Goals of the lab:

- Understand how to measure distances and displacements capacitively and with a Fabry-Perot interferometer
- Understand how a Fabry-Perot interferometer works
- Learn how to operate: oscilloscope, motor- and manual-micrometers, lock-in amplifier, laser, Fabry-Perot interferometer, photodiode, Schmitt trigger
A capacitor is a transducer which changes distance (of the capacitor plates) or the dielectric constant of the material between the plates (e.g. for gas level indicators) into a change in capacitance which can be measured electrically. The capacitance of a parallel plate capacitor is

\[ C = \frac{\varepsilon_0 \varepsilon_r A}{d} \]

where:
- \( C \) = Capacitance [farad]=[F]=[As/V]
- \( \varepsilon_0 \) = dielectric constant of free space (8.85 x 10^{-12} As/Vm)
- \( \varepsilon_r \) = relative dielectric constant
- \( A \) = area
- \( d \) = distance

The parallel plate formula is valid only to the degree that edge effects are negligible. Note the linear dependence upon \( \varepsilon_r \) and \( A \), and note the non-linear dependence upon the gap \( d \). For proximity sensing, this causes some complications which will be analyzed and discussed in the following.
Background: Capacitance Sensor

For distance measurements, the capacitance formula can be considered as a function of distance \( C = C(d) = \gamma / d \) with \( \gamma = \varepsilon_0 \varepsilon_r A \). Obviously, the capacitor used as length to capacitance transducer is not linear.

However, for small displacements \( \Delta d \) around a distance \( d_0 \), the capacitance change \( \Delta C(\Delta d) = C(d_0 + \Delta d) - C(d_0) \) can be expanded by a Taylor series yielding the expression

\[
\Delta C'(\Delta d) = \frac{\gamma}{d_0} \left[ -\frac{\Delta d}{d_0} + \left( \frac{\Delta d}{d_0} \right)^2 + O \left( \left( \frac{\Delta d}{d_0} \right)^3 \right) \right].
\]

When we assume that we deal with a small displacement (\( \Delta d \ll d_0 \)), then this expression assumes the form

\[
\Delta C_{\text{small}}(\Delta d) = -\frac{\gamma \Delta d}{d_0^2}
\]

which shows that for small relative displacements, the change in capacitance assumes a linear dependence on \( \Delta d \).
Background: Capacitance Sensor

For measurements, one needs to keep in mind that $\Delta C_{\text{small}}$ is only an approximation with finite accuracy. The absolute error introduced by this term is

$$\Delta C_{\text{error}} = |\Delta C(\Delta d) - \Delta C_{\text{small}}(\Delta d)| \approx \frac{\gamma \Delta d^2}{d_0^3}.$$  

The relative error, also called the fractional, error is

$$\Delta C_{\text{error}}^{\text{rel}} = \left| \frac{\Delta C_{\text{error}}}{\Delta C_{\text{small}}} \right| \approx \frac{\Delta d}{d_0}.$$  

These formulas need to be taken into account for the error discussion involving displacement measurements with parallel plate capacitors.
Background: Capacitance Measurement

The capacitor transduces length into a capacitance value. In order to measure the latter we measure the impedance of the capacitor for an applied AC voltage.

Note that the impedance $Z_C$ of a capacitor is:

$$Z_C = (i \omega C)^{-1}$$

with $\omega$ denoting the angular frequency for which the impedance applies and $i$ denoting the complex unit ($i = \sqrt{-1}$).

For the measurement of the impedance we use Ohm’s law:

$$V/I = Z$$

with $V$ and $I$ denoting the applied AC voltage and the measured AC current, respectively.

In order to measure a displacement (the distance change between two parallel capacitor paltes), a sinusoidal AC voltage with well known frequency is applied to the capacitor and the resulting AC current is measured!

NOTE: When you measure an AC voltage or AC current, be always aware of what you measure! Many measurement instruments (AC voltmeters, lock-in amplifiers) display measurements as rms (=root mean square) values ($V_{\text{rms}}$). Others, such as function generators use peak to peak values ($V_{\text{pp}}$) or amplitudes ($V_0$) of the harmonic function. The impedance formula given above applies only to corresponding values for voltage and current. If you get different values from different instruments, use the right conversion factors

$$\sqrt{2} V_{\text{rms}} = V_0 = V_{\text{pp}}/2$$

Example: Set $V_0$ and measure $I_{\text{rms}}$: $C = (\sqrt{2} I_{\text{rms}})/(\omega V_0)$
Background: Distance measurement with Fabry-Perot interferometer

From the lecture notes, we learn that the transmission coefficient $T$ of a Fabry-Perot interferometer depends on the wavelength $\lambda$ and the mirror spacing $d$.

For light that is perpendicular to the interferometer mirrors and under the assumption that we have air or vacuum between the mirrors (index of refraction = 1), we get transmission maxima at distances

$$d_N = \lambda \left( \frac{N}{2} - \frac{\delta_i}{4\pi} \right)$$

Thus when the mirror spacing is changed, the displacement $\delta d$ necessary to move from one fringe to the next is

$$\delta d = \frac{\lambda}{2} = \frac{c}{2\nu}$$

Since $\lambda$ can be known very accurately ($\Delta \lambda / \lambda \approx 10^{-10}$) and $\lambda$ is very small (for visible light $\lambda = 400$nm to 800nm), interferometers are useful for high resolution & high precision motion detection.

HeNe laser: $\lambda = 632.8$ nm  \[ \Rightarrow \delta d = 3164\text{Å} \]
In this lab, a relative displacement of two parallel planes is measured by means of

(i) a capacitance measurement and
(ii) an interference fringe count with a Fabry-Perot interferometer

The two parallel planes act in the experiment as both, the capacitor plates and the mirrors of the Fabry-Perot interferometer.

The experimental results of the two displacement measurement techniques are then compared.
Principal sketch of the experiment

Familiarize yourself with display (unit is micrometer, note decimal point before last digit) and operation.

Electrically controlled motor micrometer

Manual micrometer screw

The wires connecting the lock-in amplifier with the mirrors must be separated as much as possible to avoid additional stray capacitance.

Capacitor plates/interferometer mirrors

Lock-in amplifier SR830 + built in reference signal generator

Intensity detector (photodiode)

Schmitt trigger (threshold voltages adjustable inside)

Pulse counter DC504

Oscilloscope Tektronix TDS1012

HeNe laser

laser

I_{in} (amplifier input in current mode)

Sine out

Department of Physics

University of Utah
Explanation of the experimental setup:

The built-in function generator of the lock-in amplifier produces a sinusoidal voltage oscillation that is applied to one capacitor plate. The other capacitor plate is connected with the low-impedance current input of the lock-in amplifier. The connection of the function generator with the lock-in reference input is made internally. From the applied voltage (note whether an amplitude, a peak-peak value or the rms value is set) and the measured current, the capacitance can be calculated.

For the interferometric displacement measurement, the plates are used as a Fabry-Perot interferometer. A HeNe laser beam is transmitted through the two mirror plates (note that only a small area within the plates is partially transparent) and the transmitted intensity is measured by a photodiode detector with built in preamplifier (output voltage of the detector is proportional to intensity). The detector output is connected to a Schmitt trigger (= threshold switch with hysteresis) and channel 1 of the oscilloscope. The Schmitt trigger output is connected to a digital counter and channel 2 of the oscilloscope.
The fringes are counted because every transmission maximum will exceed the Schmitt trigger threshold and increase the digital counter by one. When the displacement is caused by the motor micrometer, one mirror moves at a “constant” velocity (constant in comparison to the velocity induced by manual shift) which translates at a “constant” fringe count frequency. This can be used in order to display the transmission characteristic of the Fabry-Perot interferometer on the oscilloscope. When the count pulses are used to trigger the oscilloscope and the detector output is measured as a function of time, the characteristic transmission function of the Fabry-Perot interferometer becomes visible on the oscilloscope. The time axis can be scaled into a displacement axis by identification of the period between two fringe maxima.
Overview about experiment

- laser power supply
- HeNe laser
- lock-in amplifier
- mirror 1
- mirror 2
- lateral translation stage with motor micrometer
- lateral translation stage with manual micrometer
- two parallel plates forming capacitor and Fabry-Perot interferometer
- photodiode detector
- Schmitt trigger
- optical breadboard with vibration attenuation
- digital counter
- oscilloscope
- Schmitt trigger
- digital counter
- oscilloscope
- photodiode detector
- Schmitt trigger
- optical breadboard with vibration attenuation
- digital counter
- oscilloscope
- photodiode detector
- Schmitt trigger
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- optical breadboard with vibration attenuation
- digital counter
- oscilloscope
- photodiode detector
- Schmitt trigger
- optical breadboard with vibration attenuation
Set up the experiment

1. Set up the Fabry-Perot interferometer:
   (i) Align mirror 1 so that the reflected beam from mirror 1 hits the laser at about 1/4 inch below the exit hole (to avoid laser feedback).
   (ii) Align mirror 2 so that all transmitted spots are superposed on top of each other. (Notice detector output will become highly sensitive to mirror movement when beams are aligned.)
   (iii) Translate mirror 2 while aligning mirror 2, and optimize the modulation index (contrast ratio) of the fringes seen on the oscilloscope.
   (iv) When interferometer is aligned, translate mirror 2 until it comes to an approximate distance of 1 millimeter with respect to mirror 1.

2. Set up the capacitance measurement:
   Apply a 1 volt rms signal at 1 kHz to mirror 2. Connect the lead from mirror 1 to the current input of the lock-in amplifier. Adjust the gain until a maximum signal is obtained (without saturating the lock-in amp).
Carry out the following measurements and discussions:

1. (i) Measure the absolute value of the capacitance with a gap of approximately 1 mm.

   (ii) Calculate the absolute value of the gap using the measured capacitance value, the apparent area of the capacitor plates, dielectric constant $\varepsilon_r = 1$, and the equation given on slide 2.

   (iii) Compare your measured value with the estimated gap of 1 mm.

2. (i) Using the capacitance measurement, the known area of the mirrors and the equation for a parallel plate capacitor, adjust the gap to be 1 mm.

   (ii) Use the interferometer to measure the distance change and the lock-in amplifier to measure the capacitance as the mirror spacing is changed from 1 mm to approximately 1 cm (count fringes) in 30 steps.
(iii) Plot the measured capacitance versus gap distance and compare this with what is predicted by the parallel plate equation.

(iv) Discuss why the curves do not exactly overlap.

(3)

(i) Estimate the minimum detectable displacement $\delta d_{\text{min}}$ (in the measurement bandwidth of the lock-in amplifier that can be sensed at $d = 1$ mm and $d = 1$ cm. (Use the measured noise and slope $\partial C/\partial d$ to estimate $\delta d_{\text{min}}$). For the measurement with a lock-in amplifier, how does the minimum detectable displacement depend on the chosen time constant and, hence on the measurement speed?

(ii) Calculate the non-linearity (%) of the capacitance measurement for $d = 3$ mm ± 1 mm (from theory).

(iii) What is the non-linearity of the interferometer (fringe counting approach - do not worry about sub-fringe measurement)?
(iv) What is the stability of the interferometer over a 30 second time period (i.e., how much uncertainty in a change in distance measurement is caused by variations in the interferometer output due to index of refraction variations?) Adjust the gap to the half transmission point and measure the fractional power changes over 30 seconds. Is the stability different with a 1mm gap as compared to a 1 cm gap? Why?

(v) Measure the half width of the interference fringes and estimate the finesse of the resonator. What is the reflection coefficient R of the resonator mirrors assuming it is identical for both mirrors.