Table 7.3 Rules for doing kinematic calculations.

<table>
<thead>
<tr>
<th>Part</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass ( M )</td>
<td>(kg) ( \text{Rest mass energy} \times M = (MeV) )</td>
</tr>
<tr>
<td>Momentum ( P )</td>
<td>(kg ( m \cdot s^{-1} )) ( \text{Momentum} = c \times P ) (MeV)</td>
</tr>
<tr>
<td>Velocity ( V )</td>
<td>( m \cdot s^{-1} ) ( \text{Velocity} = c \times V ) (dimensionless)</td>
</tr>
</tbody>
</table>

Then, suppressing the \( c \) everywhere we have

\[ P = \sqrt{M(2TM)} \]

\[ P = \sqrt{M(2TM)} = \delta P \]

and all the formulae given in Figs. 7.4 and 7.5 become simpler to use. The make special

check the simplicity and consistency of the equations. Of course mass is often given in \( \text{MeV}/c^2 \) or \( \text{GeV}/c^2 \); the number is the rest mass energy in MeV or GeV respectively. Thus the

velocity of a 1 MeV proton is given by

\[ V = \sqrt{2(2T)} = 0.046 \]

But bear in mind that these are non-relativistic velocities.

Ejected protons was known approximately, and a simple calculation (Problem 7.4) showed that the \( \gamma \)-rays would have to have energies of order 50 MeV, unexpectedly high for \( \gamma \)-rays emitted by nuclear; in addition, if the producing

reaction was \( \text{Be}(\alpha,\gamma)\text{C} \) for example, the known binding energies, allowing for

errors, prohibited \( \gamma \)-rays energies in excess of 14 MeV. Chadwick measured by

range and hence estimated the velocities of protons and of \( \alpha \) nuclei recoiling from impacts by this radiation. The ratio of these velocities gave the mass of the par-icles of this radiation in terms of the proton mass \( M_p \). (Problem 7.5) Chadwick found 1.1 to 1.7 \% \( M_p \). He correctly interpreted this as evidence for

the existence of the neutron. Measurements made shortly after improved the precision on

the value of the neutron to proton mass ratio.

### Definitions and Keywords

**Q-value**

This is the amount by which the sum of the rest mass energies of the initial particles of a nuclear reaction exceeds the sum of the rest mass energies of all the products of the reaction.

**Endothermic reaction**

A reaction for which \( Q < 0 \).

**Mass defect**

The mass of an atom in atomic mass units less the atomic mass number. Sometimes multiplied by 931.5 MeV to give it the dimensions of energy.

**Laboratory system**

The inertial frame in which a collision is observed.

#### Centre-of-mass system

The inertial frame in which two colliding particles have equal and opposite momentum. More generally for a system of two or more particles it is the frame in which the vector sum of their momenta is zero.

**Threshold**

An endothermic nuclear reaction cannot proceed unless the incident kinetic energy is sufficient to make the centre-of-mass kinetic energy \( E > 0 \). The equality signals the threshold.

**Laboratory threshold energy**

The kinetic energy of the incident particle in the laboratory at the threshold for an endothermic reaction.

### PROBLEMS

7.1 Prove the formula for \( \gamma, P, \) and \( E \) given in Fig. 7.4.

7.2 Prove the formulae given in the caption to Fig. 7.5.

7.3 Compute the \( Q \)-values for the reactions

\[ d + t = \text{He} \]

\[ d + n = \text{H} \]

\[ \text{mass defect of the neutron} = -0.008 \text{665 } \text{MeV} \]

\[ \text{mass defect of deuterium atom} = -0.054 \text{152 } \text{MeV} \]

\[ \text{mass defect of tritium atom} = -0.016 \text{650 } \text{MeV} \]

\[ \text{mass defect of helium 3 atom} = -0.016 \text{650 } \text{MeV} \]

\[ \text{mass defect of helium 4 atom} = -0.002 \text{603 } \text{MeV} \]

What are the maximum energies (in the laboratory) of reactions that can be produced using 4 MeV electrons incident on stationary targets of deuterium and of tritium? Use non- relativistic kinematics and assume nuclear masses are \( 1.00335 \text{ MeV}/c^2 \) (for the transforma- tions) but use the \( Q \)-values already calculated.

7.4 Show that, if the proton recoils forward with a kinetic energy of 2 MeV from photon interacting by an initially stationary proton, the incident photon energy is 32 MeV. (This problem requires a relativistic treatment.)

7.5 Show that if a particle of mass \( m \) and velocity \( v \) elastically scatters from a nucleus of mass \( M \), the greatest velocity to which it recoiling nucleus can have is \( 2m/(m+M) \).

Hence the ratio of the proton recoil velocities of protons \( V_p \) and of nitrogen nuclei \( V_N \) scattered by nucleus of the same energy is given by

\[ \frac{V_p}{V_N} = \frac{M}{M_p} \]

Where \( M, M_p \) and \( M_N \) are the masses of the neutron, proton, and nitrogen nucleus respectively. Chadwick's values were \( V_p = 3 \times 10^8 \text{ m/s} \) and \( V_N = 7 \times 10^7 \text{ m/s} \). Find his data for \( V_p/M_p \).

### 7.4 Conservation laws in nuclear collisions and reactions

We have accepted conservation of energy and momentum. In all the examples given we have made one other assumption and that is that the number of protons and the number of neutrons is separately conserved. We shall find circumstances in which this is no longer true but for this chapter, where we are considering non-relativistic induced nuclear reactions which are observable essentially, it is essentially true. Implicit in that assumption is the conservation of charge.

Angular momentum is also conserved. This influences the angular distribu- tion of the products of a collision. However, we cannot discuss this subject any further in this book except in some simple cases.

Purity is another quantity which is conserved in nuclear reactions; this leads to some selection rules which can sometimes forbid reactions which would otherwise be possible, see Problem 7.6.

These conservation laws are summarized in Table 7.4. However, when we come to consider collisions at relativistic energies or those involving the weak interactions we shall find that these rules must be qualified and extended.

### Table 7.4 Conservation laws in non-relativistic nuclear collisions

<table>
<thead>
<tr>
<th>Conservation quantity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) energy and linear momentum</td>
<td>Angular momentum is conserved.</td>
</tr>
<tr>
<td>(2) angular momentum</td>
<td>Purity is another quantity which is conserved in nuclear reactions; this leads to some selection rules which can sometimes forbid reactions which would otherwise be possible, see Problem 7.6.</td>
</tr>
<tr>
<td>(3) number of quarks and protons</td>
<td>Angular momentum is also conserved. This influences the angular distribu- tion of the products of a collision. However, we cannot discuss this subject any further in this book except in some simple cases.</td>
</tr>
<tr>
<td>(4) number of quarks and protons</td>
<td>Purity is another quantity which is conserved in nuclear reactions; this leads to some selection rules which can sometimes forbid reactions which would otherwise be possible, see Problem 7.6.</td>
</tr>
</tbody>
</table>

**Warning:** This is a list specific to low-energy nuclear scattering and reactions.