are frequently not included in discussions about the hydrogen atom because they have a very small effect, whereas in positronium and in charmonium they are very important. (Problem 10.10 asks you to discover why positronium hyperfine splitting of the ground state is nearly 150 times greater than it is for hydrogen.) Positronium (Ps, for short) and charmonium (Ch) are both bound states of two spin-\(\frac{1}{2}\) fermions, so that the classification of angular momentum states are identical from the start and the same as those we are now accustomed to using in meson spectroscopy for quark states. Figure 10.15 compares the term diagrams for the two. Let us compare some interesting features:

1. The 2\(S\) and 1\(P\) levels of Ch are not near degenerate as they are in Ps. This means that the q\(\bar{q}\) potential in Ch does not vary as 1/r, the variation known to lead to this near degeneracy in the hydrogen atom and in Ps.

2. The Ch triplet levels 1\(^3P_0\), 1\(^3P_1\), 1\(^3P_2\) are, relative to the 2\(S\)–1\(S\) spacing (589 MeV in Ch, 5.1 eV in Ps) more split than the same levels in Ps. This indicates an important spin–orbit interaction in Ch.

3. The 1\(^3S_1\) – 1\(^1S_0\) and the 2\(^3S_1\) – 2\(^1S_0\) splitting in Ch are, again relative to the 2\(S\)–1\(S\) difference, larger than those in Ps.

![Diagram](image)

**Fig. 10.15** The energy levels of charmonium (a) and of positronium (b) compared. Note that the details of the fine and hyperfine splitting of positronium have to be enlarged considerably compared with the scale set by the 2\(S\)–1\(S\), 5.1 eV splitting. Note also that we have used the atomic convention of labelling by 2\(P\) the lowest \(P\) states of positronium but have used the nuclear convention 1\(P\) for charmonium. The charmonium levels are given by broken lines when they have not been observed experimentally; their position is the result of calculations using a q\(\bar{q}\) potential such as that of equation (10.3). No such modesty has restrained us in the case of the positronium levels: there is no doubt about the ground state hyperfine splitting but in the case of the 2\(S\) and 2\(P\) levels the level positions shown are the result of quantum electrodynamic calculations and only the 2\(S_1\) → 2\(P_1\) transition has been observed. None the less, great confidence can be placed in these calculations in view of their success in other systems (see Section 9.6). Here the prediction is that the 2\(S_1\) → 2\(P_1\) splitting should be 8625 MHz and the observed value is 8628 ± 3 MHz, which reinforces that confidence. The main conclusion from the comparison is that the fine and hyperfine interactions are relatively much more important in charmonium than in positronium.