Problem Set 4

Low dimensional plasmon

1. Find the dispersion law for 1-dimensional plasma wave, propagating in the metallic wire of the radius $R$ surrounded by vacuum. The one-dimensional average electron density (per meter) is $n_0$. The charge density per meter created by ions compensate average electron density. The plasma wave propagates in the electron system. The total density of electrons in the wire with the plasma wave is $n_0 + n'$, where

$$n' = n'_0 \exp(ikz - i\omega t),$$

where $n'_0 \ll n_0$ and axis $z$ is along the wire. Electron mass is $m$. Assume that $kR \ll 1$, where $k$ is the wave vector of the wave. Assume also that the skin depth is large and the electric field $E_z$ is homogeneous in the cross-section of the wire. (10 points)

Hints: Using separation of variables in Laplace equation write solution outside the wire in cylindrical coordinates. A constant can be found by matching electric field $E_\rho$ at the surface of the wire with the field found from the Gauss theorem. Electrons move in $z$-direction only.

Useful mathematical equations for the modified Bessel function $K$:

$$\frac{dK_0(x)}{dx} = -K_1(x).$$

At $x \ll 1$

$$K_0(x) \approx \ln(2/x)$$

2. The same problem for the wire surrounded by a grounded metallic cylinder with the same axis as the wire and with the inner radius $d$, where $d \gg R$. Write a general expression for frequency and consider cases $kd \ll 1$ and $kd \gg 1$. (5 points)