>
<td>exp(x)</td>
<td>exponential function</td>
<td>e^x</td>
</tr>
<tr>
<td>ln(x) or log(x)</td>
<td>natural logarithm</td>
<td>ln x</td>
</tr>
<tr>
<td>sin(x)</td>
<td>sine function</td>
<td>sin x</td>
</tr>
<tr>
<td>cos(x)</td>
<td>cosine function</td>
<td>cos x</td>
</tr>
<tr>
<td>tan(x)</td>
<td>tangent function</td>
<td>tan x</td>
</tr>
</tbody>
</table>

Notice that Maple does arithmetic with integers exactly. That is, exact arithmetic is used instead of decimal arithmetic. Use decimal numbers if you want decimal numbers.

Examples

> 2*3+2/7;  
44/7

> 2.0*3.0+2.0/7;  
6.285714286
This principle works for formulae too. Use the \texttt{evalf} function if you want a decimal approximation.

\begin{verbatim}
> sin(Pi/3);
\end{verbatim}

\begin{verbatim}
\frac{1}{2}
\end{verbatim}

\begin{verbatim}
> evalf(");
\end{verbatim}

\begin{verbatim}
.8660254040
\end{verbatim}

There is no limit on the length of integers in Maple. It is quite common to compute with integers several hundred digits long. It is also possible to do decimal arithmetic to more than the default 10 digits of precision. You can compute \( \pi \) to a 1000 digits if you want. This is done by assigning the the global variable \texttt{Digits} to the desired precision. Note, don’t forget to reset \texttt{Digits} to 10 if you don’t need more than 10 Digits! High precision operations take longer! Here are some examples

\begin{verbatim}
> 2^100;
\end{verbatim}

\begin{verbatim}
1267650600228229401496703205376
\end{verbatim}

\begin{verbatim}
> Digits := 50:
> evalf( sin(Pi/3) );
\end{verbatim}

\begin{verbatim}
.86602540378443864676372317075293618347140262690520
\end{verbatim}

\begin{verbatim}
> Digits := 10:
\end{verbatim}

In the above examples we have used the colon to terminate a command. Use the colon : instead of a semicolon ; if you don’t want to see the output.

4 Mistakes and Errors

If you forget the semicolon don’t panic! Maple will simply print another prompt indicating that it is still waiting for more input. Just type the semicolon and return and Maple will go ahead and compute the result. Note, this means that you can enter large expressions over one or more lines. Try to input a large expression like the following

\begin{verbatim}
> f := 4*x^4+3*x^3*y+2*x^2*y^2+3*x*y^3+4*y^4;
\end{verbatim}

\begin{verbatim}
f := 4 x^4 + 3 x^3 y + 2 x^2 y^2 + 3 x y^3 + 4 y^4
\end{verbatim}

If you incorrectly input an expression, Maple will respond with a \textit{syntax error}. A common error is to forget the semicolon and then to try to correct the problem by retyping the input. For example, one user typed

\begin{verbatim}
> 2*x+1
> 2*x+1
\end{verbatim}
and Maple replied

```maple
syntax error:
2*x+1
```

The user forgot the semicolon after typing `2*x+1` and return. The user then typed in another `2*x+1`. Maple now says that there is an error and points to where it thinks the error is with the `^` character. What the user typed is equivalent to typing

```maple
> 2*x+1 2*x+1
```

Now you can see why Maple thinks the expression is wrong. To recover from a syntax error, just type a semicolon to clear the remaining input, and then reenter the command.

## 5 On-Line Help

If you need help for a Maple function, you can use the `?` command. A `?` by itself gives you general information. Try it now. E.g. `?min` gives help on the `min` function.

**FUNCTION:** max - maximum of numbers
**FUNCTION:** min - minimum of numbers

**CALLING SEQUENCE:**
- `max(x_1, x_2, ...)`
- `min(x_1, x_2, ...)`

**PARAMETERS:**
- `x_1, x_2, ...` - any expressions

**SYNOPSIS:**
- The functions `max` and `min` return the maximum or minimum respectively of one or more arguments.

- Most often the arguments will be of type numeric, that is, integers, rationals, or floats. However, the functionality is more general, allowing any type of arguments for which an unevaluated function call may be returned.

**EXAMPLES:**
```maple
> max(3/2, 1.49);
3/2
> min(3/2, 1.49);
1.49
> max(3/5, evalf(ln(2)), 9/13);
.6931471806
> max(x+1, x+2, y);
max(x + 2, y)
```

**SEE ALSO:** maximize, minimize
6 Simplification Commands

The output from Maple is not always in the simplest form. There are a number of commands such as `collect`, `combine`, `expand`, `factor` and `simplify` which can be used to simplify or rearrange the output into the desired form. For example, given the polynomial

\[(x+y)(x-y)-x^2;\]

if we multiply it out with the `expand` command the result is simpler

\[\text{expand(“);}\]

\[2\]

\[-y\]

Often factoring a polynomial yields a simpler result, e.g.

\[x^4+x^2y+2x^2+2x^3+2x^2+y^2+2x+y+1;\]

\[\text{factor(“);}\]

\[(x+1)(x+y+1)\]

The `simplify` command can simplify general expressions, e.g.

\[\text{simplify(exp(a+ln(b*exp(c)));}\]

\[b\exp(a+c)\]

In this example the `simplify` command reduces a rational function

\[(x^3-y^3)/(x^2+x*y+y^2);\]

\[\frac{x - y}{2}
\frac{x + x y + y}{2}\]

\[\text{simplify(“);}\]

\[x - y\]

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Added for 674:

Saving and Printing Out Your Results

There are times you will want to save and/or printout what you have done in `xmaple`. You may want to quit your session and resume at another time; you may want to print a log of your session; you may want to print out just the final results.
In all cases the procedure involves creating a file. The file will be created in the directory you were in when you opened xmaple. You may want to organize your work by creating a special directory for your xmaple stuff. To do this you need to create and go to the appropriate directory before starting the Maple session. You start by typing

```bash
$ mkdir maplepractice
```

in your UNIX window, and a directory with the name `maplepractice` will be created. (You can, of course, use any name you want.) Next, go to that directory by typing:

```bash
$ cd maplepractice
```

Then do

```bash
$ xmaple
```

to start the xmaple window. The xmaple window will be active when your cursor is in it.

Use the following procedures to save and print:

- If you want to save your results for later xmapling, but don’t want to print anything at the moment, make up a file name, say “someprogress,” and just before quitting type

  ```bash
  > save('someprogress.m');
  ```

  in the xmaple window. (Note: the backquotes ‘... ‘ are needed for maple to read the period in ‘someprogress.m‘.) This will store the results in a format that is efficient for Maple to read, but which cannot be read directly by you. To recover these results, when you start up Maple again, go to the same directory (“maplepractice” or wherever the file is stored) and as the first line of your Maple session do

  ```bash
  > read 'someprogress.m';
  ```

- If you want to print the results of your xmaple session, that is print the final values of all variables, then do a “save” as above, but don’t use the “.m” extension. Thus, you would do

  ```bash
  > save(someprogress);
  ```

  (No backquotes are needed.) You have now created a readable file named “someprogress” with your results in it. You can read this file on your terminal screen by going to (i.e., moving the cursor to) the UNIX window and typing

  ```bash
  $ more someprogress
  ```

  This will use the UNIX “more” facility to inspect the file. With “more” you can go forward by one page by hitting the spacebar; you can go backward one page by typing “b”(no carriage return); you can go forward one line by hitting the carriage return.
You can print the content of the file by using the UNIX “lpr” command. After you use this command on the Physics network, it will ask you for the number of the printer, as follows:

$ lpr someprogress

**PRINTER OPTIONS**

<table>
<thead>
<tr>
<th>#</th>
<th>Room</th>
<th>Type</th>
<th>Queue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NP Floor-3</td>
<td>ascii or HP/PCL graphics</td>
<td>nppcl</td>
</tr>
<tr>
<td>2</td>
<td>NP Floor-3</td>
<td>postscript</td>
<td>npps</td>
</tr>
<tr>
<td>3</td>
<td>NP Floor-3</td>
<td>ascii (text)</td>
<td>npps (after a2ps)</td>
</tr>
<tr>
<td>4</td>
<td>201 NP</td>
<td>ascii or HP/PCL graphics</td>
<td>np201</td>
</tr>
<tr>
<td>5</td>
<td>SP rotunda</td>
<td>ascii or HP/PCL graphics</td>
<td>sppcl</td>
</tr>
<tr>
<td>6</td>
<td>205 SP</td>
<td>postscript</td>
<td>spps</td>
</tr>
<tr>
<td>7</td>
<td>205 SP</td>
<td>ascii (text)</td>
<td>spps (after a2ps)</td>
</tr>
<tr>
<td>8</td>
<td>205 SP</td>
<td>postscript</td>
<td>sps2</td>
</tr>
<tr>
<td>9</td>
<td>205 SP</td>
<td>ascii (text)</td>
<td>sps2 (after a2ps)</td>
</tr>
</tbody>
</table>

If you are using a workstation in the workstation lab (205 SP) you will want your output to go to the printer in that room. Type 7 and a carriage return, and a printout of the file *someprogress* will appear at the printer.

- There is also a way of making a log of your whole session. To do this make up a file name (say: *mysession*) and type the following in the Maple session at the beginning of the part of the session that you want to have logged:

  > writeto(mysession);

  All of your xmaple output will then no longer appear on the terminal. Instead it will go to the file “*mysession*”. You can read that file by going to the UNIX window and typing

  $ more mysession

  You can print it by using the command $ lpr mysession in the same manner as described above. To end the log, and to get the output directed to your terminal again, type

  > writeto(terminal);
7 Calculus Commands

Experiment with the integration, differentiation and summation commands. The Maple syntax for these functions and examples is as follows.

<table>
<thead>
<tr>
<th>Maple notation</th>
<th>Meaning</th>
<th>Mathematical notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>diff(f(x),x);</td>
<td>Derivative</td>
<td>$\frac{df}{dx}$</td>
</tr>
<tr>
<td>int(f(x),x);</td>
<td>Indefinite integration</td>
<td>$\int f(x) dx$</td>
</tr>
<tr>
<td>sum(f(x),x);</td>
<td>Indefinite summation</td>
<td>$\sum f(n)$</td>
</tr>
<tr>
<td>int(f(x),x=a..b);</td>
<td>Definite integration</td>
<td>$\int_a^b f(x) dx$</td>
</tr>
<tr>
<td>sum(f(k),k=a..b);</td>
<td>Definite summation</td>
<td>$\sum_{k=a}^b f(k)$</td>
</tr>
</tbody>
</table>

> diff(ln(x),x);

    1/x

> diff(arcsin(a*x),x);

    a
    -----------
    2 2 1/2
    (1 - a x )

> int(x*ln(x),x);

    2 2
    1/2 x ln(x) - 1/4 x

> int(sqrt(1-x^2),x);

    2 1/2
    1/2 x (1 - x ) + 1/2 arcsin(x)

> int(ln(x),x=1..2);

    - 1 + 2 ln(2)

> sum(n^2,n);

    3 2
    1/3 n - 1/2 n + 1/6 n

> sum(a^k,k=1..n);

    (n + 1)
    a
    ---------- - -----  
    a - 1     a - 1
In the case of indefinite integration, you can test whether Maple’s answer is right by differentiating the integral and subtracting it from the integrand. The difference should be zero! But Maple may not recognize that the difference is zero immediately. You may have to help Maple simplify the difference to 0. You can make use of \texttt{expand} and \texttt{simplify}. Example:

\begin{verbatim}
> f := (x^3+2*x^2-x)/(x^3-x^2+x-1);
\end{verbatim}
\begin{verbatim}
  3  2
  x + 2 x - x
f := ---------------
  3  2
  x - x + x - 1
\end{verbatim}
\begin{verbatim}
> int(f,x);
\end{verbatim}
\begin{verbatim}
  2
x + ln(x - 1) + ln(x + 1)
\end{verbatim}
\begin{verbatim}
> diff("",x);
\end{verbatim}
\begin{verbatim}
  1 x
 1 + ----- + 2 ------
  x - 1 2
  x + 1
\end{verbatim}
\begin{verbatim}
> simplify("-f);
\end{verbatim}
\begin{verbatim}
  0
\end{verbatim}

8 The Solve Commands

You can solve single equations and systems of linear and non-linear equations exactly with the \texttt{solve} command. For approximate solutions, use the \texttt{fsolve} command. For ordinary differential equations use the \texttt{dsolve} command. And for recurrence equations use the \texttt{rsolve} command. These commands have the following syntax.

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>\texttt{solve(e, x)}</td>
<td>solve the equation $e$ for the unknown $x$</td>
</tr>
<tr>
<td>\texttt{solve({e1, ..., en}, {x1, ..., x_m})}</td>
<td>solve the equations $e_1, ..., e_n$ for $x_1, ..., x_m$</td>
</tr>
<tr>
<td>\texttt{fsolve(e, x)}</td>
<td>solve the equation $e$ numerically for $x$</td>
</tr>
<tr>
<td>\texttt{fsolve({e1, ..., en}, {x1, ..., x_m})}</td>
<td>solve numerically $e_1, ..., e_n$ for $x_1, ..., x_m$</td>
</tr>
<tr>
<td>\texttt{dsolve(e, y(x))}</td>
<td>solve the ODE $e$ for $y(x)$</td>
</tr>
<tr>
<td>\texttt{dsolve({e, i1, ..., in}, y(x))}</td>
<td>solve the ODE $e$ and initial conditions $i_1, ..., i_n$ for $y(x)$</td>
</tr>
<tr>
<td>\texttt{rsolve(e, f(n))}</td>
<td>solve the recurrence equation $e$ for $f(n)$</td>
</tr>
<tr>
<td>\texttt{rsolve({e, i1, ..., in}, f(n))}</td>
<td>solve the recurrence $e$ given initial conditions $i_1, ..., i_n$ for $f(n)$</td>
</tr>
</tbody>
</table>
For example, to solve the equation \( x^3 - 6x = 5 \) for \( x \)

\[
> \text{solve}(x^3-6*x=5, x);
\]

\[
-1, \frac{1}{2} + \frac{1}{2} \sqrt{21}, \frac{1}{2} - \frac{1}{2} \sqrt{21}
\]

To solve the ODE \( y(x) + 2y''(x) = e^x \) given initial conditions \( y(0) = 1 \) and \( y'(0) = 0 \)

\[
> \text{dsolve}\{y(x)+2*\text{diff}(y(x),x$2)=\text{exp}(x), y(0)=1, D(y)(0)=0\}, y(x));
\]

\[
y(x) = \frac{1}{2} \exp(x) - \frac{1}{2} \sin(x) + \frac{1}{2} \cos(x)
\]

9 Other Commands and Functions

Maple knows about the elementary functions \( \ln, \exp, \sin, \cos, \tan \), etc. Some other special functions that you might need are

<table>
<thead>
<tr>
<th>Maple notation</th>
<th>Function</th>
<th>Mathematical notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>erf(x)</td>
<td>Error function</td>
<td>( \text{erf}(x) )</td>
</tr>
<tr>
<td>binomial(n,k)</td>
<td>Binomial function</td>
<td>( \binom{n}{k} )</td>
</tr>
<tr>
<td>GAMMA(x)</td>
<td>Gamma function</td>
<td>( \Gamma(x) )</td>
</tr>
<tr>
<td>Psi(x)</td>
<td>Poly Gamma function</td>
<td>( \psi(x) )</td>
</tr>
<tr>
<td>Zeta(x)</td>
<td>Riemann Zeta function</td>
<td>( \zeta(x) )</td>
</tr>
<tr>
<td>BesselJ(v,x)</td>
<td>Bessel function</td>
<td>( J_v(x) )</td>
</tr>
</tbody>
</table>

See also ?integer for a list of functions for integers and ?polynom for a list of commands for computing with polynomials. See ?mod for a list of commands for computing over finite rings and fields.

10 Data Structures

Maple uses sequences, lists, sets, tables and arrays for representing more complicated data. See ?sequences, ?list, ?set, ?table, ?array for more detailed help and examples. A sequence is a sequence of expressions separated by commas. The seq function is a very useful function for creating sequences. Its syntax is

\[
\text{seq( f(i), i=a..b )}
\]

Lists group together the elements of a sequence; they are created using square brackets \([, ]\). Sets are like lists except duplicate entries are removed. Sets are created using squiggley brackets \(\{, \}\).
Some examples:

> S1 := 1,5,3;

\[ S1 := 1, 5, 3 \]

> max(S1);

\[ 5 \]

> S2 := seq(i^2, i=1..5);

\[ S2 := 1, 4, 9, 16, 25 \]

> S2[3];

\[ 9 \]

> S1 := {x,y,z,y};

\[ S1 := \{x, y, z\} \]

> S1 union {w,x};

\[ \{x, y, z, w\} \]

> L1 := [x,y,z,y];

\[ L1 := [x, y, z, y] \]

> L2 := [L1[4],L1[2..3]];

\[ L2 := [y, y, z] \]

\[ 11 \]

The Linear Algebra Package

In Maple a vector is represented as a one-dimensional array, and a matrix is represented as a two-dimensional array. See `?vector` and `?matrix` for detailed help on vectors and matrices.

In Maple there are many packages for special applications. The `linalg` package contains many functions from linear algebra for computing with vectors and matrices. In order to use a package, you must load the package using the `with` command, e.g.

> with(linalg);

Warning: new definition for `norm`
Warning: new definition for `trace`

[BlockDiagonal, GramSchmidt, JordanBlock, add, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, charmat, charpoly, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvectors, equal, exponential, extend, ffgausselim, fibonacci, frobenius, gausselim, gaussjord, gematrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, nullspace, orthog, permanent, pivot, potential, randmatrix, range, rank, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stack, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vectpotent, vectdim, vector]
You can now use any of the functions listed. The matrix command can be used to input a matrix. In the next example the inverse and determinant of a 3 by 3 matrix is computed.

\[ A := \begin{pmatrix} x - 1 & 2 & 3 \\ 0 & x - 2 & 2 \\ 2 & 1 & x - 3 \end{pmatrix} \]

\[ \text{det}(A) = x^3 - 6x^2 + 3x + 16 \]

\[ \text{inverse}(A) = \begin{pmatrix} \frac{x^2 - 5x + 4}{4} & \frac{x^3 - 6x^2 + 3x + 16}{2x - 4} & \frac{3x - 10}{2x - 2} \\ \frac{x^3 - 6x^2 + 3x + 16}{x^2 - 4x - 3} & \frac{x^3 - 6x^2 + 3x + 16}{x - 5} & \frac{x^3 - 6x^2 + 3x + 16}{(x - 1)(x - 2)} \\ \frac{-x^3 + 6x^2 - 3x + 16}{x^3 - 6x^2 + 3x + 16} & \frac{-x^3 + 6x^2 - 3x + 16}{x^3 - 6x^2 + 3x + 16} & \frac{-x^3 + 6x^2 - 3x + 16}{x^3 - 6x^2 + 3x + 16} \end{pmatrix} \]

If you type \texttt{?packages} you will get a list of all the known packages to Maple and what they are. In particular, this includes

<table>
<thead>
<tr>
<th>Package</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>numtheory</td>
<td>number theoretic functions</td>
</tr>
<tr>
<td>combinat</td>
<td>combinatorial functions</td>
</tr>
<tr>
<td>stats</td>
<td>statistical functions</td>
</tr>
<tr>
<td>geometry</td>
<td>functions for dealing with two dimensional euclidean geometry</td>
</tr>
<tr>
<td>orthopoly</td>
<td>definition of orthogonal polynomials</td>
</tr>
<tr>
<td>simplex</td>
<td>functions for linear optimization problems</td>
</tr>
</tbody>
</table>

12 Functions and Graphics

Mathematical functions of one or more variables can be defined in Maple. The function can be evaluated at either numerical or symbolic expressions. For example

\[ f := x \rightarrow \frac{\sin(x)}{x}; \]

\[ f := x \rightarrow \frac{\sin(x)}{x} \]

\[ f(2.0); \]

\[ .4546487134 \]

\[ f(t); \]

\[ \frac{\sin(t)}{t} \]

13
Functions can be graphed with the plot command which has the following syntax.

\[
\text{plot} \left( f, a..b \right)
\]

That means, that the function \( f \) will be drawn on the interval \( a \) to \( b \). Try this now

\[
> \text{plot}(f,-12..12);
\]

An example of a function in two variables

\[
g := (x,y) \rightarrow \frac{x^2 - y^2}{x^2 + y^2};
\]

\[
g := (x,y) \rightarrow \frac{x^2 - y^2}{x^2 + y^2};
\]

\[
> g(1,2);
\]

\[
-3/5
\]

\[
> g(1,x);
\]

\[
\frac{1 - x}{1 + x}
\]

Functions of two variables can be graphed with the plot3d command

\[
\text{plot3d} \left( f, a..b, c..d \right)
\]

\[
> \text{plot3d}(g, -1..1, -1..1);
\]

The examples above are plots of functions. The other possibility is to graph an expression. The syntax and examples for plotting expressions is

\[
\text{plot} \left( f(x), x=a..b \right)
\]

\[
\text{plot3d} \left( f(x,y), x=a..b, y=c..d \right)
\]

\[
> \text{plot}( \sin(x)/x, x=-12..12 );
\]

\[
> \text{plot3d}( (x^2-y^2)/(x^2+y^2), x=-1..1, y=-1..1 );
\]
Printing Your Plot

It is fairly simple to get a printout of the plots you make with `plot` or with `plot3d`. The general idea is to get Maple to make the plot, not on your terminal screen, but rather as a file in the graphics language “postscript.” You must give Maple two pieces of information: (i) you must tell it to generate a postscript output, and (ii) you must tell it to put that output in a file.

Suppose that you are plotting the function $\sin(x)/x$. If you do

```maple
> plot(sin(x)/x, x=-12..12);
```

a plot will appear on your screen. If you now want a printout, make up a file name — say “sinebyz.ps” — in which the plot will appear. The “.ps” extension identifies it as a postscript file. Now enter the following lines in Maple:

```maple
> interface(plotdevice=postscript);
> interface(plotoutput='sinebyz.ps');
> plot(sin(x)/x, x=-12..12);
```

No plot will appear on your terminal, but you will find (do `ls` in the UNIX window to get a listing of the contents of your directory) that a new file has been created with the file name “sinebyz.ps”. The final step is to print out this postscript file with the command

```
\$ lpr sinebyz.ps
```

in the UNIX window. When the terminal presents the list of printing options (see list on p. 8) you must choose a postscript printer. Type “6” if you are using the printer in 205 SP.

To reset the plotting action, so that the plots appear on your screen again, type the following in the Maple window:

```maple
> interface(plotdevice=x11);
> interface(plotoutput=terminal);
```

Note that “interface” commands for “plotdevice” and “plotoutput” can be combined. The two lines above can be combined into the single line:

```maple
> interface(plotdevice=x11, plotoutput=terminal);
```
Exercises

(674 students note: The following exercises were included by the authors of these notes; they are not assigned problems for the course.)

1. Calculate the first and second derivative of $\sin(x) \cos(x)$ wrt $x$.

2. Given the polynomial $y(x) = x^3 - 4x^2 + 4x - 1$ find the roots and any local minima and maxima of $y(x)$. Check your solutions with a plot of the polynomial.

3. Given $f = x^2 - 4$ calculate the integral of $f$ and $\frac{1}{f}$ with respect to $x$. Check that Maples answers are correct by differentiating the results.

4. Compute the following integrals

\[\int_0^\infty e^{-t} dt \quad \text{and} \quad \int_0^\infty e^{-t^2} dt\]

5. Compute the following sums

\[
\sum_{k=1}^{1000} k \quad \text{and} \quad \sum_{k=1}^\infty \frac{1}{k^2}
\]

6. Compute a formula for the sum of the first $n$ integers and also the sum of the squares of the first $n$ integers. I.e. compute and simplify the sums

\[
\sum_{k=1}^{n} k \quad \text{and} \quad \sum_{k=1}^{n} k^2
\]

7. Given the function $h(x) = 1 - x + \sin(x)$, define the function in Maple and compute the value of the function $h$ at $x = -2.0$ and then graph the function in the region $[-5,5]$.

8. Input the following Matrix $A$ into Maple

\[
A = \begin{pmatrix}
a & 0 & 5 \\
1 & 1 & 1 \\
-a & 0 & 0
\end{pmatrix}
\]

(a) Compute the characteristic polynomial of $A$. Hint: use the `charmat` and `det` commands in the linear algebra package `linalg`.

(b) Compute the eigenvalues of $A$. Hint: use the `solve` or `factor` commands to find the roots of the characteristic polynomial.

9. Use the `solve` command to solve the linear system

\[
\begin{align*}
4x - 5y &= 11 \\
2x + y &= 9
\end{align*}
\]
Printing Your Plot

It is fairly simple to get a printout of the plots you make with \texttt{plot} or with \texttt{plot3d}. The general idea is to get Maple to make the plot, not on your terminal screen, but rather as a file in the graphics language “postscript.” You must give Maple two pieces of information: (i) you must tell it to generate a postscript output, and (ii) you must tell it to put that output in a file.

Suppose that you are plotting the function $\sin(x)/x$. If you do

```
> plot(sin(x)/x, x=-12..12);
```

a plot will appear on your screen. If you now want a printout, make up a file name — say “sinebyz.ps” — in which the plot will appear. The “.ps” extension identifies it as a postscript file. Now enter the following lines in Maple:

```
> interface(plotdevice=postscript);
> interface(plotoutput='sinebyz.ps');
> plot(sin(x)/x, x=-12..12);
```

No plot will appear on your terminal, but you will find (do $\$ \texttt{ls}$ in the UNIX window to get a listing of the contents of your directory) that a new file has been created with the file name “sinebyz.ps”. The final step is to print out this postscript file with the command

```
\$ \texttt{lpr sinebyz.ps}
```

in the UNIX window. When the terminal presents the list of printing options (see list on p. 8) you must choose a postscript printer. Type “6” if you are using the printer in 205 SP.

To reset the plotting action, so that the plots appear on your screen again, type the following in the Maple window:

```
> interface(plotdevice=x11);
> interface(plotoutput=terminal);
```

Note that “interface” commands for “plotdevice” and “plotoutput” can be combined. The two lines above can be combined into the single line:

```
> interface(plotdevice=x11, plotoutput=terminal);
```